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Third-Order TRANSPORT with MAD Input
A Computer Program for Designing Charged Particle Beam Transport Systems

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Table of Contents

Mathematical Formulation of TRANSPORT **1**

General Conventions 1

Representation of Magnetic Fields 2

The Transfer Matrix R 6

Higher Orders 8

The Beam Matrix σ 9

Off-Axis Expansion of Matrix Elements 11

Fitting 13

Mispowering and Violations of Midplane Symmetry 14

Input Format for TRANSPORT **17**

General Description of Input 17

The Input Deck 17

Notation 18

Types of Elements 19

Notation for individual elements 22

Strictly MAD-Compatible Notation 22

Alternate Parameters 23

The Use of Labels 24

Activation and Deactivation of Elements 25

Insertion and Removal of Elements 26

Changing of Physical Parameters 27

Variation of Physical Parameters 27

Examples 27

Transforming a MAD Deck to a TRANSPORT Deck 33

Summaries of TRANSPORT Elements **35**

Table 1. Summary of TRANSPORT Elements in MAD Notation 35

Table 2. Summary of Other TRANSPORT Elements in Keyword Notation 36

Output Format **40**

General Appearance 40

Initial Listing 40

Listing During the Calculation 40

Calculated Quantities 41

Options 41

The Transfer Matrix 41

The Beam Matrix 43

The Layout Coordinates 45

Fitting 45

Plotting Output 46

Title, Indicator, and Comments **53**

Title Card 53

Indicator Card (0 or 1) 54

Output File Options 57

Comment Cards 63

Preliminary Specifications **64**

Input-Output Unit Sets –
UMM, UMETER, UMICR, UMAD, UTRANS 64

UNIT – Individual Units Changes 68

SPECIAL — Special Input Parameters 72

Initial Beam Line Coordinates and Direction 73

Cumulative Length of the System 75
Reference Trajectory Momentum 75
Reference Momentum for Magnetic Fields 75
Particle Mass 75
Tilt to Focal Plane 76
Random Number 77
Half Apertures of Bending Magnets 77
Fringing-Field Integrals 77
Pole-Face Curvature 79
Second-Order Term in the Midplane Field Expansion of Bending Magnets . . 81
Third-Order Term 81
Other Magnetic Field Components 81
ORDER – Higher-Order Calculations 83
RANDOM – Random Errors on Physical Parameters 89
LIMIT – Limits on the Range of Varied Physical Parameters 92
MAGNET – External Magnet Dimensions 95

Description of the Beam **99**
BEAM — Input Beam 99
Sigma Matrix Notation 99
Courant-Snyder Notation 100
The Courant-Snyder Parameters 105
R.M.S. Addition to the Beam 108
CORR(ELATION) — Beam (Rotated Ellipse) 109
CENTROID — Shift in the Beam Centroid 110
ETA – Accelerator η Function 112

Physical Elements **114**

Symbolic Constants 114

PARAMETERS 115

Algebraic Expressions 116

DRIFT 117

BEND – Sector Bending Magnet 119

ROTATION — Fringing Fields and Pole-Face Rotations for Bending Magnets . . . 129

RBEND and SBEND – Bending Magnets with Fringing Fields 134

HKICK or VKICK – Horizontal and Vertical Steering Magnets 142

KICKER – A Steering Magnet in Both Planes 145

QUADRUPOLE 146

SEXTUPOLE 150

OCTUPOLE 154

PLASMALENS – Plasma (Lithium) Lens 157

SOLENOID 159

ACCELERATION 162

SEPTUM 167

MARKER – Location markers 169

Other Transformations **170**

MATRIX — Arbitrary Transformation Input 170

SROT — Coordinate Rotation 172

SHIFT — Shift in the Reference Coordinate System 176

REALIGN — Realignment of the Reference Trajectory 178

ALIGN — Magnet Alignment Tolerances 180

Assembling Beam Lines **204**

Assembling Beam Lines with MAD Notation 204
Simple Beam Lines 204
Sub-Lines 205
Repetition and Reflection 205
Replaceable Arguments 205
USE Statement 206
Assembling Beam Lines by Listing the Elements 208
Repetition 208
Defined Section 209

Operations **212**
TRANSFORM 1 UPDATE 212
AUXILIARY TRANSFORMATION MATRIX (R2) 214
ALMARK – Alignment Marker 216
PRINT — Output Print Control Instructions 218
Summary of `PRINT` Instructions 218
Notation 219
Placement 220
Floor Coordinate Layout Control 221
Effective Bend Points 224
Transformation Matrix Print Controls 224
Beam Matrix Print Controls 227
Positions of Waists 229
Printing of the Beam Centroid 230
Beam Matrix Print in Accelerator Notation 231
Misalignment Table Print Control 233

Refer Transfer Matrix to Original Coordinate System 233
Refer Transfer Matrix to Level Coordinate System 233
General Output Format Controls 234
Precise Values of Varied Parameters 235
STORE – Storage of Matrix Elements 236
PLOT — Data for Plotting 237
Plots vs Accumulated Length 237
Floor Coordinate Layouts 240
Plots of the Beam Ellipse 241
Anything vs Anything Else 242
Sequential Runs 244
Use with TOPDRAWER 244

Variation of Parameters for Fitting **246**
The `VARY` Command 246
The `FIX` Command 247
Parameters Variable Only in First Order 248
Parameters Variable in Any Order 249
Parameters Variable Only in Second or Higher Order 251
Parameters Variable Only in Third Order 251

Available Constraints – The FIT Command **253**
Types of Constraints 253
Layout Constraints 253
First-Order Constraints 253
Second-Order Constraints 254
Third-Order Constraints 254

Notation 255
Constraints on Algebraic Expressions 256
Upper and Lower Limits 257
Placement 257
Individual Examples 258
System Length Constraint 258
Floor Coordinate Fitting Constraint 259
Elevation Fitting Constraint 260
R1 Matrix Fitting Constraints 261
R2 Matrix Fitting Constraints 263
AGS Machine Constraint 264
First Moment Constraint 266
σ (BEAM) Matrix Fitting Constraints 267
Beam Correlation Matrix (r) Fitting Constraints 268
Accelerator Function β and α Fitting Constraints 269
Accelerator Eta Function ($\eta_{x,y}$) Fitting Constraints 270
Phase Advance ($\psi_{x,y}$) Fitting Constraints 271
T1 Matrix Fitting Constraints 272
T2 Matrix Fitting Constraints 273
Second-Order Sigma (BEAM) Matrix Fitting Constraint 274
U1 or U2 Matrix Fitting Constraint 275
Third-Order Sigma (BEAM) Matrix Fitting Constraint 276
Internal Constraints 277
Corrections and Covariance Matrix 278
Stepping of Parameters and Broad-Band Fitting 279
Stepping 279

The Purpose of Stepping 279
Functional Dependence 280
Broad-Band Fitting 281

SENTINEL **284**

Original TRANSPORT Notation **285**

Positional Format 285
Element Description 285
Type Codes 287
The Use of Labels 288
Activation and Deactivation of Elements 289
Changing of Physical Parameters 290
Variation of Physical Parameters 290
Examples 291
Table 6: Summary of Elements in Positional Notation 293
Elements with Code Digits 298
Magnet Alignment Tolerances – Type Code 8 298
Repeat – Type Code 9 299
Vary Codes 300
First-Order Vary Codes 300
Vary Codes Usable in Any Order 301
Second-Order Vary Codes 301
Third-Order Vary Codes 302
Coupled Vary Codes 302
Constraints – Type Code 10 303
Print – Type Code 13 305

Punched Output Controls 306

Matrix – Type Code 14 307

Units Changes – Type Code 15 309

Special Parameters – Type Code 16 311

Defined Section – Type Code 24. 313

Acknowledgements **314**

Introduction

TRANSPORT has been in existence in various evolutionary versions since 1963. The present version of TRANSPORT is a first-, second-, and third-order matrix multiplication computer program intended for the design of static-magnetic beam transport systems.

Many people from various laboratories around the world have contributed either directly or indirectly to the development of TRANSPORT. The first-order matrix methods were introduced by the Courant and Snyder[1] followed by a paper by Penner [2]. Second-order differential equations of motion, including driving terms to represent the second-order aberrations were first derived by John Streib[3]. The mathematical formalism introduced by Streib was later adopted by K. L. Brown to develop the underlying theory for the TRANSPORT program [4]. Matrix methods to include second and higher order aberrations were conceived and developed by Brown, Belbeoch and Bounin[5] in Orsay, France in 1958-59.

The original first-order TRANSPORT computer code was written in BALGOL by C.H. Moore at SLAC in collaboration with H.S. Butler and S.K. Howry in 1963. The second-order portion of the program was developed and debugged by Howry and Brown[6], also in BALGOL. Further contributions to the second-order theory were made by R. Helm at SLAC. The BALGOL was translated into FORTRAN by S. Kowalski at MIT and free field reading routines were implemented by B. Kear at SLAC.

In 1971-72, D. C. Carey at Fermilab completely rewrote the program and developed efficient second-order matrix accumulation and fitting routines. This version was installed at SLAC by F. Rothacker in the early spring of 1972 and subsequently carried to CERN by K. L. Brown. Ch. Iselin at CERN made further contributions to the program structure and improved the convergence capabilities of the first-order fitting routines. Third-order matrix elements and fitting capabilities have been included in the latest version of TRANSPORT by D. C. Carey and L. Sagalovsky.

In April 1984, a meeting was held at SLAC where K.L. Brown, D.C. Carey, R. Servranckx, F. Rothacker, and Ch. Iselin agreed to adopt the MAD[7] input format designed by Ch. Iselin as a standard input language for their accelerator optics programs. As a consequence, the reading routines of MAD have been incorporated into the present version of TRANSPORT by D. C. Carey.

A standard version of the program has been maintained, developed, and used at SLAC, Fermilab, and CERN since 1972. This manual describes the third-order version with MAD input capability and graphics. *It is not necessarily applicable to other versions of TRANSPORT.* Copies of this manual may be obtained from

1. The Reports Office, SLAC, P.O. Box 4349, Stanford, CA 94305 (Ref. SLAC-R-95-462).
2. The Reports Office, Fermi National Accelerator Laboratory, P. O. Box 500, Batavia, IL 60510 (Ref. Fermilab-Pub-98/310).

This program may be obtained from:

D. C. Carey
Fermi National Accelerator Laboratory
P. O. Box 500
Batavia, IL 60510

Phone: (630) 840-3639
TRANSPORT_SUPPORT@FNAL.GOV

To get the TRANSPORT source from the web, follow these instructions:

1. Go to the Fermitools web page, which is at: http://www.fnal.gov/fermitools

2. Click on the word, software, at thew top of the page. That will bring you to a list of software titles that are in alphabetical order.

3. Click on Transport. This will bring you to the Transport abstract web page.

4. Click on the word, Software, at the top of the page. This will bring You to a Fermitools Terms and Conditions page.

5. After reading this page, click on: I have read and accept the Terms and Conditions. This will be at the bottom of the page.

6. This will bring you to the transport directory: ftp://ftp.fnal.gov/pub/transport/

7. Click on the highest version number.

8. This will bring you to that subdirectory and you will see a transport tar file along with some documentation files.

9. After you click on the file that you want to copy, you get a prompt asking where you would like the file and what it should be called. After you answer the question, it should copy the file.

Mathematical Formulation of TRANSPORT

General Conventions

A beam line is comprised of a set of magnetic elements placed sequentially at intervals along an assumed reference trajectory. The reference trajectory is the path of a particular charged particle with the central design momentum p_0, passing through idealized magnets. By "idealized magnets" we mean no errors in fabrication or positioning.

The particle following the reference trajectory and having momentum p_0 will henceforth be known as the reference particle. The reference trajectory is also known as the central trajectory.

In TRANSPORT,* a beam line is described as a sequence of elements. Such elements may consist not only of magnets and the spaces between them, but also of specifications of the input beam, calculations to be done, or special configurations of the magnets. All magnets are normally considered "aligned" on the central trajectory. The precise meaning of aligning the magnets is described below. Alternative configurations can be described by means of elements provided for such purposes.

The two coordinates transverse to the initial reference trajectory are labeled as horizontal and vertical. A bending magnet will normally bend in the horizontal plane. To allow for other possibilities a coordinate rotation element is provided. Because of such other possibilities, when describing bending magnets we shall often speak of the bend and nonbend planes. The transverse coordinates will also often be labeled x and y. The curvilinear distance along the reference trajectory is denoted by s. In parts of the beam line where the reference trajectory is straight, the longitudinal coordinate is sometimes also labeled z.

The reference trajectory is a straight line through all magnetic elements except a bending magnet. Through the interior field of a bending magnet the reference trajectory is the arc of a circle. Unless otherwise specified, a particle following the central trajectory through an idealized magnet experiences a uniform field which begins and ends abruptly at the effective entrance and exit faces of the magnet. To accomodate a more gradual variation of field at the ends of a bending magnet a fringing field element is provided. In order to represent an orientation with respect to the reference trajectory other than normal of a magnet or section of a beam line, a misalignment element also exists.

The program TRANSPORT will step through the beam line, element by element, calculating the properties of the beam or other quantities, described below, where requested. Therefore one of the first elements needed in a TRANSPORT input is a specification of the phase space region occupied by the beam entering the system. Magnets and intervening spaces and other elements then follow in the sequence in which they occur in the beam line.

*For a more complete description of the mathematical basis of TRANSPORT, refer to SLAC-75[4], and to other references listed at the end of this manual [8] [9] [10] [11].

Specifications of calculations to be done or of configurations other than normal are placed in the same sequence, at the point where their effect is to be made.

Representation of Magnetic Fields

The normal magnetic field of any magnet, except a solenoid, is assumed to have midplane symmetry. This means that the scalar potential expanded in transverse coordinates about the reference trajectory is taken to be an odd function of the vertical coordinate. If a coordinate rotation is included, then the potential is odd in the coordinate to which the vertical has been rotated. For a bending magnet this will always be in the nonbend plane.

We begin our discussion with the field of the combined-function bending magnet. It is the one element which, in principle, contains all the magnetic multipoles. The expansion of the magnetic field on the nominal magnetic midplane has historically been given in terms of unitless quantities n, β, and γ. The expansion may also be given in terms of multipole coefficients K_n. The two representations are as follows:

$$\begin{aligned} B_y &= B_0(1 - nhx + \beta\, h^2x^2 + \gamma\, h^3x^3 + \ldots) \\ &= (B_0\rho)\sum K_n x^n \\ B_x &= 0 \end{aligned} \tag{1}$$

The quantity h is the curvature of the reference trajectory (the reciprocal of the radius of curvature ρ). The expression $B_0\rho$ is equal to the momentum p_0 of the reference particle divided by its charge q.

The dipole component in the expansion is represented by the coefficient K_0 defined as

$$K_0 = \frac{\alpha}{L} = \frac{B_y}{B_0\rho} \tag{2}$$

where α is the total bending angle of a dipole magnet and L is its total length as measured along the reference trajectory. The quadrupole component K_1 is related to the coefficient n by

$$K_1 = -\frac{n}{\rho^2} = \frac{1}{B_0\rho}\frac{\partial B_y}{\partial x} \tag{3}$$

The sextupole component K_2 is related to the coefficients β and ϵ by

$$K_2 = \frac{\epsilon}{\rho} = \frac{\beta}{\rho^3} = \frac{1}{2B_0\rho}\frac{\partial^2 B_y}{\partial x^2} \tag{4}$$

The octupole component K_3 is related to the cubic variation of the magnetic field by.

$$K_3 = \frac{\epsilon_3}{\rho} = \frac{\gamma}{\rho^4} = \frac{1}{6B_0\rho}\frac{\partial^3 B_y}{\partial x^3} \tag{5}$$

The coefficients n, ϵ, and ϵ_3 originally served as the only form for the input data for TRANSPORT. The coefficients β and γ are also shown above, since they appear directly in the algebraic expressions for the higher order transfer matrix elements. The multipole components K_1, K_2, and K_3 are used as input to the MAD program and are also understood by the present TRANSPORT.

The definition of the multipole components K_0, K_1, K_2, and K_3 allows the orbit of a particle to be computed without knowledge of its momentum. The normalization used here is such that a multipole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. In the MAD program the values of K_2 and K_3 are defined as the normalized derivatives of the field. The value of K_2 in MAD is then a factor of 2 greater than the TRANSPORT value defined above. The MAD value of K_3 is a factor of 6 $(= 3!)$ greater than that used here. The normalization of K_n in various computer programs can be found in a table on the following page. TRANSPORT can be instructed via a UMAD command to observe the MAD conventions so as to ensure consistency of results. The `UMAD` command is described on page 64.

Physical quadrupole, sextupole, and octupole elements are single multipoles and are known as "separated function" components. The reference trajectory through such elements is a straight line, so that the radius of curvature ρ becomes infinite. The appropriate measures of the strength of these multipole components are the coefficients K_0, K_1, K_2, and K_3, since these coefficients are defined directly as field derivatives divided by $B_0\rho$. The quantity $B_0\rho$, being defined in terms of the reference momentum, is not affected by the curvature of the reference trajectory.

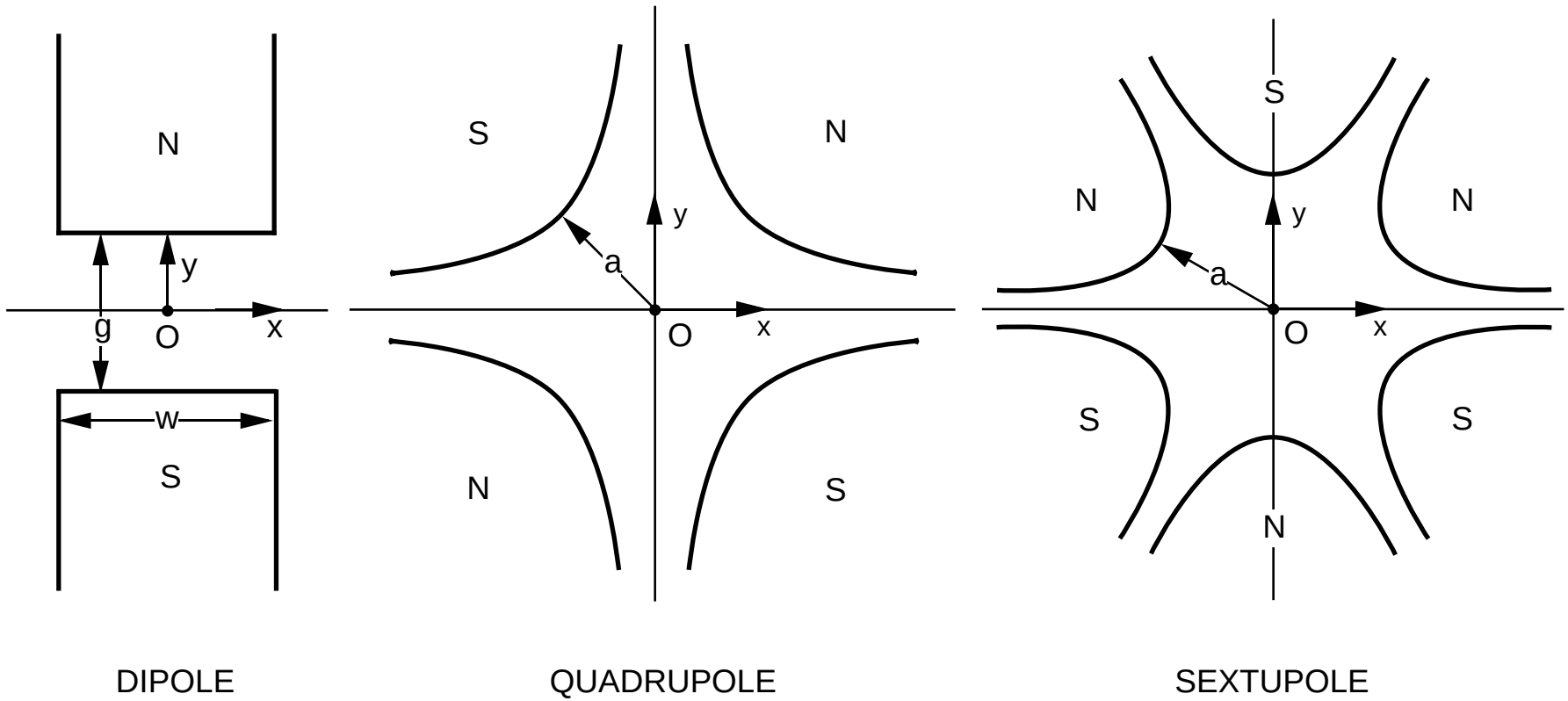


Illustration of the magnetic midplane (x axis) for dipole, quadrupole and sextupole elements. The magnet polarities indicate multipole elements that are positive with respect to each other. The coordinate system is right-handed, so that the beam particles will be emerging from the paper and moving into the face of the reader.

In TRANSPORT the multipole strength has traditionally been defined in terms of a pole tip field B_0 and a half aperture a. The half aperture is illustrated in the preceding figure. It is the distance from the optic axis (reference trajectory) to the nearest point on the pole tip.

For a quadrupole the gradient g of the field is also often used. The quantities K_1 and g are related to the pole tip field B_0 and the half aperture a by:

$$K_1 = \frac{g}{(B_0\rho)} = \frac{1}{(B_0\rho)}\frac{B_o}{a} \tag{6}$$

For a sextupole the quantity K_2 is related to the pole tip field B_o and the half aperture a by:

$$K_2 = \frac{1}{(B_0\rho)}\frac{B_o}{a^2} \tag{7}$$

For an octupole the quantity K_3 is related to the pole tip field B_o and the half aperture a by:

$$K_3 = \frac{1}{(B_0\rho)}\frac{B_o}{a^3} \tag{8}$$

Again, the definitions in the MAD program are greater by a factor of 2 for the sextupole term and 6 for the octupole term.

DEFINITION OF MULTIPOLE STRENGTHS IN VARIOUS COMPUTER PROGRAMS

TRANSPORT DIMAD	MAD SYNCH TEAPOT
$B_y(x,0,s) = (B_0\rho)\sum K_n x^n$	$B_y(x,0,s) = (B_0\rho)\sum \frac{1}{n!} K_n x^n$
$K_n(s) = \frac{1}{B_0\rho}\frac{1}{n!}\frac{\partial^n B_y}{\partial x^n}$	$K_n(s) = \frac{1}{B_0\rho}\frac{\partial^n B_y}{\partial x^n}$
$K_n = \frac{1}{(B_0\rho)}\frac{B_o}{a^n}$	$K_n = \frac{n!}{(B_0\rho)}\frac{B_0}{a^n}$

So $\quad K_n(\text{TRANSPORT}) = \frac{1}{n!} K_n(\text{MAD})$

where $\quad n = 0 \quad$ Dipole
$\quad\quad\quad n = 1 \quad$ Quadrupole

$n = 2$	Sextupole
$n = 3$	Octupole

The Transfer Matrix R

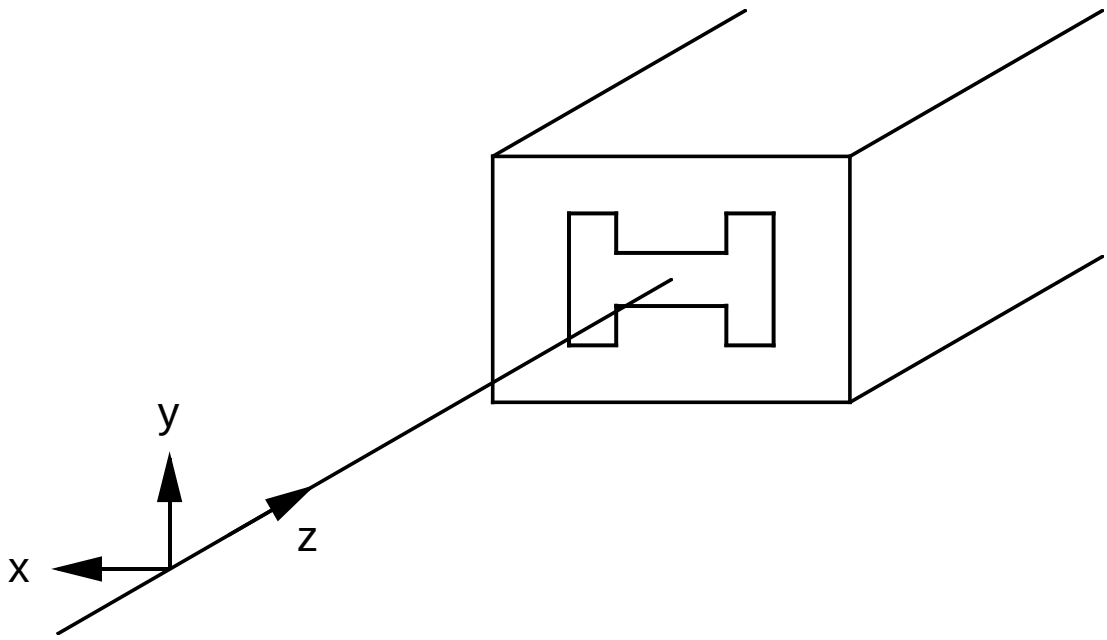
The following of a charged particle through a system of magnetic lenses may be reduced to a process of matrix multiplication. At any specified position in the system an arbitrary charged particle is represented by a vector (single column matrix) X, whose components are the positions, angles, and momentum of the particle with respect to the reference trajectory, i.e.

$$X = \begin{bmatrix} x \\ x' \\ y \\ y' \\ \ell \\ \delta \end{bmatrix} \tag{9}$$

Definitions:

x = the horizontal displacement of the arbitrary ray with respect to the assumed central trajectory.
x' = the angle this ray makes in the horizontal plane with respect to the assumed central trajectory.
y = the vertical displacement of the ray with respect to the assumed central trajectory.
y' = the vertical angle of the ray with respect to the assumed central trajectory.
ℓ = the path length difference between the arbitrary ray and the central trajectory.
δ = $\Delta p/p$ is the fractional momentum deviation of the ray from the assumed central trajectory.

This vector, for a given particle, will henceforth be referred to as a ray. The two components x' and y' are often referred to as angles, although, strictly speaking, they are not. They are really tangents of angles. However, the difference between an angle and its tangent is of third order and is generally unimportant as a measure of phase-space dimensions. We therefore occasionally lapse into this somewhat imprecise usage.



The local coordinate system is shown at the entrance to a magnet. The longitudinal coordinate is shown as z. The distance along the reference trajectory is usually denoted by the letter s. In a field-free region such as a drift, or the axis of a quadrupole, or other higher multipole, the coordinates s and z are the same. In the interior of a bending magnet the coordinate s is curvilinear and follows the reference trajectory. The coordinate z is rectilinear and is taken as tangent to the reference trajectory.

The magnetic lens is represented to first order by a square matrix R, which describes the action of the magnet on the particle coordinates. Thus the passage of a charged particle through the system may be represented to first order by the equation:

$$X(1) = RX(0), \tag{10}$$

where $X(0)$ is the initial coordinate vector and $X(1)$ is the final coordinate vector of the particle under consideration. The same transformation matrix R is used for all such particles traversing a given magnet [one particle differing from another only by its initial coordinate vector $X(0)$].

The traversing of several magnets and interspersing drift spaces is described by the same basic equation, but with R now being replaced by the product matrix

$$R(t) = R(n) \ldots R(3)R(2)R(1) \tag{11}$$

of the individual matrices of the system elements. This cumulative transfer matrix is automatically calculated by the program and is called TRANSFORM 1. It may be printed where desired, as described on page 224.

Higher Orders

This formalism may be extended to second order by the addition of another term.[4] The components of the final coordinate vector, in terms of the original, are now given as

$$X_i(1) = \sum_j R_{ij}\ X_j(0) + \sum_{jk} T_{ijk}\ X_j(0)\ X_k(0) \quad , \tag{12}$$

where T is the second-order transfer matrix. It too is accumulated by the program as one traverses a series of elements. At each point the series is again truncated to second order. Normally the program will calculate only the first-order terms and their effect. If it is desired to include second-order effects in a beam line, an element is provided which specifies that a second-order calculation is to be done. For more information on the T matrix, see the references at the end of the manual.

Third-order terms may also be included in the transformation of the particle coordinate vector. The third-order terms are given in terms of the U matrix so that our transformation equation becomes

$$X_i(1) = \sum_j R_{ij}\ X_j(0) + \sum_{jk} T_{ijk}\ X_j(0)\ X_k(0) + \sum_{jk\ell} U_{ijk\ell}\ X_j(0)\ X_k(0)\ X_\ell(0) \quad , \tag{13}$$

The comments about second order apply also to third order. The addition of a parameter to the element which specfies second order allows calculations to be done to third order.

The following of a charged particle via TRANSPORT through a system of magnets is thus analogous to tracing rays through a system of optical lenses. The difference is that TRANSPORT is a matrix calculation which truncates the problem to either first-, second- or third- order in a Taylor's series expansion about a central trajectory. *For studying beam optics to greater precision than a third-order TRANSPORT calculation permits, ray-tracing or differential algebra programs which directly integrate the basic differential equation of motion are recommended.*[†]

The Beam Matrix σ

In accelerator and beam transport systems, the behavior of an individual particle is often of less concern than is the behavior of a bundle of particles (the beam), of which an individual particle is a member. An extension of the matrix algebra of Eq. (1) provides a convenient means for defining and manipulating this beam. TRANSPORT assumes that the beam may be correctly represented in phase space by an ellipsoid in the six-dimensional coordinate system described above. Particles in a beam are assumed to occupy the volume enclosed by the ellipsoid, each point representing a possible ray. The sum total of all phase points, the phase space volume, is commonly referred to as the "phase space" occupied by the beam.

The validity and interpretation of this phase ellipse formalism must be ascertained for each system being designed. However, in general, for charged particle beams in, or emanating from accelerators, the first-order phase ellipse formalism of TRANSPORT is a reasonable representation of physical reality. For other applications, such as charged particle spectrometers, caution is in order in its use and interpretation.

The equation of an n-dimensional ellipsoid may be written in matrix form as follows:

$$X(0)^T \ \sigma(0)^{-1} \ X(0) = 1 \quad , \tag{14}$$

where $X(0)^T$ is the transpose of the coordinate vector $X(0)$, and $\sigma(0)$ is a real, positive definite, symmetric matrix.

The volume of the n-dimensional ellipsoid defined by sigma is $[\pi^{m/2}/\Gamma(n/2+1)]\ (det\ \sigma)^{1/2}$. The area of the projection in one plane is $A = \pi(det\ \sigma_1)^{1/2}$, where σ_1 is the submatrix corresponding to the given plane. This is the "phase space" occupied by the beam.

As a particle passes through a system of magnets, it undergoes the matrix transformation of Eq. (10). Combining this transformation with the equation of the initial ellipsoid, and using the identity $RR^{-1} = I$ (the unity matrix), it follows that

$$X(0)^T \ (R^T \ R^{T^{-1}}) \ \sigma(0)^{-1} \ (R^{-1} \ R) \ X(0) = 1 \tag{15}$$

[†]A list of such programs is given at the end of this manual.

from which we derive

$$[RX(0)]^T \; [R\sigma(0) \; R^T]^{-1} \; [RX(0)] = 1 \quad . \tag{16}$$

The equation of the new ellipsoid after the transformation becomes

$$X(1)^T \; \sigma(1)^{-1} \; X(1) = 1 \quad , \tag{17}$$

where

$$\sigma(1) = R\sigma(0) \; R^T \quad . \tag{18}$$

It can readily be shown that the square roots of the diagonal terms of the sigma matrix are a measure of the "beam size" in each coordinate. The off-diagonal terms determine the orientation of the ellipsoid in n-dimensional space (for TRANSPORT, n = 6).[‡] Thus, we may specify the beam at any point in the system via Eq. (18), given the initial "phase space" represented by the matrix elements of $\sigma(0)$.

The initial beam is specified by the user as one of the first elements of the beam line. Normally it is taken to be an upright ellipse centered on the reference trajectory; that is, there are no correlations between coordinates. Both correlations and centroid displacements may be introduced via additional elements.

The phase ellipse may be printed wherever desired. For an interpretation of the parameters printed see page 107.

When a second- or third-order calculation is specified, the appropriate matrix elements are included in the beam matrix. In this case the beam is no longer an ellipse. The elements of the sigma matrix may be interpreted as second moments of the beam about its center of gravity. However, a large second moment may be caused by many factors, including a very long but sparsely populated tail in one direction. Detailed analysis should be done with a ray tracing program.

A simplification occurs if the two transverse planes are independent, the momentum spread in the beam phase space is zero, and we are not interested in longitudinal motion. Then the significant part of the σ matrix can be reduced to two two-by-two matrices, one for each transverse plane. The quantities used in the two-dimensional parameterization we shall cann the Courant-Snyder parameters.

A two-dimensional representation of an ellipse is:

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \epsilon$$

The four parameters, γ, α, β, and ϵ, are not independent but are related by:

$$\beta\gamma - \alpha^2 = 1$$

[‡]See the Appendix of this report (available under separate cover or as part of Report CERN 80-04), for a derivation of these statements.

The area of the ellipse is given by

$$A = \pi\epsilon$$

The relationship between a two-dimensional segment of the σ matrix and the matrix of Courant-Snyder parameters is given by

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \epsilon \begin{bmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{bmatrix}$$

The momentum dependence of the optical system is represented by a separate trajectory designated as η. In a circular machine η is typically defined so as to be periodic. The δ component of η is one unit. In TRANSPORT, the default values of all the components of η except the δ component are taken as zero. Other values may be specified by the user, as explained on page 112.

Off-Axis Expansion of Matrix Elements

If the beam is off axis, then the beam centroid will then no longer coincide with the reference trajectory. An off axis beam can be achieved in several ways.

1. The initial beam centroid may not coincide with the reference trajectory.
2. The beam centroid may be displaced by the misalignment of a magnet or section of beam line [12].
3. The beam may be steered off axis by an excess field on a bending magnet.
4. The beam may be steered off axis by a midplane-symmetry-violating vertically bending component of a bending magnet.
5. The beam may be deliberately steered by the use of steering magnets.
6. The beam may be deflected by an electrostatic septum.
7. The reference coordinate system can be shifted.

The transformation through a single element can then be represented in the same manner as in on-axis configurations, except that there is now a zeroeth order term. This new term represents the displacement of the six coordinates of a ray that lies on the reference trajectory as it enters the magnetic element. The net transformation can be represented to third order by

$$X_1 = X_{1s} + RX_o + TX_oX_o + UX_oX_oX_o \tag{19}$$

The effect of the magnetic system on neighboring trajectories can be represented by a first-, second-, or third-order transformation. We start with equation (19) for the transformation of an arbitrary trajectory. If we reexpress the coordinates relative to the transformed original reference trajectory X_r, and denote the difference by ΔX, then we have

$$\begin{aligned} X_{1r} + \Delta X_1 &= X_{1s} + R(X_{or} + \Delta X_o) + T(X_{or} + \Delta X_o)(X_{or} + \Delta X_o) \\ &+ U(X_{or}\Delta X_o)(X_{or} + \Delta X_o)(X_{or} + \Delta X_o) \end{aligned} \tag{20}$$

Subtracting the equation for the reference trajectory, we derive

$$\begin{aligned} \Delta X_1 &= R\Delta X_o + T(2X_{or}\Delta X_o + \Delta X_o\Delta X_o) \\ &+ U(3X_{or}X_{or}\Delta X_o + 3X_o\Delta X_o\Delta X_o + \Delta X_o\Delta X_o\Delta X_o) \\ &= (R + 2TX_{or} + 3UX_{or}X_{or})\Delta X_o \\ &+ (T + 3UX_{or})\Delta X_o\Delta X_o + U\Delta X_o\Delta X_o\Delta X_o \end{aligned} \tag{21}$$

From equation (21), we can define new first- and second-order transfer matrices by

$$\begin{aligned} R^* &= R + 2TX_{or} + 3UX_{or}X_{or} \\ T^* &= T + 3UX_{or} \end{aligned} \tag{22}$$

These redefined matrices for each element can then be accumulated to produce transfer matrices for the entire magnetic optical system. The transformation of a particle trajectory through the system can now be represented by a transformation similar in appearance to equation (19).

$$X_1 = X_{1r} + R(t)X_o + T(t)X_oX_o + U(t)X_oX_oX_o \tag{23}$$

Here the matrices $R(t)$, $T(t)$, and $U(t)$ are calculated as products of the matrices R^*, T^*, and U, as defined in equation (22). They are expressed relative to the transformed original reference trajectory X_{1r}. The transformed original reference trajectory is determined from equation (19) on an element-by-element basis.

Since the fractional deviation from the reference momentum is one of the trajectory coordinates, the off-axis expansion can be used to explore chromatic effects. If a nonzero δ is specified, the first-order transfer matrix will be with respect to the off-momentum centroid. If there is dispersion in the system, the centroid will, at some point, be displaced from the reference trajectory. The result from an off-axis expansion can differ substantially from that of an on-axis calculation. The effect of many orders higher than second or even third can be seen.

The most straightforward example is that of a very long focusing system, with many intermediate foci. For a slight deviation from the reference momentum, the final beam

spot size will grow linearly with the size of this deviation. As the momentum is increased, however, one of the intermediate foci will move downstream and the system as a whole will once again be focusing.

An on-axis expansion to second order will show only the linear growth in spot size, no matter how great the momentum deviation. The third-order on-axis expansion will show very little difference from the second. The off-axis expansion will show the system once again coming to a focus as the momentum is increased. Even for very long systems, a third-order off-axis chromatic calculation will be indistingushable from a ray-tracing calculation. The ray tracing program uses unexpanded momentum dependence in the equations of motion through the magnetic fields.

The beam centroid displacement is printed next to the first-order transfer matrix when the beam is off axis. For a zeroth-order calculation the centroid displacement is printed alone. For a first- or lower-order calculation the same vector of six quantities will appear as the leftmost column in the beam matrix representation.

For second- and third-order calculations, the column in the beam matrix may differ slightly from the column given with the transfer matrix. That is because the numbers given with the beam matrix represent the position of the center of gravity of the beam. Second- and higher-order effects introduce distortions into the beam ellipse, so that it is no longer an ellipse. The center of gravity at a given beam location is no longer the optical image of the center of gravity at the beginning of the system. Steering magnets should always be set so that the centroid (not the center of gravity) is realigned along the beam axis. Higher-order multipoles should then be used to make the center of gravity coincide with the centroid.

Fitting

Several types of physical elements have been incorporated in TRANSPORT to facilitate the design of very general beam transport systems. Included are an arbitrary drift distance, bending magnets, quadrupoles, sextupoles, octupoles, solenoids, and an accelerator section (to first-order only). Provision is made in the program to vary some of the physical parameters of the elements comprising the system and to impose various constraints on the beam design. In a first-order run one may fit either the TRANSFORM (R) matrix representing the transformation of an arbitrary ray through the system and/or the phase ellipse (sigma) matrix representing a bundle of rays by the system as transformed. In a second-order run one may fit either the second-order TRANSFORM (T) matrix or minimize the net contribution of second-order terms to the beam (sigma) matrix. Similar constraints in third order may be placed on the third-order TRANSFORM (U) or the sigma matrix.

The program will normally make a run through the beam line using values for the physical parameters as specified by the user and printing the results. If constraints and parameters

to be varied are indicated, it will attempt to fit. To do this it will make an additional series of runs through the beam line. Each time it will calculate corrections to be made to the varied parameters from the previous step to try to satisfy the indicated constraints. When the constraints are satisfied (or the fitting procedure has failed) TRANSPORT will make a final run through the beam line again printing the results. In this final run the values of the physical parameters used are those which are the result of the fitting procedure.

Thus, in principle, the program is capable of searching for and finding the first-, second-, or third-order solution to any physically realizable problem. In practice, life is not quite so simple. The user will find that an adequate knowledge of geometric magnetic optics principles is a necessary prerequisite to the successful use of TRANSPORT. He (or she) should possess a thorough understanding of the first-order matrix algebra of beam transport optics and of the physical interpretation of the various matrix elements.

In other words, the program is superb at doing the numerical calculations for the problem but not the physics. The user must provide a reasonable physical input if he (or she) expects complete satisfaction from the program. For this reason a list of pertinent reprints and references is included at the end of this manual. They should provide assistance to the inexperienced as well as the experienced user.

TRANSPORT can also do broad-band fitting. In this case, one of the input physical parameters is stepped over an interval. For each value of this physical parameter, a run is made through the beam line. The chi-squared is then the sum of the chi-squareds from the different runs through the beam line. When the set of runs is complete, corrections are calculated to the varied parameters. The corrections are made, and the process is repeated. Thus, in broad-band fitting, the set of runs with the different values of the stepped parameter replace the single run with fixed values of all physical parameters. More details can be found on page 279.

Mispowering and Violations of Midplane Symmetry

The reference trajectory of a correctly aligned bending magnet is the path followed by the reference particle only if the magnet is also correctly powered. If the magnet is mispowered, the reference trajectory can still be defined geometrically as if the magnet were correctly powered. The field expansion given in equation (1) will then be about this geometrically defined reference trajectory. The parameterization of the magnetic field will contain a multiplicative factor representing the degree of mispowering.

Violations of midplane symmetry can also be included in the representation of the magnetic field. Individual magnetic elements are typically designed to have midplane symmetry. Errors of fabrication, misalignments, and correction windings can cause the actual midplane of an element not to coincide with its intended midplane. For a magnet which corresponds to a pure multipole, such as a zero gradient bending magnet (dipole), quadrupole, sextupole,

or octupole, the required procedure is straightforward. The departure of the midplane from its intended location can be represented by the rotation and misalignment elements.

The combined function bending magnet however, contains several multipole components in the same magnet. A bending magnet may have multipole components in addition to the dipole component either by design or by errors of fabrication. These different multipoles may have different planes of symmetry. The bending magnet may also have a midplane defined by the geometry of the beam. This geometric midplane may or may not coincide with a plane of symmetry of even the dipole component.

A single reference plane can still be selected for the purpose of defining the magnetic field. This reference plane can be geometrically defined by some significant physical characteristic, such as the intended plane of symmetry of the iron pole tips of the magnet. We shall continue to refer to this reference plane as the midplane, without implying any field symmetry.

A complete expansion of the magnetic field about this reference plane then includes midplane-symmetry-violating terms. The reference trajectory may be defined as if these terms were not present. The reference trajectory will then <u>not</u> be the possible trajectory of an actual particle. The field configuration, however, can still have translational invariance as one follows the reference trajectory.

We therefore introduce midplane-symmetry-violating dipole, quadrupole, and sextupole components of the field. The dipole component will cause the beam centroid to deviate from the geometrically defined reference trajectory.

$$\begin{aligned} B_y &= B_o(1+r_s)(1-nhx+\beta\ h^2x^2+\gamma\ h^3x^3+\ldots) \\ &= (B_o\rho)(1+r_s)\sum K_n x^n \\ B_x &= B_o r_a(v_r - n'hx+\beta' h^2x^2+\ldots) \\ &= (B_o\rho)r_a\sum K'_n x^n \end{aligned} \tag{24}$$

The consequences of the centroid being diverted off axis are described below.

The coefficient r_s represents the fractional degree of mispowering. The corresponding coefficient r_a is a multiplicative factor for the midplane-symmetry-violating terms. These two coefficients are independent and are defined so that they are both zero in the correctly powered, midplane symmetric case.

The coefficients v_a, n', and β' represent the relative strengths of the midplane-symmetry violating multipole coefficients. The coefficients K'_n provide an alternate parameterization just as in the midplane symmetric expansion. The K'_n coefficients include K'_0 for the midplane symmetry violating dipole term.

There is a certain redundancy in the definition or r_a, since the multipole coefficients

provide a complete specification of the magnetic field. The parameter r_a is included in the parameterization so that the midplane-symmetry-violating terms can be scaled simultaneously by a single multiplicative factor.

The midplane-symmetry-violating multipole components are defined similarly to the midplane-symmetric terms. The skew quadrupole component n' is related to the coefficient K_1' by

$$K_1' = -\frac{n'}{\rho^2} = \frac{1}{r_a}\frac{1}{B\rho}\frac{\partial B_x}{\partial x} \tag{25}$$

The skew sextupole component β' is related to the coefficients ϵ' and K_2' by

$$K_2' = \frac{\epsilon'}{\rho} = \frac{\beta'}{\rho^3} = \frac{1}{r_a}\frac{1}{2B\rho}\frac{\partial^2 B_x}{\partial x^2} \tag{26}$$

The midplane-symmetry-violating formalism in TRANSPORT has thus far been extended to include only the second-order or skew sextupole terms. Just as in the midplane symmetric case, the coefficient K_2' in MAD is defined as being twice that in TRANSPORT. The command UMAD (see page 64) instructs TRANSPORT to use the MAD definitions.

Input Format for TRANSPORT

General Description of Input

The Input Deck

By the TRANSPORT input DATA SET is meant the totality of data read by the program in a single job. A DATA SET may consist of one or more problems placed sequentially. A problem specifies a beam line and a calculation or set of calculations to be done on that beam line.

A problem, in turn, may consist of one or more problem steps. The data in the first step of a problem specify the beam line and calculations to be made. The data in succeeding steps of the same problem specify only changes to the data given in the first step.

A common example of a problem with several steps is sequential fitting. In the first step one may specify that certain parameters are to be varied to satisfy certain constraints. Once the desired fit has been achieved the program will then proceed to the next step. The data in this step now need specify only which new parameters to vary, or old ones no longer to vary, or which constraints to add or delete. The values of the varied parameters that are passed from one step to the next one are those that are the result of the fitting procedure.

A problem step contains three kinds of DATA cards: the TITLE card, the INDICATOR card, and the ELEMENT cards. The term "card", indicating a single line of an input data set, may be a bit archaic. However, users have come to be comfortable with it and continue to speak in terms of "card"s. We shall therefore continue this usage with the understanding that we should not be taken too literally.

The TITLE card contains a string of characters and blanks enclosed by single quotes. Whatever is between the quotes will be used as a heading in the output of a TRANSPORT run.

The second line of the input is the INDICATOR card. If the data which follow describe a new problem, a zero (0) is placed in any column on the card. If the data which follow describe changes to be made in the previous problem step, a one (1) is placed in any column on the card. For further explanation see page 54.

The remainder of the data for a given problem step consist of the ELEMENT cards. These data describe the units to be used, the physical objects making up the beam line, and the calculations to be done. Unless otherwise specified, the order of the elements is the same as encountered as one proceeds down the beam line. Provision also exists for listing the elements first, then using other commands to assemble them into beam lines.

Each element specifies a magnet or portion thereof or other piece of equipment, a drift space, the initial beam phase space, a calculation to be done, or a print instruction. Calcu-

lation specifications, such as misalignments and constraints, are placed in sequence with the other beam line elements where their effect is to take place.

The input format of the cards is "free-field," which means that exact positioning of data items is not important. A tabbing mark is also taken as a space. The input is case-insensitive, which means that both upper case and lower case alphabetic characters can be used, and no distinction is made between them. More information is given below.

The data for a given problem step are terminated by the word `SENTINEL`, which need not be entered on a separate line.

Notation

Two different input data formats have been used for TRANSPORT as it was developed over the years. The MAD (Methodical Accelerator Design) program of Christoph Iselin (see refs) has an input data format which is both easy to use and clear in its meaning. However, since the program TRANSPORT existed long before MAD was originated, TRANSPORT had its own original format. Both formats are recognized by TRANSPORT and will continue to be. There is no possibility that at some future point users will be required to convert old data decks to a new format.

The input format in MAD notation is described in this section and in most of the remainder of the manual. The original notation is described in a section entitled "Original TRANSPORT Notation," beginning on page 285.

The MAD program also uses a certain set of parameters to describe a physical element. Elements and notation for elements which are MAD compatable will be identified below.

Strict MAD compatability requires using the same set of parameters. However, since the original TRANSPORT used a different set of parameters, the current version has been made to recognize both possibilities, as well as several others. Details are given both in this section and in the sections for the individual elements.

The TRANSPORT input data for a single element can be described as follows.

1. Each element is given a proper name, henceforth to be known as a label. The label is the first item in the element specification and is followed by a colon. If the elements are to be assembled into a beam line by additional program instructions, then the label <u>must</u> be present. If the beam line consists simply of the sequence of elements as listed, then, in TRANSPORT, the label is optional.

2. The next item in the specification of an element is a mnemonic which identifies what sort of entity (such as a bending magnet, quadrupole, or drift space) is represented. The mnenomic follows the label if one is present, or begins the element if there is no

label. The mnenomic is an alphanumeric character string, usually a recognizable name such as `BEND` or `QUAD`. The mnemonic is followed by a comma.

3. The element parameters are designated by keywords . The parameters are identified by name (such as `L` and `ANGLE`). The parameter name is followed by an equals sign, which, in turn, is followed by the value to be given to the parameter. If another parameter keyword is to follow, the value must be followed by a comma. The entire sequence is terminated by a semicolon or the end of a line.

Thus a typical element representing a bending magnet might look like.

```
BM1:   BEND,  L = 10.,  B = 20.,  N = 0.5  ;
```

The meaning of the parameters `L` and `B` is obvious, while `N` represents a normalized field gradient.

In MAD the elements are specified first, then assembled into an optical system by additional program instructions. TRANSPORT also allows the option of having the optical system consist of the elements as listed.

These aspects of TRANSPORT input are explained in greater detail, with examples, in the immediately following sections. The choice of a particular notation for the specification of the elements does not dictate how the beam line is to be assembled. For example, in TRANSPORT, elements specified in MAD keyword notation may simply be listed sequentially to describe a beam line. *However, if a beam line specification is to be read by the MAD program, it must be entirely in MAD notation.* The method of assembling a beam line in MAD is used because accelerators typically have a high degree of repetition and symmetry. Secondary beam lines sometimes have very little repetition or symmetry.

Types of Elements

The mnemonic identifies the element, indicating what sort of entity (such as a magnet, drift space, constraint, etc.) is represented. If a minus sign is placed immediately before the mnemonic with no intervening spaces, the element will be ignored in the given problem step. However, storage for that element will be allocated by the program, so that the element may be introduced in a later step of the same problem. Elements may also be introduced in a later step, without previous specification, by reference to elements already present. The procedure for doing this will be described below, in the subsection on "The Use of Labels." For MAD notation the use of labels is described on page 24.

The mnemonic names chosen for the elements are the same as occur in the printed output of the TRANSPORT program. These names for elements which are MAD compatible are the same as used in the MAD program. Some of the names occuring in the printout have been changed slightly from older versions of TRANSPORT. The following is a list of mnemonics

recognized by TRANSPORT. Also included are the meaning of the mnemonics, and whether they are MAD compatible.

Mnemonic	MAD Compatible	Element
UNITS		Input-output units
SPECIAL		Special input parameters
ORDER		Zeroeth, first, second, or third order
RANDOM		Random errors on physical parameters
LIMIT		Limit on allowed range of varied physical parameter
MAGNET		Magnet external dimensions for plotting
BEAM		Input phase space dimensions and reference trajectory momentum.
CORRELATION		Beam rotated ellipse
CENTROID		Shift in the beam centroid
ETA		Accelerator eta function
PARAMETER	Yes	Parameter value
DRIFT	Yes	Drift space – a field free region
BEND		Sector bending magnet (without fringing fields)
ROTATION		Fringing fields and pole-face rotations for bending magnets
RBEND	Yes	Rectangular bending magnet (with fringing fields)
SBEND	Yes	Sector bending magnet (with fringing fields)
HKICKER	Yes	Horizontal steering magnet
VKICKER	Yes	Vertical steering magnet
KICKER	Yes	Steering magnet in both planes
QUADRUPOLE	Yes	Quadrupole magnet
SEXTUPOLE	Yes	Sextupole magnet
OCTUPOLE	Yes	Octupole magnet
PLASMALENS		Plasma (lithium) lens
SOLENOID	Yes	Solenoid magnet
ACCELERATOR	Yes	Travelling wave linear accelerator
SEPTUM		Electrostatic septum
HORN		Neutrino horn
MARKER	Yes	Position marker
MATRIX		Arbitrary matrix input
SROT	Yes	Coordinate Rotation
SHIFT		A shift in the reference coordinate system
REALIGN		Realignment of the reference trajectory on the centroid.
ALIGN		Magnet alignment tolerances
REPEAT		Repetition of a segment of a beam line
SECTION		Defined section of beam line
UPDATE		Transfer matrix update
ALMARK		Alignment marker

PRINT	Output print control instructions
STORE	Storage of matrix element values
PLOT	Plotting instruction
FIT	Fitting constraint
BROAD	Broad-band fitting

The first four letters of a mnemonic are necessary for its specification. More may be used for clarity if desired. For example, the mnemonic SECTION may provide greater clarity than just SECT, which, at first glance, might be taken to mean sector magnet. If more than four characters are used, they must be correct. A mnemonic of SECTIO will be recognized but SECTOIN will not. The same restrictions apply to the keywords. The use of parentheses in mnemonics in this report is to show the characters which are non-essential.

A MAD specification for a sector bending magnet with fringing fields and with the individual name of BM1 might then appear as:

```
BM1:   SBEND,  L = 10.,  ANGLE = 10.,  K1 = 0.5E-6 ;
```

Here, the only physical parameter whose meaning is not obvious is K1, which is a measure of field gradient. Expressed in terms of an alternate set of physical parameters (length and magnetic field instead of length and bending angle), the same element might now be:

```
BM1:   SBEND,  L = 10.,  B = 20.,  N = 0.5  ;
```

The quantity indicated by the keyword N is an alternate measure of field gradient. More details about these various notations are given in the sections below.

Notation for individual elements

Strictly MAD-Compatible Notation

The strict MAD input notation, which is compatible with the MAD program of Christoph Iselin, can be used in TRANSPORT only for the specification of the charged-particle optical system. Other instructions such as units specifications, methods of calculation, printing instructions, and fitting constraints may be specified in a keyword notation which is similar to MAD notation. The options available depend on the type of element.

The strictly MAD notation for the specification of elements has four distinctive characteristics:

1. As explained above, some TRANSPORT elements are compatable with MAD while others aren't. Only physical elements (such as an `SBEND` or a `QUAD`) are MAD compatable. TRANSPORT recognizes those MAD elements which are used to specify the optical system. However, TRANSPORT also has some other elements used for the same purpose but are not in MAD. Items such as units specifications or fitting instructions are also not MAD compatable.

2. There may be several equivalent sets of keywords which may be used to describe a given element. Strict MAD notation requires the use of a particular set of parameters for the specification of any physical element. The parameters to be used for any particular type of element are described in the section for that element. An example might be a sector bending magnet. In MAD notation a sector bending magnet is specified in terms of its length, bending angle, and normalized field gradient K_1. The normalized field gradient K_1 is defined in the earlier section on the mathematical formulation of TRANSPORT.

3. Strict MAD notation also requires the use of a particular set of units for the specification of any physical parameter. For a sector bending magnet the length is given in meters, the bending angle in radians, and the normalized field gradient K_1 in inverse meters squared.

4. In MAD the individual elements are defined first, then assembled into beam lines. In TRANSPORT, the beam lines may also be defined by simply listing the elements.

A complete specification for a sector bending magnet with fringe fields would then look like:

```
BM1:   SBEND,  L = 10.,  ANGLE = 10.,  K1 = 0.5E-6 ;
```

In strict MAD notation the units for length `L` are meters, those for the bend angle `ANGLE` are radians, and those for `K1` are in inverse square meters.

TRANSPORT will also accept data using MAD parameters which are not in MAD units. For example, an SBEND can be parameterized in terms of `L`, `ANGLE`, and `K1`, while using feet for distance, degrees for angle and inverse squared centimeters for `K1`. In the sections describing the individual elements, we shall refer to the parameterization of elements in terms of MAD variables as being MAD notation. However, strict MAD notation requires the use of MAD units, while TRANSPORT can use alternative sets of units even for MAD parameters.

MAD notation has the advantage that it does not require a separate specification of the beam momentum in order to calculate the transfer matrix. The beam momentum is incorporated in the definition of K_1.

If the physical parameters of an element are specified in keyword notation, then any physical parameters to be adjusted in a fitting procedure are identified by the `VARY` command. The `VARY` command is placed in the input data after the complete beam line specification. See page 246 for an explanation of the VARY command.

Alternate Parameters

A physical elements may be specified in several different ways in TRANSPORT by sets of physical parameters. One such element is the sector bending magnet we have been using as an example. Instead of the length and bend angle, we might use the length and magnetic field. We might also use a different normalized gradient n, defined previously. The use of the magnetic field as part of the magnet description now requires knowledge of the momentum to calculate the transfer matrix. For a 600 GeV beam, our bend magnet would then be specified as

```
BM1:   SBEND,  L = 10.,  B = 20.,  N = 0.5  ;
```

In the original TRANSPORT there is provision for a sector bending magnet which does not contain the fringing fields. Such an element is useful in the longitudinal segmentation of a bending magnet to obtain the transfer matrix or beam profile at interior points. The following `BEND` element is physically the same as the above `SBEND` except that the fringe fields are not included:

```
BM1:   BEND,  L = 10.,  B = 20.,  N = 0.5  ;
```

The order of the parameters is not significant in keyword notation. For example, the above specification is equivalent to

```
BM1:   BEND,  B = 20.,  L = 10.,  N = 0.5 ;
```

The `BEND` element as described above, with no fringing fields, is not found in the MAD program. However, it can be described by the same parameters as the `RBEND` (rectangular bending magnet with fringe fields) or the `SBEND` (sector bending magnet with fringe fields), which are found in MAD.

In the specification of a bending magnet still other sets of parameters may be substituted for or added to those mentioned. Other elements such as quadrupoles, sextupoles, solenoids, etc. also have alternate or additional sets of parameters. Complete lists will be found in the sections describing each individual element. Not all elements or possible sets of keyword parameters are compatible with the MAD program. The MAD compatible elements or sets of keyword parameters are indicated in the sections describing the individual elements.

In any keyword notation, the `VARY` command may be used to indicate a parameter to be varied in a fitting procedure. The use is the same as with strict MAD notation, and is explained on page 246.

The Use of Labels

Labels are available for identification of individual elements. There are two circumstances where such identification by the use of labels is necessary.

1. When using explicit `LINE` commands in the MAD format to assemble beam lines from previously defined components, labels are required. In the `LINE` command, the element or subline to be included in a beam line is identified by its label.
2. If an element is to be activated or deactivated or if the parameters of an element are to be changed between steps of a given problem, a label is also required. The label identifies the element in the earlier step to which the changes specified in the latter step are to apply.

Labels may also be inserted for the user's convenience. In this case the placement of a label on a given element is optional.

The label is placed before the mnemonic. It is then identified by being immediately followed by a colon. The maximum length of a label is fifteen non-blank characters. It may contain letters, numerals, underscore, percent (%), and dollar sign ($).

As an example, a drift space labelled `DRF` would appear as:

```
DRF:  DRIFT,  L = 1.5  ;
```

The use of labels to assemble elements into a beam line or subline is completely described on page 204. If the beam line is specified simply by listing the elements and no `LINE` statement is used, then only those elements to be changed in later steps need to be labeled. In this

section we will restrict our description of labels to their use in changing an element from one problem step to the next.

An element may be altered in any of several ways in passing from one step of a problem to the next. An element may be deactivated by placing a minus sign in front of its literal mnemonic (`QUAD`, `BEND`, etc.) or activated by removing the minus sign. New elements can be inserted and old elements removed from the beam line. The values of the parameters can also be changed.

Activation and Deactivation of Elements

For an element to be activated or deactivated from one step to the next of a given problem the element must be included in both steps, having the same label each time. All elements to be activated in a later step must be included in the first step (indicator card 0) of that problem. In later steps (indicator card 1) of a problem only those elements to be changed are specified. The elements to be changed are identified by their labels.

If the mnemonic of an element is immediately preceded by a minus sign in a given step of a problem, that element will be ignored when the calculation is performed. However, storage space in the computer will be allocated for the element for possible activation in later steps of the problem. For example, if a fitting constraint is to be ignored in the first step of a problem, but activated in a latter step, it should be indicated in both steps. In the first step such an element might appear as

```
FIT1:   -FIT,  S12 = 0.0,  TOLER = .001  ;
```

in the first problem step, and the element

```
FIT1:   FIT  ;
```

in the later step. Here the sigma matrix element σ_{12} is fit to zero, causing a waist constraint to be imposed on the beam.

The storage space used for a `FIT` element is fixed. Therefore, in the first step, we can simply indicate that storage is to be reserved for a `FIT` element. In an ensuing problem step, the remainder of the `FIT` element can be specified. The above procedure is therefore equivalent to placing the element:

```
FIT1:   -FIT  ;
```

in the first problem step. In the later step one would then insert

```
FIT1:   FIT,  S12 = 0.0,  TOLER = .001  ;
```

Insertion and Removal of Elements

Elements may also be inserted without the need to be specified in the first of a set of problem steps. They may also be removed so as not to be passed on to further problem steps. A given problem might contain the print statement:

```
PR1:   PRINT,  TRANS  ;
```

To insert two fitting constraints for the σ matrix after the print command on a later problem step, the later step would contain the lines

```
PR1:   AFTER  ;
FIT,  S12 = 0.0,  TOLER = 0.001  ;
FIT,  S34 = 0.0,  TOLER = 0.001  ;
ENDINSERT  ;
```

The `FIT` elements would then remain in the data unless removed in a subsequent step.

If it is desired to place the `FIT` commands before the `PRINT` command, then the first line should say `BEFORE` instead of `AFTER`. The `FIT` elements need have labels only if they are to be modified or removed in a later step.

In the user wishes to remove the print command with the label `PR1` in a subsequent step of a problem, he/she should include in that step the command:

```
PR1:   REMOVE  ;
```

The print command then cannot be activated in a still later step, but must be reinserted. In order to be removed by a `REMOVE` command, an element must be given a label when it is introduced, either in the first step or by an `AFTER` or `BEFORE` command.

If only one element is to be inserted then it can be placed on the same line as the `BEFORE` or `AFTER` statement without an intervening semicolon.

```
PR1:   AFTER,  FIT,  S12 = 0.0,  TOLER = 0.001  ;
```

No `ENDINSERT` command is then necessary to terminate the list of elements to be inserted.

The value of the `BEFORE`, `AFTER`, and `REMOVE` commands is that they allow one to keep a standard data set representing a beam line. One can then effectively modify the data set to do various calculations while not having to alter the original version.

Changing of Physical Parameters

The physical parameters of an element may be changed between steps of a problem. In the first step, all the parameters to be used should be specified. In the later step, only those parameters to be changed need be specified. The storage space allocated for a given element is determined by the element type and the set of parameters given. For the `BEAM` and `MATRIX` elements, included among the data may be continuation codes which will also affect the amount of storage space taken by the element.

In the first step a bending magnet may be given a magnetic field of 10 kilogauss.

```
B1:   BEND,  L = 5.0,  B = 10.0,  N = 0.0  ;
```

In a succeeding step, its magnetic field could be increased to 15 kilogauss by inserting the element:

```
B1:   BEND,  B = 15.0  ;
```

Variation of Physical Parameters

The command `VARY` is used to identify a parameter to be varied in a given problem step. The parameter is left variable until its value is again fixed by the `FIX` command. The commands identify the parameter to be varied or fixed by means of the element label and the parameter name. Directions for the use of the `VARY` command may be found on page 246 and those for the `FIX` command on page 247.

The commands `VARY` and `FIX` are not elements, but rather instructions modifying the specification of elements. What this means is that they are not stored in the computer as part of the beam line representation. Instead they affect the representation of other elements, indicating whether certain of their parameters are to be varied or not in the present problem step. They may appear in a problem step with indicator (1), without having appeared in a previous problem step with indicator (0). <u>There is no reason for the commands `VARY` and `FIX` ever to have labels or to appear with a minus sign in front of the word `VARY` or `FIX`</u>.

Examples

The sample problem below contains two problem steps, each beginning with title and indicator cards and terminating with a SENTINEL. The first step causes TRANSPORT to do a first-order calculation with fitting. The second step initiates a second-order calculation with the data that is a result of that fitting. Elements which are considered to be the same in the two steps are identified by having the same label.

Here we present three versions of the same example. The part of the first example which describes the beam line itself is MAD compatible. The elements are described first, then assembled into a beam line by use of the `LINE` command. Any elements which represent operations, and are therefore not MAD compatible, are placed after the specification of the beam line and refer to `MARKER` elements by means of `LOCATION` specifications. In this example the `PRINT` and `FIT` elements are so specified.

In the second example, the beam line is described by simply listing the elements in order. In the third example, keyword notation is used but with a different set of physical parameters describing each element. In this case the physical parameters are those used in the original TRANSPORT notation. Each of the three formulations of the beam line is a TRANSPORT readable set of data.

A MAD-Compatible TRANSPORT Input Deck

The example may be written with the specification of the optical lattice entirely in MAD notation. The elements – the SBEND, the DRIFTs, – are first given, then assembled into an optical lattice by use of the LINE, command. The USE statement specifies the lattice to be used for the calculation. The other TRANSPORT commands now appear either before of after the part of the data where the optical lattice is given. The complete set of data is as follows:

```
'FORTRAN H CHECK ON BETA FIT'
0
UMAD
BEAM  X = .5,  XP = 1,  Y = .5,  YP = 1,  L = .5,  DEL = 1,  P0 = 1  ;
SEC1:  -ORDER   ;  PRINT,  BEAM,  ON   ;
DRI:  DRIFT,  L = 2.745 ;
SB:  SBEND,  L = 9.879,  ANGLE = 2.96,  K1 = .0449 ;
MRKA:  MARKER ;
EXAMPLE:  LINE =  (DRI,SB,DRI,MRKA)  ;
USE,  EXAMPLE ;
PRINT,   TRANS,  LOCATION = MRKA  ;
FIT1:   FIT,  R12 = 0.0,  TOLER = 0.0001,  LOCATION = MRKA  ;
VARY, DRI[L] ;
SENTINEL
'SECOND ORDER'
1
SEC1:    ORDER  ;
FIT1:    -FIT  ;
FIX,   DRI[L]   ;
SENTINEL
SENTINEL
```

Since, MAD elements are being used, the parameters are in MAD units. The element UMAD then specifies that MAD units (meters, radians) are to be assumed throughout.

The element SBEND includes the fringing fields and assumes that the pole-face-rotation angle is zero unless explicitly specified otherwise. The two drifts are given by a single specification. Consequently they are automatically varied in unison.

The USE command indicates that the line to be used is the one with the proper name EXAMPLE. Several different lines may be assembled into a single set of data. TRANSPORT will run through the line specified by the USE statement. If there is no USE statement, TRANSPORT will run through the elements in order as they occur in the data.

The FIT element indicates an operation to be performed at a certain location in the optical system. However, it is not a physical element, and, in MAD notation, is not included

directly in the specification of the beam line. Instead we define a marker labelled with the word `MRKA`. The marker is included directly in the beam line specification. The `FIT` element is included in the data after the beam line specification and contains a reference to the marker. The `PRINT` command can now also be related to the marker. When TRANSPORT goes through the beam line and comes to the marker labeled `MRKA`, it will execute all operations referring to that marker.

The `VARY` command indicates that in the first step the length of the drift space with label `DRI` is to be varied so as to satisfy the constraint imposed by the `FIT` command. The `FIX` command indicates that in the second step the length of the drift is to be held fixed. The second step will be executed with the value of the drift length as given by the fitting procedure of the first step.

`VARY` and `FIX` commands are considered to be instructions to modify the data and not part of the data itself. Therefore, they can be introduced in later steps without allocating space for them in the first step.

A TRANSPORT Input Deck Using MAD Elements

The physical elements specifying the beam line can be specified using MAD notation. However, the beam line is specified by simply listing the elements in order. MAD requires that the beam be explicitly assembled by the use of `LINE` commands. The result is a set of data which appears as:

```
'FORTRAN H CHECK ON BETA FIT'
0
UMAD
BEAM  X = .5,  XP = 1,  Y = .5,  YP = 1,  L = .5,  DEL = 1,  P0 = 1  ;
SEC1:    -ORDER   ;   PRINT,   BEAM,   ON   ;
DRI:  DRIFT,   L  =  2.745   ;
SBEND,   L  =  9.879,   ANGLE  =  2.96,   K1  =  .0449   ;
DRI:   DRIFT,   L  =  2.745   ;
PRINT,   TRANS   ;
FIT1:    FIT,   R12  =  0,   TOLER  =  .0001   ;
VARY,   DRI[L]   ;
SENTINEL
'SECOND ORDER'
1
SEC1:    ORDER   ;
FIT1:    -FIT   ;
FIX,   DRI[L]   ;
SENTINEL
SENTINEL
```

Notice that the `DRIFT` element with the proper name `DRI` occurs twice. In this example, the duplication effort is inconsequential. However, in a large machine with much repetition, the duplication can cause much unnecessary effort. It is often useful to be able to define each different element only once and then assemble them into a beam line.

A TRANSPORT Input Deck with Keyword Notation

The same set of data can also be represented in an alternate keyword notation. Here the variables describing the beam line are explicitly identified. The variables used here are the same as with original TRANSPORT notation. The complete set of variables available for specifying elements is described in the sections for the individual elements.

```
'FORTRAN H CHECK ON BETA FIT'
0
UTRANS
BEAM  X = .5,  XP = 1,  Y = .5,  YP = 1,  L = .5,  DEL = 1,  P0 = 1  ;
SEC1:    -ORDER   ;   PRINT,   BEAM,   ON   ;
DRI:  DRIFT,   L  =  2.745   ;
ROTAT,   ANGLE  =  0   ;
BEND,   L  =  9.879,   B  =  10,   N  =  .5   ;
ROTAT,   ANGLE  =  0   ;
DRI:   DRIFT,   L  =  2.745   ;
PRINT,   TRANS   ;
FIT1:    FIT,   R12  =  0,   TOLER  =  .0001   ;
VARY,   DRI[L]   ;
SENTINEL
'SECOND ORDER'
1
SEC1:    ORDER   ;
FIT1:    -FIT   ;
FIX,   DRI[L]   ;
SENTINEL
SENTINEL
```

The combination `ROTAT`, `BEND`, `ROTAT` replaces the element `SBEND` . The element `BEND` can be specified in MAD variables, but it is not a MAD element, and will not be recognized by the MAD program.

Transforming a MAD Deck to a TRANSPORT Deck

So far we have been emphasizing the versatility of the TRANSPORT input format. It is now time that we emphasize its compatability with the MAD input. We shall show how, given a complete MAD deck, to extract that portion which describes the physical structure of the beam line, and from this to construct a complete TRANSPORT deck. It is the part which describes the physical structure of the beam line which is compatible between the input of the two programs. An example can be taken from the Synchrotron Light Source Data Book, by Jim Murphy [13]. The MAD input has the following form:

```
TITLE,"SOR RING, TOKYO"
!         DATA COURTESY OF Y.KAMIYA AT KEK, JUNE 1988
QF        :QUADRUPOLE,L=.1,K1=6.08691786
QD        :QUADRUPOLE,L=.1,K1=6.2452322
BD        :SBEND,L=.863937,K1=.3719008,ANGLE=TWOPI/8
D1        :DRIFT,L=.655
D2        :DRIFT,L=.325
D3        :DRIFT,L=.13
HSUP        :LINE=(D1,BD,D2,QF,D3,QD)
USE,HSUP,SYMM,SUPER=4
PRINT,# S/E
TWISS
STOP
```

The elements specified consist of two quadrupoles, one sector bending magnet, and three drifts of differing lengths. The generic names of the elements are preceded by their proper names `QF`, `QD`, `BD`, `D1`, `D2`, and `D3`. The `LINE` command combines these elements into a single line with the proper name `HSUP`. The `USE` command indicates that the line `HSUP` is indeed the optical system to be used for calculations.

Since TRANSPORT is basically designed for single pass systems, the TRANSPORT `USE` command does not contain the modifiers `SYMM` and `SUPER`. The `SYMM` command indicates that a duplicate of the beam line with the elements in reverse order is created and placed immediately following the original beam line. The `SUPER` command indicates that the combined system is repeated four times.

In both MAD and TRANSPORT the same effect can be achieved by defining a `LINE` made up of the appropriate number of repetitions of `HSUP`, both forwards and backwards. To accomplish that effect, we need to replace the above `LINE` defining `HSUP` with two `LINE` statements defining first an intermediate step `HSIN`, and then the final line `HSUP`.

The `PRINT`, `TWISS`, and `STOP` commands are particular to MAD.

A complete TRANSPORT deck from the same data would be:

```
"SOR RING, TOKYO"
       0
UMAD
PRINT, BEAM, ON
BEAM, P0 = .380
!         DATA COURTESY OF Y.KAMIYA AT KEK, JUNE 1988
QF        :QUADRUPOLE,L=.1,K1=6.08691786
QD        :QUADRUPOLE,L=.1,K1=6.2452322
BD        :SBEND,L=.863937,K1=.3719008,ANGLE=TWOPI/8
D1        :DRIFT,L=.655
D2        :DRIFT,L=.325
D3        :DRIFT,L=.13
HSIN        :LINE=(D1,BD,D2,QF,D3,QD)
HSUP        :LINE=(4*(HSIN,-HSIN))
USE,   HSUP
SENTINEL
```

Summaries of TRANSPORT Elements

Table 1. Summary of TRANSPORT Elements in MAD Notation

Physical Element	Mnemonic	Keywords					
Drift	DRIFT	L					
Bending Magnets with Fringing Fields	RBEND or SBEND	L	ANGLE	K1	K2	K3	TILT
Horizontal and Vertical Steering Magnets	HKICK or VKICK	L	KICK				
Steering Magnet in Both Planes	KICKER	L	HKICK	VKICK			
Quadrupole	QUAD	L	K1	TILT			
Sextupole	SEXT	L	K2	TILT			
Octupole	OCTU	L	K3	TILT			
Plasma Lens	PLAS	L	K1				
Solenoid	SOLE	L	KS				
Accelerator	ACCE	L	VOLT	LAG	FREQ		
Coordinate Rotation	SROT	ANGLE					
Marker	MARKER						

The meanings of the various keywords are given in more detail in the sections of this manual describing the individual physical elements

Table 2. Summary of Other TRANSPORT Elements in Keyword Notation

Preliminary Specifications in Keyword Notation

Physical Element	Mnemonic	Keywords		
Units	UNIT	Unit type	Unit name	SIZE
Mass of Particles in beam	SPEC	PMASS		
Half Aperture of Bending Magnet in x Plane	SPEC	HWIDTH		
Half Aperture of Bending Magnet in y Plane	SPEC	HGAP		
Length of System	SPEC	LENGTH		
Reference Trajectory Momentum	SPEC	PO		
Random Number	SPEC	RANNO		
Focal Plane Rotation	SPEC	FOTILT		
Initial Beam Line x-Coordinate	SPEC	XBEGIN		
Initial Beam Line y-Coordinate	SPEC	YBEGIN		
Initial Beam Line z-coordinate	SPEC	ZBEGIN		
Initial Beam Line Horizontal Angle	SPEC	YAW		
Initial Beam Line Vertical Angle	SPEC	PITCH		
Reference Momentum for Magnetic Fields	SPEC	PREF		
Order of Calculation	N1	N2	N	

Random Errors	RANDOM	Element Type	Parameter Name	VARIATION
Limits on Varied Parameters	LIMIT	Element Type	Parameter Name	LOWER or UPPER or ABSOLUTE
Magnet Dimensions	MAGNET	WIDTH	HEIGHT	
Broad Band Fitting	BROAD			

Beam Description in Keyword Notation

Physical Element	Mnemonic	Keywords							
Beam	BEAM	X	XP	Y	YP	L	DEL	PO	
RMS Addition	BEAM	X	XP	Y	YP	L	DEL	PO	RMS
Beam (Accelerator Notation)	BEAM	BETAX	ALPHAX	EPSX	BETAY	ALPHAY	EPSY	PO	
Beam (Rotated Ellipse)	CORR	C21, C43,	C31, C53,	C41, C63,	C51, C54,	C61, C64,	C32, C65	C42,	C52, C62,
Beam Centroid Shift	CENT	X	XP	Y	YP	L	DEL		
Eta	ETA	ETAX	DETAX	ETAY	DETAY	ETAL	ETAP		

Physical Elements in Keyword Notation

Physical Element	Mnemonic	Keywords				
Drift Space	DRIFT	L				
Wedge Bending Magnet without Fringing Field	BEND	L	ANGLE	K1	K2	K3
		L	ANGLE	N	EPS	EPS3
		L	B	N	EPS	EPS3
		L	RADIUS	N	EPS	EPS3
		B	ANGLE	N	EPS	EPS3
		RADIUS	ANGLE	N	EPS	EPS3
Pole Face Rotation including Fringing Field	ANGLE	HGAP	FINT	H		
Bending Magnets with Fringing Fields	RBEND or SBEND	L	ANGLE	N	EPS	EPS3
		L	B	N	EPS	EPS3
		L	RADIUS	N	EPS	EPS3
		B	ANGLE	N	EPS	EPS3
		RADIUS	ANGLE	N	EPS	EPS3
Horizontal and Vertical Steering Magnets	HKICK or VKICK	L	B			
		L	ANGLE			
Quadrupole	QUAD	L	B	APER	TILT	
		L	GRAD	TILT		
Sextupole	SEXT	L	B	APER	TILT	
Octupole	OCTU	L	B	APER	TILT	
Plasma Lens	PLAS	L	B	APER	TILT	
		L	GRAD	TILT		
Solenoid	SOLE	L	B			
Electrostatic Septum	SEPTUM	L	VOLTAGE	APER	TILT	
		L	E			
Accelerator	ACCEL	L	VOLT	LAG	WAVEL	

Other Transformations in Keyword Notation

Physical Element	Mnemonic	Keywords						
Alignment	ALIGN	X	RX	Y	RY	Z	RZ	Other Keywords
Arbitrary Matrix	MATRIX	Rij	Tijk	Uijkℓ				
Coordinate Rotation	ANGLE							
Coordinate System Shift	SHIFT	X	XP	Y	YP	L	DEL	

Assembling Beam Lines

Physical Element	Mnemonic	Keywords			
Repeat control	REPEAT	N			
Defined Section	SECTION	BEGIN	END	FORWARD	BACKWARD

Operations in Keyword Notation

Physical Element	Mnemonic	Keywords				
Update	UPDATE	R1	R2			
Alignment Marker	ALMARK	R1	R2			
Print Instruction	PRINT	FLOOR R2INV ALIGN ELEMENTS PRECISE	BP BEAM REFER ALL ON	R1 WAIST LEVEL MARKER OFF	R2 CENTROID ONELINE ONLY	R1INV TWISS NOPARA NARROW
Storage of Parameters	STORE	Parameter Name				
Fitting Constraints	FIT	See page 253	VALUE	TOLER		
Plotting	PLOT	See page 237				

Output Format

General Appearance

Here we give a brief description of the general appearance of the output and its meaning. The user may refer to the sample output shown at the end of this section. It is the printed output resulting from the sample data shown on page 29. In a simple example it is not possible to show each of the different elements. We therefore refer the user to the sections on the various elements for an explanation of any features particular to a given element.

The output for each step of a given problem is printed separately. The printing for one step is completed before that for the next step is begun. Therefore we will describe the output for a single problem step. The output shown starting on page 48 is from a problem with two steps.

Initial Listing

For each problem step, the program begins by printing out the user's input data. The total number of elements in the beam line after each line is interpreted is printed out after each element. The number may increase by more than one after certain lines for two reasons:

1. Several elements may be placed on a single line, separated by semicolons.
2. A data element may result in several stored elements. One example of that is an element containing an algebraic expression. The instructions to retrieve the quantities in the expression and the algebraic operations are stored in the program as elements.

In the earliest versions of TRANSPORT, the input data was reformatted into neat columns. This aspect of the program eventually interfered with other, more useful features and was abandoned. Nevertheless, it is recoverable by the placement of the word `REFORMAT` on the indicator line of the input data. For more details, see the following section.

Listing During the Calculation

The program now begins the calculation. If there is no fitting, one listing of the beam line will be made. If there is fitting there will normally be two listings. The first will represent the beam line before any fitting has occurred. The second will be based on the new values of the physical parameters which were altered by the fitting process. If sequential fitting is employed, and the keyword `NOBEFORE` is included on the indicator card, the first run will be omitted. For further explanation of the various print control keywords, see page 54.

In any listing the elements are printed in order with their labels and physical parameters. Elements with a minus sign before the element name or type code number are ignored. The listing for each element begins with the element number. By use of this number a correspondence may be made between the elements in the listing during the calculation and those in the initial listing.

Following the element number is the name of that type of element, enclosed in asterisks. All physical elements are listed in this way. Some of the other elements are not explicitly listed but produce their effect in either the calculated quantities or the listing of the beam line. For descriptions of individual cases, the reader should consult the sections on the individual elements.

Calculated quantities appear in the listing as requested in the input data. Important cases will be described in greater detail below. The physical parameters for each element are printed with the appropriate units. For some elements a calculated quantity, not in the input data, will appear, enclosed in parenthesis. Such quantities are explained in the sections under the individual elements.

Calculated Quantities

Options

The important cases of calculated quantities which appear in the output are the transfer matrices, the beam matrix, the layout coordinates, and the results of the fitting procedure. The transfer and beam matrices and layout coordinates appear as requested in the listing of the beam line. The results of the fitting procedure appear between the two listings. All these quantities are explained in greater detail below.

The transfer and beam matrices appear only where requested, either at specific locations or globally. A request for the printing of layout coordinates can be made only globally. Any global print request should be made at the beginning of the beam line. The requested item will then be printed after each physical element. In all cases the quantities printed are the values at the interface between two elements. They are evaluated at a point after the element listed above them and before the element listed below. For further explanation of calculated quantities the user should read the section on the mathematical formulation of TRANSPORT and the sections describing the appropriate elements. The transfer matrix, beam matrix, and layout coordinates are all printed by various forms of the `PRINT` command.

The Transfer Matrix

The six coordinates of the transfer matrix correspond to x, x', y, y', ℓ, and δ. The matrix will then have the appearance

$$
\begin{array}{cccccc}
(x|x_0) & (x|x'_0) & (x|y_0) & (x|y'_0) & (x|\ell) & (x|\delta) \\
(x'|x_0) & (x'|x'_0) & (x'|y_0) & (x'|y'_0) & (x'|\ell) & (x'|\delta) \\
(y|x_0) & (y|x'_0) & (y|y_0) & (y|y'_0) & (y|\ell) & (y|\delta) \\
(y'|x_0) & (y'|x'_0) & (y'|y_0) & (y'|y'_0) & (y'|\ell) & (y'|\delta) \\
(\ell|x_0) & (\ell|x'_0) & (\ell|y_0) & (\ell|y'_0) & (\ell|\ell) & (\ell|\delta) \\
(\delta|x_0) & (\delta|x'_0) & (\delta|y_0) & (\delta|y'_0) & (\delta|\ell) & (\delta|\delta)
\end{array}
$$

If the centroid is off axis, an extra column will appear to the right, giving the position of the beam centroid. The centroid given is the optical image of the centroid at the beginning of the beam line. In first order it will coincide with the center of gravity of the beam ellipse. In higher orders the two will differ due to nonlinear distortions of the beam ellipse.

$$
\begin{array}{cccccccc}
(x|x_0) & (x|x'_0) & (x|y_0) & (x|y'_0) & (x|\ell) & (x|\delta) & & x_c \\
(x'|x_0) & (x'|x'_0) & (x'|y_0) & (x'|y'_0) & (x'|\ell) & (x'|\delta) & & x'_c \\
(y|x_0) & (y|x'_0) & (y|y_0) & (y|y'_0) & (y|\ell) & (y|\delta) & & y_c \\
(y'|x_0) & (y'|x'_0) & (y'|y_0) & (y'|y'_0) & (y'|\ell) & (y'|\delta) & & y'_c \\
(\ell|x_0) & (\ell|x'_0) & (\ell|y_0) & (\ell|y'_0) & (\ell|\ell) & (\ell|\delta) & & \ell_c \\
(\delta|x_0) & (\delta|x'_0) & (\delta|y_0) & (\delta|y'_0) & (\delta|\ell) & (\delta|\delta) & & \delta_c
\end{array}
$$

For a static magnetic system with midplane symmetry, a good deal of simplification occurs. Many of the matrix elements become zero. For the case of an on-axis centroid, the transfer matrix then becomes:

$$
\begin{array}{cccccc}
(x|x_0) & (x|x'_0) & 0.0 & 0.0 & 0.0 & (x|\delta) \\
(x'|x_0) & (x'|x'_0) & 0.0 & 0.0 & 0.0 & (x'|\delta) \\
0.0 & 0.0 & (y|y_0) & (y|y'_0) & 0.0 & 0.0 \\
0.0 & 0.0 & (y'|y_0) & (y'|y'_0) & 0.0 & 0.0 \\
(\ell|x_0) & (\ell|x'_0) & 0.0 & 0.0 & 0.0 & (\ell|\delta) \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & (\delta|\delta)
\end{array}
$$

If the beam line is midplane symmetric and there is no vertical bending, then the transfer matrix can be printed on a single line. To do this, one should include the command (`PRINT, ONELINE ;`) described below. Only those matrix elements which can be nonzero under the stated conditions are printed. The quantities printed are:

$$
L_c \ \texttt{M} \qquad R_{11} \ R_{12} \ R_{21} \ R_{22} \ R_{33} \ R_{34} \ R_{43} \ R_{44} \ R_{16} \ R_{26}
$$

The units for only the accumulated length L_c are shown and are here taken to be meters `M`.

Second- and third-order matrices can also be printed. The second- and third-order matrix elements are individually identified in the output. The second-order matrix is identified with the letter "T". The elements of the T matrix have one dependent index and two independent indices. The indices apply to the six coordinates of the transfer matrix, described above.

The dependent index is written first and followed immediately by the independent indices. Thus the matrix element T_{116} can be written in Dirac notation as the coefficient $(x|x_0\delta)$. It is the coefficient of the product of x_0 and δ in the Taylor series expansion for x.

The second-order transfer matrix is printed in triangular form. That means that there is no coefficient T_{161} or $(x|\delta x_0)$. The individual matrix elements are identified by the dependent index followed by a space and then the dependent index. For example, if the matrix element T_{116} had the value `1.124E-03`, as in the example below, it would be listed as

```
1 16  1.124E-03
```

The third-order matrix is identified by the letter "U". It has one dependent index and three independent indices. Thus the matrix element U_{1166} or $(x|x_0\delta^2)$ would be the coefficient in the expansion for x which multiplied the product $x_0\delta^2$. The U matrix is printed in pyramidal form, in analogy with the triangular form for the second-order matrix.

More details are given on page 224.

The Beam Matrix

The beam matrix, as printed will have the following appearance:

x_g	Δx	CM					
x'_g	$\Delta x'$	MR	r_{21}				
y_g	Δy	CM	r_{31}	r_{32}			
y'_g	$\Delta y'$	MR	r_{41}	r_{42}	r_{43}		
ℓ_g	$\Delta \ell$	CM	r_{51}	r_{52}	r_{53}	r_{54}	
δ_g	$\Delta \delta$	PC	r_{61}	r_{62}	r_{63}	r_{64}	r_{65}

The first column with the subscript "g" represents the coordinates of the center of gravity of the beam phase space. In first order the center of gravity is the same as the centroid, which is the geometric center of the phase ellipsoid. In second and higher order, the ellipsoid becomes distorted and the center of gravity is no longer the beam centroid. If the centroid does not coincide with the reference trajectory, then the centroid coordinates are printed along with the transfer matrix (see below).

The second column gives the half widths of the beam ellipsoid in each of the six beam coordinates. Once again, in higher orders the beam phase space is not an ellipsoid, but some distorted shape like a banana. The quantities given are then the square roots of the second moments of the beam phase space, taken about the beam center of gravity. The units used are given in the third column.

The remaining quantities r_{ij} are the correlations between coordinates. They are defined in terms of the beam matrix elements σ_{ij} by: where

$$r_{ij} = \frac{\sigma_{ij}}{[\sigma_{ii}\ \sigma_{jj}]^{1/2}} \quad .$$

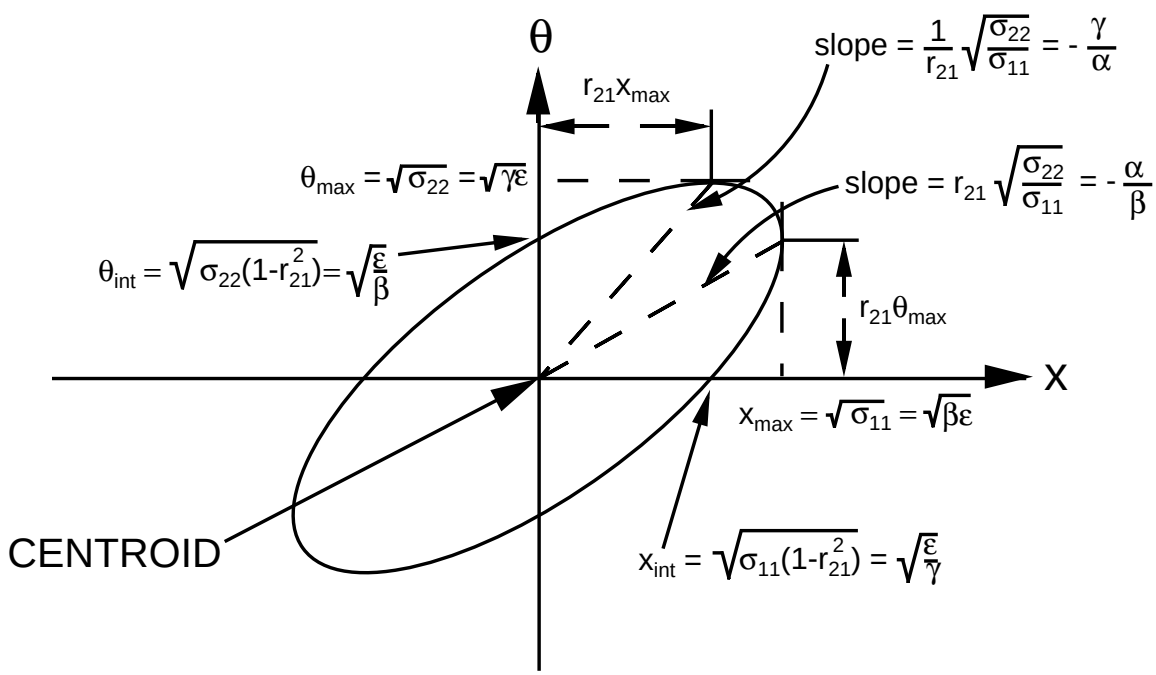
Their initial values may be specified by the `CORRELATION` element described on page 109.

The beam matrix may also be printed in a single line. The command which causes the printing to occupy a single line is (`PRINT, ONELINE ;`). This command, described below, does not print anything, but simply affects the style of how quantities are printed. A single line printing of the beam matrix would look like:

L_c `M` Δx `CM` $\Delta x'$ `MR` Δy `CM` $\Delta y'$ `MR` $\Delta \ell$ `CM` $\Delta \delta$ `PC` r_{21} r_{43}

The accumulated length L_c along the reference trajectory is shown first.

An example of an (x, x') plane ellipse is illustrated below.



A Two-Dimensional Beam Phase Ellipse

The Layout Coordinates

The floor coordinates of the beam reference trajectory can be printed after every beam line component. One can produce a layout of a beam line in any Cartesian coordinate system one chooses.

The coordinates printed represent the x, y and z position, and the yaw, pitch, and roll angles, respectively, of the reference trajectory at the interface between two elements. The y-axis will point up and the x-axis to the left. The *yaw* angle is between the floor projection of the reference trajectory and the floor z axis. The *pitch* angle is the vertical pitch, the angle the reference trajectory makes with a horizontal plane. The *roll* angle is a rotation about the reference trajectory. In the printed output the values given are those at the exit of the element listed above and at the entrance of the element listed immediately below.

The calculation of the coordinates is done from the parameters of the physical elements as given in the data. Therefore, if effective lengths are given for magnetic elements, the coordinates printed will be those at the effective field boundary.

The floor coordinates will be printed after every physical element. The accumulated length L_c of the reference trajectory, the three Cartesian floor positions and the three orientation angles will be printed on a single line, along with the units in which these quantities are expressed. The single line will have the following appearance:

$$L_c \ \texttt{M} \qquad xfloor \ \ yfloor \ \ zfloor \ \ \texttt{M} \qquad yaw \ \ pitch \ \ roll \ \ \texttt{MR}$$

Fitting

Quantities relevant to the fitting appear between the two listings of the beam line. At each iteration of the fitting procedure a line is printed containing the value of the relaxation factor used, the value of chi-squared before the iteration was made, and the corrections made to each of the varied parameters. Once the fitting is complete the final chi-squared and the covariance matrix are printed.

The appearance of the chi-squared and covariance matrix is:

*COVARIANCE (FIT χ^2)

$$
\begin{array}{lllll}
\sqrt{C_{11}} & & & & \\
r_{12} & \sqrt{C_{22}} & & & \\
\cdot & \cdot & & & \\
\cdot & & \cdot & & \\
\cdot & & & \cdot & \\
r_{1n} & \cdot & \cdot & \cdot & r_{n,n-1}\sqrt{C_{nn}}
\end{array}
$$

If MAD notation is used parameters may be given names and be declared independently of any physical element which might refer to them. Such parameters can be varied in a fitting procedure. The new values of all such varied parameters are printed out after the covariance matrix. For further details on the variation of parameters see page 246.

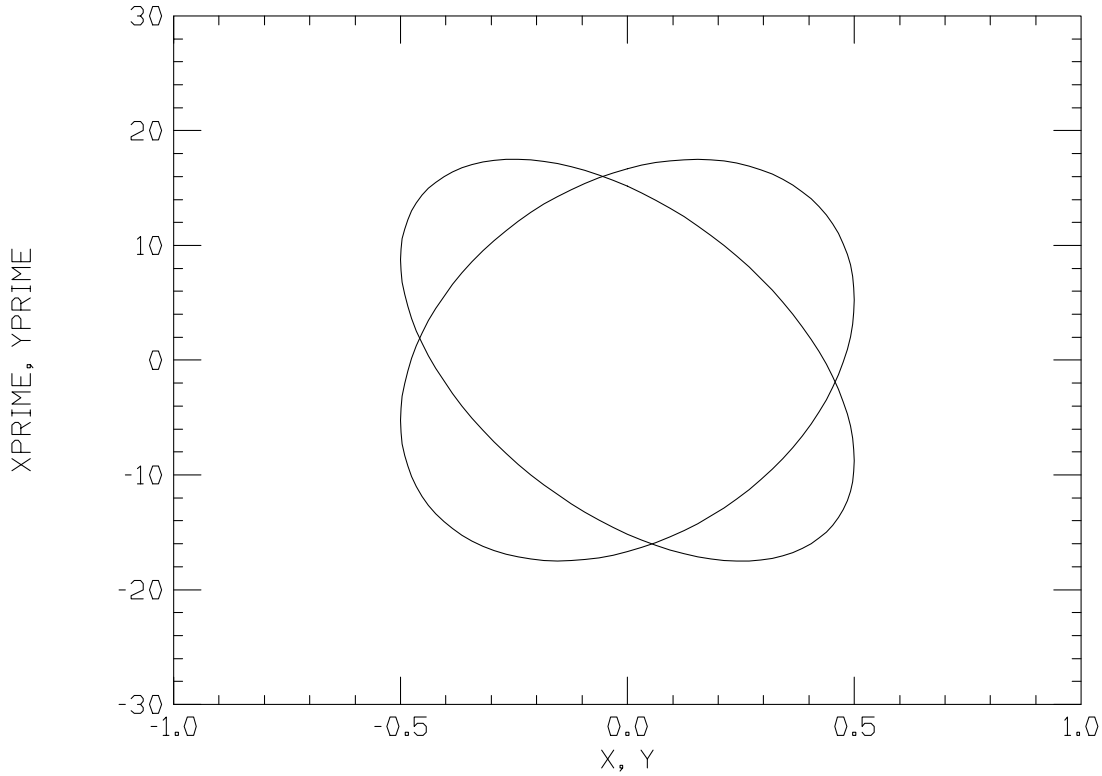
Plotting Output

A PLOT instruction in the input data causes a separate file to be produced in the output. This file consists of lines of sets of values of whatever quantities are specified on the plot instruction. It can be incorporated into the input data of a plotting program like TOP DRAWER via an auxiliary procedure. The procedure TOP DRAWER can be invoked and plots produced.

There are currently four types of plot that can be made with TRANSPORT. They are:

1. Elements of the transfer or beam matrices or any algebraic combination of such quantities vs accumulated distance along the beam line.
2. The beam line reference trajectory floor coordinates. Views from the top and from the side can be made. The magnet external dimensions can be specified so that the magnets are drawn into the plot.
3. The beam ellipse at any point in the beam line. The two coordinates can be any two of the six beam coordinates.
4. Anything vs anything else with stepping. Any input parameter used in the description of the beam line may be stepped over a specified interval. For more information see page 279. Each point of the plot then corresponds to one of the values of the stepped parameter. Each of the coordinates of the plot can be either an element of a transfer or beam matrix at a particular point in the beam line, or one of the parameters used to describe the beam line. Fitting may be included in the stepping so that each point plotted represents a system where any imposed constraints are satisfied.

A plot of the beam ellipse in x vs x' and y vs y' on the same plot might look like:



For more details, see the section of the `PLOT` instruction on page 237.

```
      "CHECK ON BETA FIT                                                    "
          0
      UMM
      BEAM, X=.5, XP=1, Y=.5, YP=1, DL=.5, DEL=1, P0=1  ;
 5    SEC1: -ORDER  ;  PRINT, BEAM, ON  ;
      DR1: DRIFT,  L = 2.745  ;
      ROTAT, ANGLE = 0  ;
      BEND, L = 9.879, B = 10, N = .5  ;
      ROTAT, ANGLE = 0  ;
10    DR2: DRIFT,  L = DR1[L] ;
      PRINT, R1  ;
      FIT1: FIT,  R12 = 0, TOLER = .0001  ;
      VARY, DRL ;
      SENTINEL
11 ELEMENTS USED OUT OF A MAXIMUM ALLOWABLE 4001
36 NUMBERS USED OUT OF A MAXIMUM ALLOWABLE 12001
```

```
CHECK ON BETA FIT

(   2)  *BEAM*                      1.00000 GEV/C
            0.000 M                                                  0.000   0.500 MM
                                                                     0.000   1.000 MR      0.000
                                                                     0.000   0.500 MM      0.000  0.000
                                                                     0.000   1.000 MR      0.000  0.000  0.000
                                                                     0.000   0.500 MM      0.000  0.000  0.000  0.000
                                                                     0.000   1.000 PM      0.000  0.000  0.000  0.000  0.000
(   5)  *DRIFT*        DR1          2.74500 M
            VARIED:  L
            2.745 M                                                  0.000   2.790 MM
                                                                     0.000   1.000 MR      0.984
                                                                     0.000   2.790 MM      0.000  0.000
                                                                     0.000   1.000 MR      0.000  0.000  0.984
                                                                     0.000   0.500 MM      0.000  0.000  0.000  0.000
                                                                     0.000   1.000 PM      0.000  0.000  0.000  0.000  0.000
(   6)  *ROTAT*                     0.00000 MR
(   7)  *BEND *                     9.87900 M        10.00000 KG        0.50000      (     3.336 M   ,2961.64917 MR  )
(   8)  *ROTAT*                     0.00000 MR
            12.624 M                                                 0.000  10.370 MM
                                                                     0.000   1.586 MR      0.581
                                                                     0.000   2.725 MM      0.000  0.000
                                                                     0.000   1.008 MR      0.000  0.000 -0.983
                                                                     0.000  17.708 MM     -0.828 -0.026  0.000  0.000
                                                                     0.000   1.000 PM      0.965  0.772  0.000  0.000 -0.654
(   9)  *DRIFT*        DR1          2.74500 M
            15.369 M                                                 0.000  13.378 MM
                                                                     0.000   1.586 MR      0.776
                                                                     0.000   0.504 MM      0.000  0.000
                                                                     0.000   1.008 MR      0.000  0.000  0.173
                                                                     0.000  17.708 MM     -0.650 -0.026  0.000  0.000
                                                                     0.000   1.000 PM      0.999  0.772  0.000  0.000 -0.654
        *TRANSFORM 1*
            -1.00383  -0.04179   0.00000   0.00000   0.00000  13.36811
            -0.18361  -1.00383   0.00000   0.00000   0.00000   1.22488
             0.00000   0.00000  -1.00383  -0.04179   0.00000   0.00000
             0.00000   0.00000  -0.18361  -1.00383   0.00000   0.00000
            -1.22488 -13.36811   0.00000   0.00000   1.00000 -11.58647
             0.00000   0.00000   0.00000   0.00000   0.00000   1.00000
(  11)  *FIT*          FIT1         R12            0.0     /0.00010        (    -0.04179 )
        *LENGTH*         15.36900 M
        *CORRECTIONS*
        *NUMBER OF VARIED PARAMETERS =     1 *
        *NUMBER OF CONSTRAINTS       =     1 *

 0.10000E+01 ( 0.17462E+06)   -0.0208
 0.10000E+01 ( 0.63033E+00) -0.40E-04
        *COVARIANCE  (FIT 0.56843E-05 )
       0.000
```

```
(   5)  *PARAM*        "DRL   "      0.27241464E+01
            VARIED
CHECK ON BETA FIT

(   2)  *BEAM*                       1.00000 GEV/C
             0.000 M                                                  0.000   0.500 MM
                                                                      0.000   1.000 MR      0.000
                                                                      0.000   0.500 MM      0.000  0.000
                                                                      0.000   1.000 MR      0.000  0.000  0.000
                                                                      0.000   0.500 MM      0.000  0.000  0.000  0.000
                                                                      0.000   1.000 PM      0.000  0.000  0.000  0.000  0.000
(   6)  *DRIFT*        DR1           2.72415 M
             2.724 M                                                  0.000   2.770 MM
                                                                      0.000   1.000 MR      0.984
                                                                      0.000   2.770 MM      0.000  0.000
                                                                      0.000   1.000 MR      0.000  0.000  0.984
                                                                      0.000   0.500 MM      0.000  0.000  0.000  0.000
                                                                      0.000   1.000 PM      0.000  0.000  0.000  0.000  0.000
(   7)  *ROTAT*                      0.00000 MR
(   8)  *BEND *                      9.87900 M        10.00000 KG        0.50000      (     3.336 M   ,2961.64917 MR  )
(   9)  *ROTAT*                      0.00000 MR
            12.603 M                                                  0.000  10.373 MM
                                                                      0.000   1.584 MR      0.582
                                                                      0.000   2.736 MM      0.000  0.000
                                                                      0.000   1.004 MR      0.000  0.000 -0.983
                                                                      0.000  17.689 MM     -0.829 -0.028  0.000  0.000
                                                                      0.000   1.000 PM      0.965  0.773  0.000  0.000 -0.655
(  10)  *DRIFT*        DR1           2.72415 M
            15.327 M                                                  0.000  13.352 MM
                                                                      0.000   1.584 MR      0.775
                                                                      0.000   0.500 MM      0.000  0.000
                                                                      0.000   1.004 MR      0.000  0.000  0.091
                                                                      0.000  17.689 MM     -0.653 -0.028  0.000  0.000
                                                                      0.000   1.000 PM      0.999  0.773  0.000  0.000 -0.655
        *TRANSFORM 1*
            -1.00000   0.00000   0.00000   0.00000   0.00000  13.34257
            -0.18361  -1.00000   0.00000   0.00000   0.00000   1.22488
             0.00000   0.00000  -1.00000   0.00000   0.00000   0.00000
             0.00000   0.00000  -0.18361  -1.00000   0.00000   0.00000
            -1.22488 -13.34257   0.00000   0.00000   1.00000 -11.58647
             0.00000   0.00000   0.00000   0.00000   0.00000   1.00000
(  12)  *FIT*          FIT1         R12            0.0     /0.00010        (  -0.238E-06 )
        *LENGTH*         15.32729 M
```

```
         "SECOND ORDER                                                                  "
             1
         SEC1:  ORDER   ;
         FIX,  DR1[L]  ;
         FIT1:  -FIT ;
   20    SENTINEL

   11 ELEMENTS USED OUT OF A MAXIMUM ALLOWABLE 4001
   29 NUMBERS USED OUT OF A MAXIMUM ALLOWABLE 12001
SECOND ORDER

(   2)  *BEAM*                      1.00000 GEV/C
            0.000 M                                                  0.000   0.500 MM
                                                                     0.000   1.000 MR      0.000
                                                                     0.000   0.500 MM      0.000  0.000
                                                                     0.000   1.000 MR      0.000  0.000  0.000
                                                                     0.000   0.500 MM      0.000  0.000  0.000  0.000
                                                                     0.000   1.000 PM      0.000  0.000  0.000  0.000  0.000
(   3)  *2ND ORDER*    SEC1             GAUSSIAN DISTRIBUTION    2.    2.
(   5)  *DRIFT*        DR1          2.72415 M
            2.724 M                                                  0.000   2.770 MM
                                                                     0.000   1.000 MR      0.984
                                                                     0.000   2.770 MM      0.000  0.000
                                                                     0.000   1.000 MR      0.000  0.000  0.984
                                                                    -0.003   0.500 MM      0.000  0.000  0.000  0.000
                                                                     0.000   1.000 PM      0.000  0.000  0.000  0.000  0.000
(   6)  *ROTAT*                     0.00000 MR
(   7)  *BEND *                     9.87900 M        10.00000 KG        0.50000      (     3.336 M   ,2961.64917 MR  )
(   8)  *ROTAT*                     0.00000 MR
           12.603 M                                                  0.003  10.373 MM
                                                                     0.000   1.584 MR      0.582
                                                                     0.000   2.736 MM      0.000  0.000
                                                                     0.000   1.004 MR      0.000  0.000 -0.983
                                                                    -0.017  17.689 MM     -0.829 -0.028  0.000  0.000
                                                                     0.000   1.000 PM      0.965  0.773  0.000  0.000 -0.655
(   9)  *DRIFT*        DR1          2.72415 M
           15.327 M                                                  0.003  13.352 MM
                                                                     0.000   1.584 MR      0.775
                                                                     0.000   0.501 MM      0.000  0.000
                                                                     0.000   1.004 MR      0.000  0.000  0.091
                                                                    -0.022  17.689 MM     -0.653 -0.028  0.000  0.000
                                                                     0.000   1.000 PM      0.999  0.773  0.000  0.000 -0.655
```

/normalsize

```
*TRANSFORM 1*
   -1.00000   0.00000   0.00000   0.00000   0.00000  13.34257
   -0.18361  -1.00000   0.00000   0.00000   0.00000   1.22488
    0.00000   0.00000  -1.00000   0.00000   0.00000   0.00000
    0.00000   0.00000  -0.18361  -1.00000   0.00000   0.00000
   -1.22488 -13.34257   0.00000   0.00000   1.00000 -11.58647
    0.00000   0.00000   0.00000   0.00000   0.00000   1.00000

*2ND ORDER TRANSFORM*
  1 11  1.124E-04
  1 12  1.225E-03   1 22  6.673E-03
  1 13  0.000E+00   1 23  0.000E+00   1 33 -4.871E-04
  1 14  0.000E+00   1 24  0.000E+00   1 34 -2.042E-03   1 44 -1.112E-02
  1 15  0.000E+00   1 25  0.000E+00   1 35  0.000E+00   1 45  0.000E+00   1 55  0.000E+00
  1 16  2.065E-03   1 26  3.066E-02   1 36  0.000E+00   1 46  0.000E+00   1 56  0.000E+00   1 66  7.908E-03

  2 11  6.128E-09
  2 12  1.335E-07   2 22  6.126E-04
  2 13  0.000E+00   2 23  0.000E+00   2 33 -5.504E-05
  2 14  0.000E+00   2 24  0.000E+00   2 34 -2.999E-04   2 44 -1.021E-03
  2 15  0.000E+00   2 25  0.000E+00   2 35  0.000E+00   2 45  0.000E+00   2 55  0.000E+00
  2 16  3.097E-04   2 26  3.564E-03   2 36  0.000E+00   2 46  0.000E+00   2 56  0.000E+00   2 66  3.834E-04

  3 11  0.000E+00
  3 12  0.000E+00   3 22  0.000E+00
  3 13 -7.500E-05   3 23 -2.042E-03   3 33  0.000E+00
  3 14 -2.042E-03   3 24 -2.224E-02   3 34  0.000E+00   3 44  0.000E+00
  3 15  0.000E+00   3 25  0.000E+00   3 35  0.000E+00   3 45  0.000E+00   3 55  0.000E+00
  3 16  0.000E+00   3 26  0.000E+00   3 36 -1.064E-03   3 46 -3.414E-03   3 56  0.000E+00   3 66  0.000E+00

  4 11  0.000E+00
  4 12  0.000E+00   4 22  0.000E+00
  4 13  5.503E-05   4 23  5.994E-04   4 33  0.000E+00
  4 14 -2.999E-04   4 24 -2.042E-03   4 34  0.000E+00   4 44  0.000E+00
  4 15  0.000E+00   4 25  0.000E+00   4 35  0.000E+00   4 45  0.000E+00   4 55  0.000E+00
  4 16  0.000E+00   4 26  0.000E+00   4 36  5.756E-05   4 46  4.367E-04   4 56  0.000E+00   4 66  0.000E+00

  5 11 -2.641E-04
  5 12 -3.564E-03   5 22 -1.533E-02
  5 13  0.000E+00   5 23  0.000E+00   5 33 -1.031E-04
  5 14  0.000E+00   5 24  0.000E+00   5 34 -4.367E-04   5 44  1.707E-03
  5 15  0.000E+00   5 25  0.000E+00   5 35  0.000E+00   5 45  0.000E+00   5 55  0.000E+00
  5 16 -5.338E-04   5 26 -1.916E-02   5 36  0.000E+00   5 46  0.000E+00   5 56  0.000E+00   5 66 -7.857E-03

*LENGTH*         15.32729 M
```

Title, Indicator, and Comments

Title Card

The title card is the first card in every problem step of a TRANSPORT data set. *The title card is always required and must be followed by the indicator card* (see next section). The indicator card indicates whether the data to follow is new (0 card) or a continuation of a previous data set (a 1 card or a 2 card).

The title must be enclosed within quotation marks (’). The string may begin and end in any column (free field format), for example

```
'SLAC 8 GeV/c SPECTROMETER'
```

<u>Example of a DATA SET for a Single Problem Step</u>

Title
0 or 1
Elements
_____
_____
_____
`SENTINEL`

Indicator Card (0 or 1)

The second line of the input for each step of a problem is the indicator card. If the data which follow describe a new problem, a zero (0) is placed anywhere on that line. If the data which follow describe changes to be made in the previous step of a given problem, a one (1) is placed on that line.

If a given problem step involves fitting, the program will normally list the beam line twice. In each listing the program will print the sequence of elements along with transfer or beam matrices where specified. The first listing uses the parameters of each element before any fitting has taken place. The second shows the results of the fitting. If a problem involving fitting has several steps, the second run of a given step often differs little from the first run of the following step. The user may wish to suppress it.

Similarly, a user may wish to delete the initial listing, and/or to print only the chi-squared and changes in parameter values from the fitting. Alternatively, he or she may wish to print only that portion of the beam line where the fitting occurs.

Various print controls, can be specified via keywords. The keywords are placed on the indicator card, following the indicator number. If there is more than one, they must be separated by commas. Some examples are given on the following pages. The complete listing of such keywords is

Keyword	Meaning
`NOLIST`	The initial listing is suppressed.
`REFORMAT`	The initial listing is reformatted. The output from this initial listing now resembles the uniformly spaced sequence of numbers used in older versions of TRANSPORT. The default is now to simply reprint the input data. The `REFORMAT` option exists simply for reasons of compatability, so that the user can obtain the same style output as with the eariler versions of the program. The `REFORMAT` option should not be used with keyword input as this option assumes positional parameter input and the use of original TRANSPORT variables.
`NOBEFORE`	The printing from the run through the beam line before fitting is suppressed. If there is no fitting to be done, this option will have no effect.
`NOPRINT`	The printing from the run through the beam line both before and after fitting is suppressed. The printing during fitting remains. The output will still contain the sequence of chi-squared values and the changes in the varied parameters.

`NOSOLVE` The printing from the run through the beam line before, during, and after fitting is suppressed. The fitting is itself suppressed. The `NOSOLVE` option is useful on a 1 indicator card after the fitting has been completed in previous steps. The initial deck listing using the `REFORMAT` command can sometimes be used as data for subsequent TRANSPORT runs.

`BWRITE` The common blocks are written onto logical unit 7 on a disk. Subsequent runs of TRANSPORT may then be made without having to read and decipher the data for the first step of a problem. Instead the common blocks can be read directly using the `BREAD` instruction, described below. Clearly, the mnemonic `BWRITE` should be placed only on the last step of a problem. The job should contain the appropriate JCL to identify logical unit 7 with the file the user wishes to contain the output.

`BREAD` The common blocks are read from logical unit 7 on a disk. The first step of a problem then need consist of only the TITLE card and the indicator card with the mnemonic `BREAD`. All the data, including all the elements and the `SENTINEL` card, are, in effect, read from the disk file. This initial problem step, with indicator card 0, may then be followed by steps with indicator card 1. The additional steps may contain modifications to the original data. Thus a charged particle optical system may be stored as a binary file, and the only data that need be decoded from the new job are modifications to the stored file. The job should contain the appropriate computer instructions to identify logical unit 7 with the file containing the input.

If the options `NOLIST` and `NOSOLVE` are both used, there is no output. For example, if one wanted only the listing printed after fitting is complete, then the indicator card might read

```
0,  NOLIST,  NOBEFORE
```

The use of these indicator options along with the two commands

```
PRINT,  ELEMENTS,  OFF  ;
PRINT,  ONLY  ;
```

allows for great flexibility in determining the output to be printed. The two commands are placed near the beginning of the elements which specify the beam line. The first command suppresses the printing of the physical elements. Only the transfer and beam matrices and the results of various other print commands will appear in the output. The second command allows only the varied elements and the constraints to be printed. These commands are described on pages 234 and 235. If the final step of a problem has the word `BWRITE` on

the indicator line, the common blocks of the program are written on a disk. A subsequent problem can read the common blocks. The entire first problem step can then be:

```
'EXAMPLE OF READING THE DATA FROM COMMON BLOCKS'
 0,  BREAD
```

All the elements, including the `SENTINEL`, are read as common blocks. Subsequent steps, with a new title and indicator 1, can follow. They need contain only changes to the data from the first step. The changes are specified exactly as if the first step had been read from a complete data listing. That complete data listing is clearly that which would result from the step of a previously run problem where the word `BWRITE` appeared on the indicator.

A sample problem input was shown previously in the section on input format. In this example TRANSPORT does a first-order calculation with fitting (0 indicator card) and then a second-order calculation (1 indicator card) with the data that are the result of the fitting.

Output File Options

A number of new output file options have been added by Norman M. Gelfand. Two of the files may be used for other processing of data. Five of them serve as input to various programs, including TRANSPORT itself.

As an example, a suitable combination of keywords which can be used to produce a MAD input file is

`PRINT`, `MAD`, filename

If input for programs other than MAD is desired, the keyword `MAD` is replaced by the keyword for the desired program.

`PRINT`	An additional file or files is created at the end of the run with the fitted results. The keyword or keywords indicating specific files must also appear. These keywords are the next six described, namely `MAD`, `TRANSPORT`, `LATDEF`, `STRUCT`, `ACAD`, and `FILE`.
`MAD`	An optional MAD file is created at the end of the run with the fitted results. The final file extension will be .mad
`TRANSPORT`	An optional TRANSPORT file is created at the end of the run with the fitted results. The final file extension will be .tran
`LATDEF`	An optional LATDEF input file is created at the end of the run with the fitted results. The final file extension will be .latdef
`STRUCT`	An optional STRUCT input file is created at the end of the run with the fitted results. The final file extension will be .struct
`ACAD`	An optional AUTOCAD input file is created at the end of the run with the fitted results. The final file extension will be .scr
`FILE`	Two new files are written at the end of the run. The first is a fixed format formatter output file, while the second is a fixed format, unformatted (*binary*) output file.
`filename`	If a name is found which is not one of the listed indicator keywords, then it is used to make up the name of the output file, if the `PRINT` keyword and the keyword indicating which file is to be created are present. The actual output filename will have .tout.ext appended to the name. The characters ".ext", will be replaced by the true final extension, which is given above with the keywords for the different files.

Table 3
Description of the New Formatted Output File (extension.tnout)

Record Number	Format	Contents
1	Text[1]	"Output from TRANSPORT."
2	Text	"Title-" "the title for this run"
3	Text	"Run on" "date" at "time" "."
4	Mixed Int, Text	(INT1) "elements" (INT2) "in beam line" (INT3) "items in list." (The current length of output array is 60.)
5	Int	Number of elements in the transport array dspec.(=33)
6 to 38	Text, F[2]	*"name of the element in dspec", value of the element.* A value of -1.0e+10 is used to indicate that no value has been set for this element.
39	Int	Number of elements in the transport array dbeam.(=18)
40 to 57	Text, F	"*name of the element in dbeam", value of the element.* A value of -1.0e+10 is used to indicate that no value has been set for this element.
58	Int	Number of elements in the transport array deta.(=6)
59 to 64	Text, F	"*name of the element in deta", value of the element.* A value of -1.0e+10 is used to indicate that no value has been set for this element.
65 to 124	Int, Text	(INT) "*description.*" If the value of the integer is *n* then the record contains a short descriptor of the contents of item *n* in the output vectors to follow.
	Varies	For each of the INT2 items in the list there are two records:
r1		*n1 n2 n3 "Name" "<label>" "Type" "<type>".* *n1* A counter running from 1 to INT2 *n2* A counter equal to *n1 - 1* *n3* The transport type code *label* The label for the element in transport. *type* The type of the element in transport. The first record in the list is generated in the routine which writes the data to the file. This first record contains the initial values of the coordinates and lattice functions. It has *n1=1, n2= 0, n3= 0, label= Start* and *type= Start.* *n1, n2* and *n3* are written with format (' ',3I6,...
r2	(' ',1p, 4g16.8)	The output array described below. The number of elements in the array is INT3

[1]All text is included between double quotes.

[2]F will be used to indicate a floating point number.

Table 4
Description of the New Unformatted Output File (extension.tbin)

Record Number	Contents
1	***ltitle,title(1:ltitle),lcdate, cdate(1:lcdate),lctime,ctime(1:lctime),nel,icount, loutar, loutdes*** *ltitle* Length of the character variable *title*. *title* The title for this run. *lcdate* Length of the character variable *cdate*. *cdate* The date of the run. *lctime* Length of the character variable *ctime*. *nel* Number of elements defined. *icount* Number of elements in the current beamline. The count includes the start record. *loutar* The length of the vector containing the output data. This vector is described in Table 5. The current value is 60. *loutdes* The number of characters in the elements in the character array *outdesc*.
2	***nname, 'dspec', ldspec, ndspec, (dspec(i dspec_ val(i),i=1,ldspec)*** *nname* Number of characters in the name of the following **transport** array; in this case 5. *'dspec'* Name of the **transport** array, in this case *dspec*. *ldspec* Length of the array *dspec*; in this case 33. *dspec* The **transport** array *dspec* containing the names of the elements of *dspec*. *dspec_ val* The values of the elements in *dspec*. Undefined elements have a value of -1.0e+10.
3	***nname, 'dbeam', ldbeam, ndbeam, (dbeam(i), dbeam_ val(i),i=1,ldbeam)*** *nname* Number of characters in the name of the following **transport** array; in this case 5. *'dbeam'* Name of the **transport** array, in this case *dbeam*. *ldbeam* Length of the array *dbeam*, in this case 18. *dbeam* The **transport** array *dbeam* containing the names of the elements of *dbeam*. *dbeam_ val* The values of the elements in *dbeam*. Undefined elements have a value of -1.0e+10.
4	***nname, 'deta', ldeta, ndeta, (deta(i), deta_ val(i),i=1,ldeta)*** *nname* Number of characters in the name of the following **transport** array; in this case 4.

	'*deta*'	Name of the **transport** array, in this case *deta*.
	ldeta	Length of the array *deta*, in this case 6.
	deta	The **transport** array *deta* containing the names of the elements of *deta*.
	deta_ val	The values of the elements in *deta*. Undefined elements have a value of -1.0e+10.
5	***(outdesc(i),i=1,loutar)***	
	outdesc(i)	A text string (of length 8) which describes the contents of the output array described in Table 5.
6	***pmass, pmom, energy,*** $B\rho$***.***	
	pmass	The proton mass in GeV/c^2 .
	pmom	The momentum of the beam in GeV/c.
	energy	The beam energy in GeV.
	$B\rho$	$[B\rho]$
7...	For each element in the beam line we have a record containing:	
	n1,n2,n3,llbl,label(1:llbl),lkel,type(1:lkel), loutar,(outnew(i),i=1,loutar)	
	n1	A counter running from 1 to INT2
	n2	A counter equal to *n1 - 1*
	n3	The transport type code
	llbl	The length of the string containing the name of the element.
	label	The label for the element in transport.
	lkel	The length of the string describing the type of element.
	type	The type of the element in transport.
	loutar	The length of the output array (as above).
	outnew	The array described in Table 5.
	The first record in the list is generated in the routine which writes the data to the file. This first record contains the initial values of the coordinates and lattice functions. It has *n1=1, n2= 0, n3= 0, label= Start and type= Start.*	

Table 5
Description of output array from transport.

i	Name	Note
1	l	Element length
2	q	Bend angle
3	$k1$	$B'/[B\rho]$
4	$k2$	$B''/[B\rho]$
5	$k3$	$B'''/[B\rho]$
6	tilt	
7	α_x	
8	β_x	
9	μ_x	
10	η_x	
11	η'_x	
12	α_y	
13	β_y	
14	μ_y	
15	η_y	
16	η'_y	
17	x	x of centroid
18	x'	x' of centroid
19	y	y of centroid
20	y'	y' of centroid
21	suml	Length from start of line.
22	x	Floor coordinates at end of element
23	y	Floor coordinates at end of element
24	z	Floor coordinates at end of element
25	yaw	Floor coordinates at end of element
26	pitch	Floor coordinates at end of element
27	roll	Floor coordinates at end of element
28	σ_{11}	Beam σ.
29	σ_{12}	Beam σ.
30	σ_{13}	Beam σ.
31	σ_{14}	Beam σ.
32	σ_{15}	Beam σ.
33	σ_{16}	Beam σ.
34	σ_{22}	Beam σ.
35	σ_{23}	Beam σ.

36	σ_{24}	Beam σ.
37	σ_{25}	Beam σ.
38	σ_{26}	Beam σ.
39	σ_{33}	Beam σ.
40	σ_{34}	Beam σ.
41	σ_{35}	Beam σ.
42	σ_{36}	Beam σ.
43	σ_{44}	Beam σ.
44	σ_{45}	Beam σ.
45	σ_{46}	Beam σ.
46	σ_{55}	Beam σ.
47	σ_{56}	Beam σ.
48	σ_{66}	Beam σ.
49		Space left for future expansion
50		” ” ” ” ”
51		” ” ” ” ”
52		” ” ” ” ”
53		” ” ” ” ”
54		” ” ” ” ”
55		” ” ” ” ”
56		” ” ” ” ”
57		” ” ” ” ”
58		” ” ” ” ”
59		” ” ” ” ”
60		” ” ” ” ”

Comment Cards

Comment cards may be introduced anywhere in the deck where an element would be allowed. Comments are identified differently in MAD and in original TRANSPORT. Both methods of identifying comments are available in the present version of TRANSPORT. The comments are not stored, but appear only in the initial listing of the given problem step.

In MAD anything on a line after an exclamation mark (!) is considered to be a comment. For example, the bending magnet element described earlier can have an attached comment as follows:

```
BM1:   SBEND,  L = 10.,  ANGLE = 10.,  K1 = 0.5E-6 ; !  THIS IS A COMMENT
```

In original TRANSPORT the comments made on any line are enclosed within single parentheses. No parentheses are allowed within the parentheses of any comment card.

Example of the Use of Comment Cards in a Data Set

```
'TITLE CARD'
0
(THIS IS A TEST PROBLEM TO ILLUSTRATE THE)
(USE OF COMMENT CARDS)
elements
(COMMENTS MAY ALSO BE MADE BETWEEN ELEMENTS)
elements
SENTINEL
```

Preliminary Specifications

Input-Output Unit Sets – UMM, UMETER, UMICR, UMAD, UTRANS

TRANSPORT allows a great deal of flexibility in the choice of units for representation of physical quantities. Several different sets of units may be specified by means of single unit set commands. The choice of units may be made independently of the choice of notation for the elements.

Further changes may be made in individual units changes. The unit set commands are described in this section. The format for individual unit changes is given in the next section.

The unit sets `UMETER` and `UMAD` use meters for transverse dimensions and radians for angles. The unit set `UMM` uses millimeters for transverse dimensions and milliradians for angles. The unit set `UMICRON` uses microns (not micrometers – that is a measuring instrument, not a unit) for transverse dimensions and microradians for angles.

Since transfer matrix elements are ratios of transverse measures of the beam, the elements of the transfer matrix are numerically the same in the four unit sets `UMETER`, `UMAD`, `UMM`, and `UMICRON`. The accelerator phase space parameters beta and alpha will also be the same in all four of these unit sets.

The unit set `UMM` has transverse distances in millimeters and transverse angles in milliradians. It also has longitudinal distance im meters. These units are on the order of the dimensions usually found in accelerators and beam lines. The unit set `UMM` is therefore a very convenient unit set to work with. All further examples in this manual will be given in the unit set `UMM`, unless there is good reason to mention more than one unit set.

The command `UMAD` allows compatability with the MAD program and facilitates the performance of accelerator calculations. Both unit sets `UMAD` and `UMETER` cause all lengths to be expressed in meters and all angles (except for accelerator phase advance) to be expressed in radians. The fractional momentum deviation δ is expressed as a fraction, not as a percent. One reason these units are preferred by accelerator designers is that they are consistent. The angular measure milliradians is intrinsically self-inconsistent since it is the ratio of two lengths expressed in different units. The unit percent is self-inconsistent since a quantity in percent must be divided by 100 to be used in any equation.

The use of a particular set of units, such as `UMAD` is specified by placing the appropriate command, such as `UMAD`, at the beginning of the sequence of elements in the data deck. This command should immediately follow the title and indicator cards and precede any `UNIT` command, described in the following section.

Example of the Use of UMAD in a Data Set

```
'TITLE CARD'
0
UMAD
other elements
SENTINEL
```

A different set of units was originally considered standard in TRANSPORT. This original set of units is still available and is specified by substituting the command `UTRANS` for `UMAD` in the above example. Previous manuals were written in terms of this original set of units. Now there is a choice among several basic sets. The units to be used in specifying the physical parameters of the various elements are given in the sections describing those elements. Appropriate units are given for `UMM` and sometimes `UMETER`.

The `UMAD` command is very similar to `UMETER`. The two specifications differ in that with `UMAD` the accelerator phase advance is in radians/2π while the coordinate rotation about the longitudinal axis is specified in radians. In all other unit sets, both quantities are specified in degrees.

With `UMETER` the multipole strengths are also specified in the standard TRANSPORT convention as the normalized field derivative divided by $n!$, where n is the order of the multipole. Compared to `UMETER`, the `UMAD` sextupole strength should be multiplied by a factor of two. The `UMAD` octupole strength should be multiplied by a factor of six. Details may be found on page 2 in the section on Representation of Magnetic Fields under the Mathematical Formulation of TRANSPORT.

Any of the five unit sets, `UMAD`, `UTRANS`, `UMETER`, `UMM`, or `UMICRON` can be used for any of the physical parameters on any element. The units of a specific type of quantity, such as length, transverse distance, angle, magnetic field, particle mass can also be set individually. See the next section describing the `UNIT` command. Units can be anything the user desires, such as furlongs, parsecs, or cubits for length, arc seconds for angles, stone or solar masses for mass, and Webers per square foot for magnetic field.

The TRANSPORT units and their meanings are:

Keyword	Quantity	Unit in Unit set `UMM`	Symbols used in SLAC-75
`X`	horizontal and vertical transverse dimensions, magnet apertures, and misalignment displacements	mm	x,y
`XP`	horizontal and vertical angles and misalignment rotation angles	mr	θ, ϕ
`DL`	pulsed beam length and wave length in accelerator	mm	ℓ
`DP`	momentum spread	permil	δ
`ANGLE`	bend, pole face rotation, and coordinate layout angles	mr	
`L`	length (longitudinal) of elements, layout coordinates, and bending magnet pole face curvatures	meters	t
`B`	magnetic fields	kG	B
`MASS`	mass	electron mass	GeV
`P`	momentum and	GeV/c	p(0)
	energy gain in accelerator section	GeV	ΔE
`PHASE`	Betatron phase shift	degrees	ψ
`ROLL`	Rotations about the reference trajectory	degrees	
`V`	Electric fields and voltages	MV	

The units that are specified by the various unit sets are as follows. Any further units changes should refer to these units:

Code Digit	UMM	UMETER	UMICR	UMAD	UTRANS
X	mm	meters	μ	meters	cm
XP	mr	radians	μr	mradians	mr
DL	mm	meters	μ	meters	cm
DEL	pm	fraction	pμ	fraction	pc
ANGLE	mr	radians	μr	radians	degrees
L	meters	meters	meters	meters	meters
B	kG	kG	kG	kG	kG
MASS	GeV	GeV	GeV	GeV	Electron mass
P	GeV/c	GeV/c	GeV/c	GeV/c	GeV/c
PHASE	degrees	degrees	degrees	radians/2π	degrees
ROLL	degrees	degrees	degrees	radians	degrees
V	MV	MV	MV	MV	MV

The meaning of the abbreviations used above is as follows:

cm = centimeters
mm = millimeters
μ = microns
mr = milliradians
μr = microradians
pc = percent
pm = permil (one part in a thousand)
pμ = permic (one part in a million)
kG = Kilogauss
MV = Megavolts

The command UMAD also affects the definition of the multipole component (see above).

UNIT – Individual Units Changes

Strict MAD input requires no individual units changes. It requires merely the specification of the global unit set `UMAD`, described on the previous few pages. Elements described in terms of `MAD` parameters, in other keyword notation, or in original TRANSPORT notation may require individual units changes from one of the global unit set specifications.

The individual units changes are placed in the data at the beginning of the ELEMENTS. They follow only the global units specifications.

The individual units changes are written in keyword notation. A `UNIT` element which changes the `UMM` transverse unit of meters to inches would look like:

```
UNIT,  X,  IN  ;
```

Since TRANSPORT recognizes the mnemonic `IN`, a units conversion factor is not needed. If TRANSPORT did not recognize the mnemonic `IN`, then the above units conversion element would have to be written as:

```
UNIT,  X,  IN,  SIZE = 25.4  ;
```

In addition to the element name `UNIT`, there are three items to be specified:

1. The unit type (`X`, `XP`, `P`, etc.) as indicated on the charts on the previous two pages.
2. The abbreviation of the new unit (see examples below). This will be printed on the output listing. It is written as a simple keyword.
3. The scale factor (if needed). The scale factor is needed only TRANSPORT does not recognize the abbreviation of the new unit. If a new unit (such as furlongs or stone) is introduced, TRANSPORT needs to be told the size of the new unit. The scale factor is the size of the new unit relative to the unit in the prevailing unit set. It is indicated by the keyword `SIZE`. Consider, for example, the case when the new transverse unit is inches and the unit set `UMM` has been specified. The `UMM` transverse unit is mm, so the scale factor is 25.4.

Examples

In the following examples, it will be assumed that the prevailing unit set is `UMM`. Thus, unless otherwise specified, longitudinal units are in meters, transverse units in millimeters and milliradians, and momentum in GeV/c.

To change length to feet, width to inches, and momentum to MeV/c, add to the front of the deck the elements

```
UNIT,  L,  FT  SIZE = 0.3048;
UNIT,  X,  IN  SIZE = 25.4;
UNIT,  P,  MEV  SIZE = 0.001;
```

The scale factor, 0.3048, gives the length of the new unit, feet, in terms of the reference unit, meters, etc. Since the unit names `FT`, `IN`, and `MEV` are all recognized by TRANSPORT, the size specification is unnecessary, and may be omitted.

Qualifications

The keywords `ELEMENT`, `BEAM`, `FLOOR`, `ALIGN`, `INPUT`, and `OUTPUT` may be used to restrict the units change to a limited application. The keywords are placed on the units element, and are separated by commas. The keyword `ELEMENT` indicates that the units change is to be used only for the description of a physical element. The application can be further restricted to the input data alone or the printed output alone by use of the additional keywords `INPUT` or `OUTPUT`. The data may then be input in one set of units and output in another. Two beam lines may also be concatenated longitudinally, even if they are specified in different units. A new set of units specifications must be placed between the two beam line descriptions.

The keyword `BEAM` refers to the beam dimensions and the transfer matrix elements. The keyword `FLOOR` refers to the floor coordinates. The keyword `ALIGN` refers to the misalignment parameters. These three keywords apply both to input and output and are not subject to further restriction by the `INPUT` and `OUTPUT` keywords.

The unit elements are the first cards in a deck (immediately following the title card and the 0 and 1 indicator card) and should not be inserted in any other location. The unit set specification (`UTRANS`, `UMAD`, `UMETER`, `UMM`, or `UMICR`) should appear first, followed by any `UNIT` elements. Units specifications produce no printed output during the calculation, their effect being visible only in the output from other elements.

More Examples

If the linear measures for the units changes above were to apply only to the input data for the physical elements, then the example would be modified to:

```
UNIT,  L,  FT,  SIZE = 0.3048,  ELEMENT,  INPUT ;
UNIT,  X,  IN,  SIZE = 25.4,  ELEMENT,  INPUT ;
UNIT,  P,  MEV,  SIZE = 0.001  ;
```

If the beam dimensions are to be expressed in microns and microradians, the following elements should be added to the front of the deck.

```
UNIT,  X,  MU,  SIZE = 1.0E-3 ,  BEAM ;
UNIT,  XP,  MUR,  SIZE = 1.0E-3 ,  BEAM ;
```

If the floor coordinates are to be expressed in feet and degrees, the following elements are used.

```
UNIT  ANGLE,  DEG,  FLOOR ;
UNIT  L,  FT,  FLOOR ;
```

Finally, if the misalignment parameters are to be expressed in mils (thousandths of an inch) and arc minutes, then the following units changes should be inserted.

```
UNIT  X,  MIL,  SIZE = .0254,  ALIGN ;
UNIT  XP,  MIN,  SIZE = .290888,  ALIGN ;
```

The specifications of sets of units is equivalent to a set of `UNIT` specifications, with one small caveat. The specification of a unit set (`UTRANS`, `UMAD`, etc.) not only specifies the units, but also the reference for further unit changes. The specification of the unit size on the `UNIT` element is done relative to the prevailing unit set.

Without further units changes then, the single command

```
UMAD
```

is equivalent to the series of commands

```
UTRANS ;
UNIT,  X,  M ;
UNIT,  XP,  R ;
UNIT,  DL,  M ;
UNIT,  DEL,  N ;
UNIT,  ANGLE,  R ;
UNIT,  P,  GEV ;
UNIT,  PHASE,  TUNE ;
UNIT,  ROLL,  R ;
```

The mnemonics `M` for meters and `R` for radians are recognized by TRANSPORT and the conversion factor is supplied automatically. The units conversion containing the symbol `N` indicates that the momentum deviation δ is measured fractionally (not per-anything).

For the conventional units listed below, it is sufficient to stop with the unit name (the conversion factor is automatically inserted by the program). If units other than those listed below are desired, then the unit name and the appropriate conversion factor must be included. The conversion units below are based on the assumption that the prevailing unit set is `UMM`.

Input-output units: `UNIT` element – For use with `UMM`
(Conversion factors for dimension changes versus unit type mnemonic and unit)

Unit	Unit Type Mnemonic											
	X	XP	DL	DEL	ANGLE	L	B	MASS	P	PHASE	ROLL	V
M	1000.	- - -	1000.	- - -	- - -	1.	- - -	- - -	- - -	- - -	- - -	- - -
CM	10.	- - -	10.	- - -	- - -	.01	- - -	- - -	- - -	- - -	- - -	- - -
MM	1.	- - -	1.	- - -	- - -	.001	- - -	- - -	- - -	- - -	- - -	- - -
MICR	.001	- - -	.001	- - -	- - -	.001	- - -	- - -	- - -	- - -	- - -	- - -
FT	304.8	- - -	304.8	- - -	- - -	.3048	- - -	- - -	- - -	- - -	- - -	- - -
IN	25.4	- - -	25.4	- - -	- - -	.0254	- - -	- - -	- - -	- - -	- - -	- - -
R	- - -	1000.	- - -	- - -	1000.	- - -	- - -	- - -	- - -	(a)	(a)	- - -
MR	- - -	1.	- - -	- - -	1.	- - -	- - -	- - -	- - -	(b)	(b)	- - -
MUR	- - -	.001	- - -	- - -	.001	- - -	- - -	- - -	- - -	(c)	(c)	- - -
DEG	- - -	(d)	- - -	- - -	(d)	- - -	- - -	- - -	- - -	1.	1.	- - -
PC	- - -	- - -	- - -	10.	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
PM	- - -	- - -	- - -	1.	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
PMIC	- - -	- - -	- - -	.001	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
N	- - -	- - -	- - -	1000.	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
MEV	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	.001	- - -	- - -	- - -
GEV	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	1.	- - -	- - -	- - -
KG	- - -	- - -	- - -	- - -	- - -	- - -	1.	- - -	- - -	- - -	- - -	- - -
G	- - -	- - -	- - -	- - -	- - -	- - -	.001	- - -	- - -	- - -	- - -	- - -
TUNE	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	$1/2\pi$	- - -	- - -

(a) The size of a radian in degrees is $180./\pi$
(b) The size of a milliradian in degrees is $.180/\pi$
(c) The size of a microradian in degrees is $.000180/\pi$
(d) The size of a degree is milliradians is $1000.*\pi/180.$
PC is an abbreviation for percent
PM means permil which is one part in a thousand or one-tenth of a percent
PMIC means permic which is one part in a million or one-ten-thousandth of a percent
N means fractional measure or parts per 100 percent

SPECIAL — Special Input Parameters

A number of constants are used by the program which do not appear as parameters of any element. Many of these are initial values, such as floor coordinates, to be set at the beginning of the beam line. Others are the default values of certain parameters which do appear in the description of physical elements. A special element has been provided to allow the designer to set the values of these various constants.

Most of these special parameter elements will appear at the beginning of the data, after any units changes. In any case, the special parameters must always precede the physical element(s) to which they apply. *Once introduced, they apply to all succeeding elements of the appropriate type in the beam line unless reset to zero or to new values.* Many of the `SPECIAL` parameters apply to bending magnets. In such cases, the value of a parameter as given by the `SPECIAL` element may be overridden for a single magnetic element by including the parameters in the description the element itself.

There is no strictly MAD format for the `SPECIAL` element. However, any `SPECIAL` element can be written in keyword notation.

Keyword Notation

A large number of keywords are available for use with the `SPECIAL` element and are listed below. However, each use of the `SPECIAL` element involves a single keyword. Separate special parameters must be set by separate uses of the `SPECIAL` element. An example might be the specification of the quadratic dependence of a bending magnet field. Using keyword notation, it could also be written as

```
SPEC,  EPS = .001  ;
```

When `EPS` is specified by the special parameter element, the value given is applied to all subsequent bending magnets. This particular special parameter can also be written directly on any element to which it applies. If so, the value given on the element will momentarily override the value given by the `SPECIAL` element. The `RBEND`, `SBEND`, and `BEND` elements will recognize the special parameter `EPS`. When `EPS` is given on a bending magnet element, it applies only to that bending magnet.

Keywords and Their Meanings

The keywords which can be used in specifying the special input parameters are given below. In each minisection, is given a description of one or more special parameters and their use. Included also may be equations, diagrams, sample data, and tables of possible values. The listing of all the parameters makes up a little table, parts of which are distributed among the minisections. The heading for the table can be considered to be:

Symbol	Keyword	Description

Initial Beam Line Coordinates and Direction

When requesting a beam line coordinate layout via a (`PRINT, FLOOR ;`) element one can employ any coordinate system one desires. The position and direction of the beginning of the reference trajectory in this coordinate system are given on a set of five `SPEC` elements. If any are omitted, the default value of zero is used Such cards should be placed before the beam card, but after any units changes. Their meanings are as follows:

x_0	`XBEGIN`	respectively, the coordinates of the
y_0	`YBEGIN`	initial point of the reference trajectory in the
z_0	`ZBEGIN`	units chosen for longitudinal length.
θ_0	`YAW`	the initial horizontal and vertical angles
ϕ_0	`PITCH`	of the reference trajectory.

The units of longitudinal length are normally meters. The angular units are milliradians in `UMM` and radians in `UMETER`. Both the length and the angle units may be changed by a `UNIT` command as described on page 68.

The angle ψ, corresponding to a rotation about the longitudinal axis, may be set by using the `SROT` element.

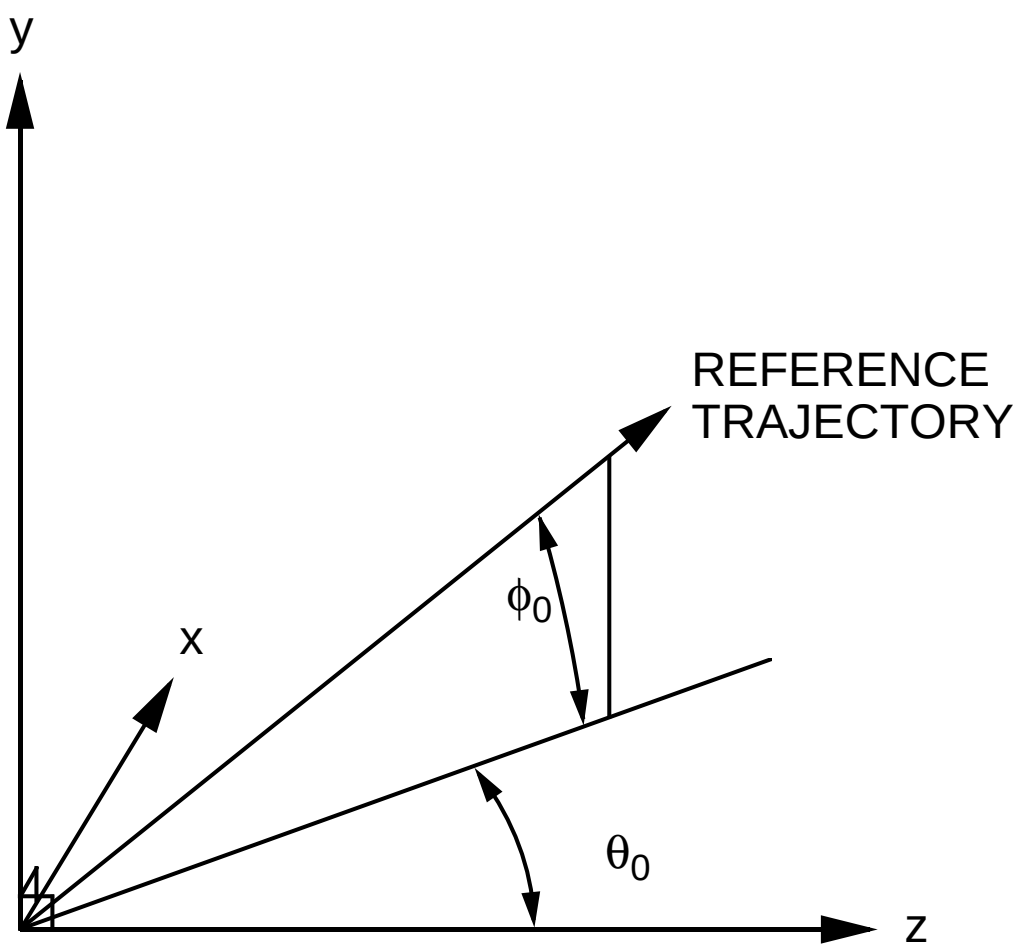
When specifying the initial orientation of the reference trajectory via the two angles θ_0 and ϕ_0, one must give the horizontal angle (θ_0) first. Both angle specifications must be preceded by the (`PRINT, FLOOR ;`) command. The meaning of the two angles is given in the following figure. Any of the above five parameters not explicitly specified will be taken to equal zero.

The initial coordinates may be varied in first-order fitting. Their values will affect only the beam line floor coordinates and not any beam or transfer matrix element.

A complete initial coordinate specification might then look something like:

```
SPEC,  XBEGIN = 6.325  ;
SPEC,  ZBEGIN = 182.111  ;
SPEC,  YAW = 32.4  ;
SPEC,  PITCH = 15.0  ;
```

Here the angles must be in degrees since 15 radians does not make much sense.



Specification of Initial Angles θ_0 and ϕ_0 for Beam Line Layout.

Cumulative Length of the System

L	`LENGTH`	Cumulative length of system, in the same units as system length (normally meters). Possible units changes are described on page 68. The cumulative length is set to zero initially, then increased by the length of each element, and finally printed at the end of the system. This element allows the cumulative length to be reset as desired.

Reference Trajectory Momentum

p_0	`P0`	The momentum of the central trajectory. This is the same item as is found on the `BEAM` element (normally GeV/c). It may be redefined at any point in the beam. In this sense it is not really a preliminary specification, but is included here because it is given by a `SPEC` element. The momentum of the beam particles are not changed, but just the momentum of the central trajectory. If, for example, the central momentum is increased, then clearly the beam centroid, as expressed in percent, is shifted downward. This element is useful when a beam is transferred from one optical system to another with a different design momentum.

Reference Momentum for Magnetic Fields

p_{ref}	`PREF`	The reference momentum (normally GeV/c) to which the magnetic fields are normalized. If this parameter is omitted it is taken to be the central momentum p_0 on the `BEAM` element. If it is included, all subsequent magnetic fields will be scaled by the ratio p_0/p_{ref}. The purpose of this parameter is to allow the user to change the momentum of the beam without retyping the values of all the magnetic fields.

A statement setting the magnetic field reference momentum to 100 GeV might then be:

```
SPEC,  PREF = 100.   ;
```

Particle Mass

(M/m)	`PMASS`	Mass of the particles comprising the beam, (GeV in `UMM` and `UMETER`; default 0). The selection of still other units is described on page 68. A non-zero mass introduces the dependence of pulse length ℓ on velocity, an important effect in low-energy pulsed beams.

Tilt to Focal Plane

α	`FOTILT`	The tilt of the focal plane (in degrees).

Very often it is desired to have a listing of the higher-order aberrations along the focal plane of a system rather than perpendicular to the optic axis, i.e. along the x coordinate. If the focal plane makes an angle α with respect to the x axis (measured clockwise) then provision has been made to rotate to this focal plane and print out the higher-order aberrations. This is achieved by the following procedures:

The symbol α represents the focal-plane tilt angle, measured from the perpendicular to the optic axis (α is normally zero).

The programming procedure for a tilt in the x(bend)-plane (rotation about y axis) is:

```
SPEC,  FOTILT = α   ;
DRIFT,  L = 0.    ;       (a necessary do-nothing element)
PRINT,  TRANS   ;
SPEC,  FOTILT = −α   ;  (rotate back to zero)
DRIFT,  L = 0.    ;       (a necessary do-nothing element)
SPEC,  FOTILT = 0.    ;  (to turn off rotation element)
```

The programming procedure for a tilt in the y-plane (rotation about x-axis) is:

```
SPEC,  FOTILT = α   ;
SROT,  ANGLE = 90.    ;
DRIFT,  L = 0.    ;
SROT,  ANGLE = -90.    ;
PRINT,  TRANS   ;
SPEC,  FOTILT = −α   ;  (rotate back to zero)
SROT,  ANGLE = 90.    ;
DRIFT,  L = 0.    ;
SROT,  ANGLE = -90.    ;
SPEC,  FOTILT = 0.    ;  (to turn off rotation element)
```

Random Number

	`RANNO`	The input value of the random number used for error calculation. In the IBM versions of TRANSPORT if this element is included but left blank, the input value will be determined by the computer clock. It will also be printed on the element in the initial data listing. Its value will then be available for further runs with the same beam line configuration.

Half Apertures of Bending Magnets

$W/2$	`HWIDTH`	Horizontal half-aperture of bending magnet, in the same units as horizontal beam width, normally 0 (i.e. effect of horizontal half aperture is ignored). The units of horizontal beam width are millimeters in `UMM` and meters in `UMETER`. Other unit sets or units may be selected as described on pages 64 and 68.
$g/2$	`HGAP`	Vertical half-aperture of bending magnet, in the same units as vertical beam height. *This parameter must be inserted if the effect of the spatial extent of the fringing fields upon transverse focusing is to be taken into account. It is essential for producing an accurate floor layout of the beam line if the quantity $g^2/2\rho$ is a large enough transverse dimension to have a detectable effect on the magnet locations.* Here ρ is the trajectory radius of curvature in the central body of the bending magnet. More details can be found in the section on fringing fields starting on page 129. The `HGAP` parameter can be specified directly on the `ROTATION` element indicating a pole-face rotation for a bending magnet or on the `RBEND` or `SBEND` elements which specify a bending magnet with fringing fields. <u>This parameter must be given a nonzero value in any third-order calculation.</u> (See the `ROTATION` and `BEND` elements as a cross reference) The units for `HGAP` are determined in the same manner as those for `HWIDTH`. The default value of the half aperture is 0.

Fringing-Field Integrals

These three integrals * are denoted by the symbols κ_0, κ_1, and κ_2 for historical reasons. They should not be confused with the symbols which are used in MAD notation to denote respectively the dipole, quadrupole, and sextupole components of the interior field of a bending magnet. They are all dimensionless.

*See page 131 and SLAC-75 [4] page 74 for further explanation.

κ_o	`FINTO`	An integral used to calculate the reference orbit displacement caused by the spatial extent of the fringing field. The default value is 0.
κ_1	`FINT`	An integral related to the extent of the fringing field of a bending magnet. If the (`SPEC, HGAP =` g/2. ;) element has been inserted, the program inserts a default value of $\kappa_1 = 1/2$ unless a (`SPEC, FINT =` κ_1. ;) element is introduced, in which case the program uses the κ_1 value selected by the user. The parameter `FINT` may be specified directly on the `ROTATION` (described on page 129) element indicating a pole-face rotation for a bending magnet or on the `RBEND` or `SBEND` elements (described on page 134) which specify a bending magnet with fringing fields. The preferred use of the special parameter element in this case is to specify a different default value at the beginning of the beam line. The table below shows typical values for various types of magnet designs.
κ_2	`FINT2`	A second integral related to the extent of the fringing field. Default value of $\kappa_2 = 0$ unless specified by a (`SPEC, FINT2 =` κ_2. ;) element. At present, this integral has no effect on any matrix element calculated by TRANSPORT.

Typical values of κ_1 and κ_2 are given below for four types of fringing field boundaries:

a) a linear drop-off of the field,
b) a clamped "Rogowski" fringing field,
c) an unclamped "Rogowski" fringing field,
d) a "square-edged" nonsaturating magnet.

Model	κ_1	κ_2
Linear	$b/6g$	3.8
Clamped Rogowski	0.4	4.4
Unclamped Rogowski	0.7	4.4
Square-edged Magnet	0.45	2.8

where b is the extent of the linear fringing-field. †

For most applications κ_2 is unimportant. If you find it is important to your result, you should probably be making a more accurate calculation with a differential equation ray-

†See page 98 of Steffen's book [9].

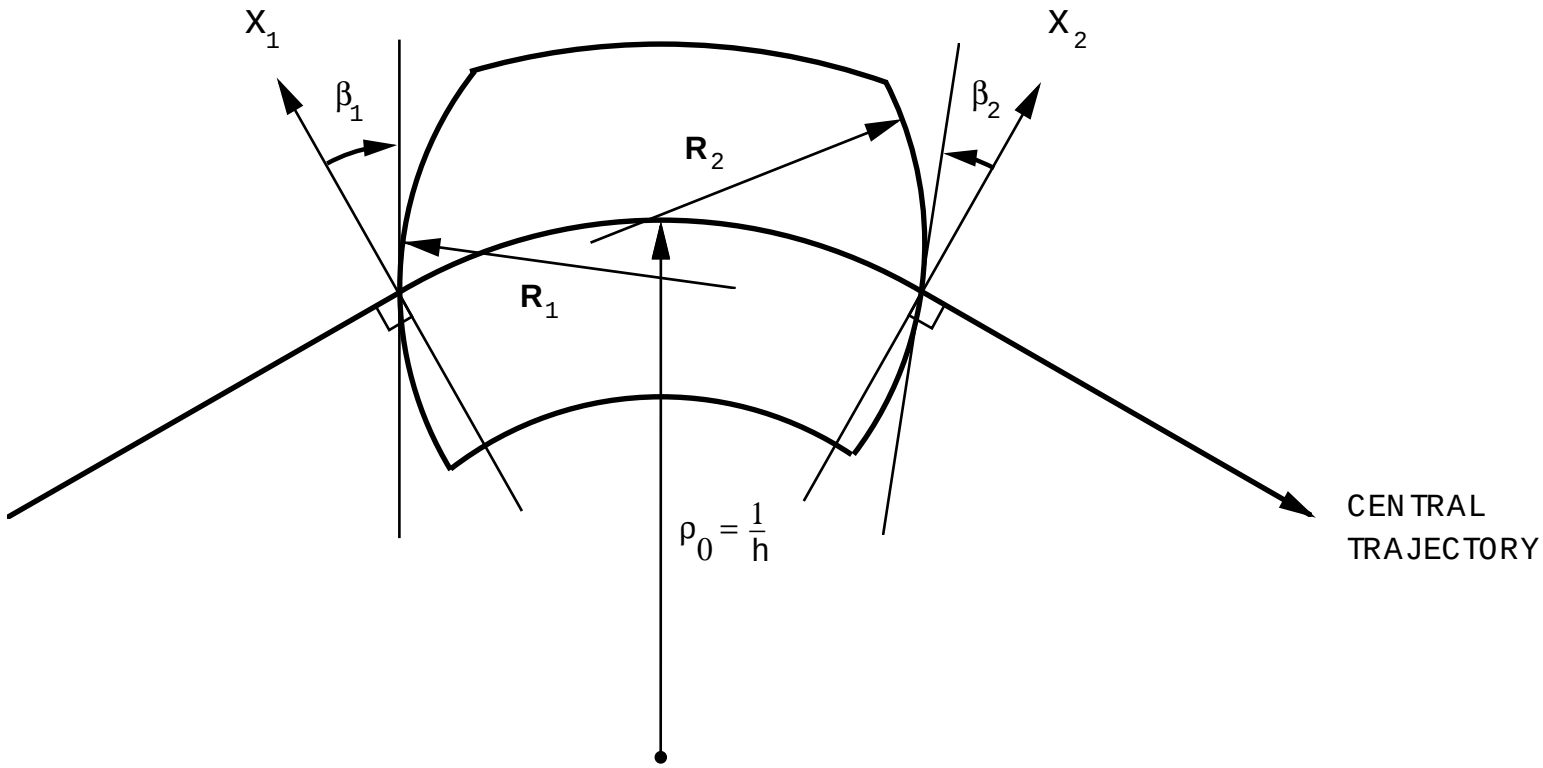
tracing program. A list of such programs is given after the references at the end of this manual.

Pole-Face Curvature

$1/R_1$	`H1`	Where R_1 is the radius of curvature (in units of longitudinal length, normally meters) of the entrance face of bending magnets. (See the figure on the following page.)
$1/R_2$	`H2`	Where R_2 is the radius of curvature of the exit face of bending magnets (in units of longitudinal length, normally meters). (See the figure on the following page.)

The pole face curvatures $(1/R_1)$ and $(1/R_2)$ affect the system only in second or higher order, creating an effective sextupole component in the neighborhood of the magnet. If the parameters are not specified, they are assumed to be zero, i.e. no curvature and hence no sextupole component. The pole-face curvature may also be specified directly on the `ROTATION` element, indicating a pole-face rotation for a bending magnet, or on the `RBEND` or `SBEND` elements, which specify a bending magnet with fringing fields.

Either parameter (or both) may be varied in second- or higher-order fitting.



Field Boundaries for Bending Magnets

The TRANSPORT sign conventions for x, β, R and h are all positive as shown in the figure. The positive y direction is out of the paper. Positive β's imply transverse focusing. Positive R's (convex curvatures) represent negative sextupole components of strength $S = (-h/2R)\ sec^3\beta$. (See SLAC-75 [4], page 71 or SLAC-PUB-3381 [11], page 22.)

Second-Order Term in the Midplane Field Expansion of Bending Magnets

$\epsilon(1)$ `EPS` A second-order measure of magnetic field inhomogeneity in bending magnets. If

$$B(x) = B(0)[1 - n(x/\rho_0) + \beta(x/\rho_0)^2 - \ldots]$$

is the field expansion in the median (y = 0) plane, then $\epsilon(1)$ is defined as

$$\epsilon(1) = \beta(1/\rho_0)^2$$

(where ρ_0 is measured in unit of horizontal beam width – millimeters in `UMM`, meters in `UMETER`–for other units choices see pages 64 and 68). The ϵ parameter may also be specified directly on the bending magnet element. This parameter affects second- and higher-order calculations only. The TRANSPORT default value for $\epsilon(1)$ is 0. It may be varied in second- or third-order fitting.

Third-Order Term

ϵ_3 `EPS3` Cubic term for the field of a bending magnet. The quantity ϵ_3 is equal to γh^3. The units are the inverse cubed of the horizontal beam width unit.

Other Magnetic Field Components

The previous description of a magnetic field has been based on the two assumptions of the existence of a reference orbit and of midplane symmetry. To allow the representation of field errors and deliberate violations of these assumptions, additional field components are introduced. In terms of them, the field expansion on a reference plane (nominally the magnetic midplane) is given by

$$B_y = B_o(1 + r_s)(1 - nhx + \beta\ h^2x^2 + \gamma\ h^3x^3 + \ldots)$$

$$B_x = B_o r_a(v_r - n'hx + \beta' h^2x^2 + ...)$$

The additional parameters required can be specified by the following SPECIAL elements:

r_s	`RMPS`	The fractional excess bend field on a bending magnet. This excess field will not change the reference trajectory, but simply steer the beam about the reference trajectory.
r_a	`RNMS`	Overall scaling factor for the midplane-symmetry-violating field components of a bending magnet.
v_r	`VR`	The vertical (midplane-symmetry-violating) bend field of a bending magnet.
n'	`NP`	The midplane-symmetry-violating normalized gradient of a bending magnet.
ϵ'	`EPSP`	The midplane-symmetry-violating sextupole component of a bending magnet. The quantity ϵ' is equal to $\beta' h^2$. The units are the inverse square of the horizontal beam width unit. The horizontal beam width unit is millimeters with `UMM` or meters with `UMETER`. Other units selections are described on pages 64 and 68.

These parameters all may be specified directly on the `BEND` element, indicating the central body of a bending magnet, or on the `RBEND` or `SBEND` elements, which specify a bending magnet with fringing fields. The preferable use of the special parameter specification is to alter the default values of zero.

ORDER – Higher-Order Calculations

The parameter or parameters associated with the `ORDER` element are essentially integers. The keywords are simply for consistency with the rest of the data.

As an example, a second-order run could be specified by the insertion of the element

```
ORDER,  N = 2.    ;
```

near the beginning of the data, after all `UNIT` elements and before the `BEAM` element. The default is first order if the order element is not present. The default is second order if the `ORDER` element is present with no keywords.

Notation

Three keywords may be specified:

Symbol	Keyword	Description
n	`N`	The order n of the expansion about the reference trajectory in each element.
n_1	`N1`	The order n_1 of the expansion about the reference trajectory in each element. The two keywords, N and N1 are identical in meaning. If there is only one order parameter to be specified, it is more natural simply to call it N. If there are two, then it is more natural to label them N1 and N2.
n_2	`N2`	The order n_2 of the expansion about the beam centroid that is calculated, accumulated, and displayed. It is necessary that $n_1 \geq n_2$. If unspecified, then n_2 is set equal to n_1.

An equivalent specification, but one which shows the two parameters explicitly would be

```
ORDER,  N1 = 2.   N2 = 2.    ;
```

If the beam is always on axis, i.e. if the beam centroid always follows the reference trajectory, it is not meaningful to have separate numbers. In that case, if two numbers are given, the first is reset to equal the second.

If the beam is off axis, then higher-order matrix elements about the beam line axis may contribute to lower-order matrix elements in an expansion about the off-axis centroid position. Some explanation is found later in this section under **Usage**. For more details the user should refer to the explanation of the off-axis expansion of matrix elements on page 11.

The first number n_1 represents the order to which the transfer matrices are calculated about the reference trajectory in each element. If the centroid is off axis, the origin of the expansion can then be shifted from the reference trajectory to the beam centroid. The second number n_2 represents the order to which the transfer matrices are calculated and displayed in the expansion about the beam centroid. Thus if the second-order matrices are calculated about the reference trajectory, but only the first order expansion about the beam centroid is printed in the output, then the `ORDER` specification would be

```
ORDER,  N1 = 2.,  N2 = 1.   ;
```

If the second of the two order parameters is omitted, it is taken as equal to the first. If both are omitted but the `ORDER` element is included in the data, they are both taken to equal two.

Usage

If no `ORDER` element is present, then TRANSPORT calculates the transfer matrix only to first order. The `ORDER` specification allows the order of the calculation to be changed to zeroeth, second, or third. Calculating to zeroeth order simply means tracing the beam centroid through the system. The linearization about the centroid, or transfer matrix, is not calculated.

The tracing of the beam centroid has meaning only if it is not coincident with the reference trajectory. The beam centroid is displaced from the beam line reference trajectory when there is:

1. A centroid displacement (`CENTROID`)
2. A known misalignment (`ALIGN`)
3. An excess horizontal bending field (special parameter `RMPS`)
4. A midplane-symmetry-violating vertical component in a horizontally bending magnet (Special parameters `RNMS` and `VR`)
5. A horizontally and/or vertically steering magnet (`KICKER` or `HKICK` or `VKICK`)
6. An electrostatic septum (`SEPTUM`).
7. A reference coordinate system shift (`SHIFT`).

If the beam is off axis, the transfer matrix as printed will be expanded about the off-axis beam centroid. The displacement of the beam centroid from the reference trajectory will be printed with the first-order transfer matrix. An additional column will appear to the right of the transfer matrix and will contain the coordinates of the beam centroid. It will appear as follows:

$$
\begin{array}{cccccc}
(x|x_0) & (x|x'_0) & (x|y_0) & (x|y'_0) & (x|\ell) & (x|\delta) \\
(x'|x_0) & (x'|x'_0) & (x'|y_0) & (x'|y'_0) & (x'|\ell) & (x'|\delta) \\
(y|x_0) & (y|x'_0) & (y|y_0) & (y|y'_0) & (y|\ell) & (y|\delta) \\
(y'|x_0) & (y'|x'_0) & (y'|y_0) & (y'|y'_0) & (y'|\ell) & (y'|\delta) \\
(\ell|x_0) & (\ell|x'_0) & (\ell|y_0) & (\ell|y'_0) & (\ell|\ell) & (\ell|\delta) \\
(\delta|x_0) & (\delta|x'_0) & (\delta|y_0) & (\delta|y'_0) & (\delta|\ell) & (\delta|\delta)
\end{array}
\qquad
\begin{array}{c}
x_c \\ x'_c \\ y_c \\ y'_c \\ \ell_c \\ \delta_c
\end{array}
$$

The order of matrix which is calculated may differ from that printed. A second-order transfer matrix can contribute to a first-order off-axis expansion. For example, the off-axis traversing of a quadrupole can give a dipole component and therefore cause dispersion. Similarly, third-order terms can contribute to both first and second order. The order of the calculation must be at least as great as that displayed. The mathematical details of calculating the off-axis expansion are given on page 11.

An off-axis beam through a multipole element of a given order will induce additional multipole components of lower order, but never of higher order. Thus, the off-axis traversing of a sextupole will induce both dipole and quadrupole components. Similarly, the off-axis traversing of an octupole will induce dipole, quadrupole, and sextupole components.

If the centroid is displaced horizontally in a midplane symmetric (normal) element, then all the lower-order induced multipoles will also be midplane symmetric. If the centroid is displaced vertically, the induced component of order one less than the original multipole will be skew. The induced multipoles will alternate in order with being midplane symmetric and midplane antisymmetric (skew). Thus a vertically displaced centroid in a normal quadrupole will induce a vertically bending (skew) dipole component.

A vertically displaced centroid in a normal sextupole will induce a horizontally bending (normal) dipole component, and a skew quadrupole component. A skew quadrupole is one which has been rotated about its axis by 45 degrees from the normal configuration. A vertically displaced centroid in a normal octupole will induce a vertically bending dipole component, a normal quadrupole component, and a skew sextupole component. A skew sextupole is one which has been rotated about its axis by 30 degrees from the normal configuration.

Skew components result in coupling between the two transverse coordinates. A skew quadrupole results in first order coupling in x and y. A skew sextupole causes coupling but in second and higher orders. A skew octupole results in coupling in third and higher orders.

Examples

Let us examine first the case of an on-axis beam. A single number may then be given. It specifies the order of the calculation. If this number is absent, it is assumed to equal two. The `ORDER` card then reverts to the original TRANSPORT usage of specifying second order. The original TRANSPORT element

```
ORDER   ;
```

is equivalent to

```
ORDER,  N = 2.   ;
```

Second-order matrices are included in the program for combined-function bending magnets (including fringing fields), quadrupoles, sextupoles, solenoids, and the arbitrary matrix. They have not been calculated for the acceleration (`ACCEL`) element.

A third-order calculation is specified by setting the number on the `ORDER` element to three. The command for a third-order run is then

```
ORDER,  N = 3.   ;
```

Third-order matrix elements are now included for quadrupoles (including fringing fields), sextupoles, octupoles, and combined function bending magnets (also including fringing fields).

If the beam is off axis, two numbers are required for an order specification. The first gives the order to which the transfer matrix is calculated internally in the program. The second gives the order to which the transfer matrix is printed. When the reference point for the expansion is shifted by having the beam centroid off axis, the higher orders can contribute to the lower orders about the new reference. For example, if in the data we include the element

```
ORDER,  N1 = 3,  N2 = 1  ;
```

then the first order transfer matrix will be calculated about the off-axis reference and printed. The second- and third-order matrices about the original reference will be used in calculating the off axis expansion.

This new feature brings up another possibility. It is now possible to calculate the positions of the beam centroid alone without also calculating a linear expansion about the centroid. One is then calculating the progress of a single trajectory through the system. The transfer matrices may be calculated internally about the reference trajectory, but are not printed in the expansion about the beam centroid.

In other words, if we insert the element

```
ORDER,  N1 = 3.,  N2 = 0.   ;
```

then the progress of the centroid through the beam line is calculated to third order. Ordinarily, when the beam centroid is off axis, the transfer matrix is printed with a column to the right of it giving the coordinates of the beam centroid. If the second of the order parameters is set to zero, then at any location in the output where the transfer matrix would ordinarily be printed, only the coordinates of the centroid are printed, No transfer matrix appears.

The element

```
ORDER,  N1 = 3.,  N2 = 2.   ;
```

will cause the transfer matrix to be printed to second order, but with third-order components used in calculating the off axis expansion.

To calculate the centroid position alone using first-order transfer matrices the command

```
ORDER,  N1 = 1.,  N2 = 0.   ;
```

is used.

Results

To print out the second-order T1 matrix terms at a given location in the system, the (`PRINT, TRANS ;`) print control card is used. For T2, the (`PRINT, TRAN2 ;`) print control card is used. The update rules are the same as those for the corresponding first-order R matrix.* The third-order matrix elements $U_{ijk\ell}$ are defined similarly, but with an additional index.

The second- and third-order transfer matrices will also affect the beam ellipse. *For the beam matrix to be calculated correctly, there should be no elements, other than the initial beam specification, which update the R2 matrix.* The complete list of elements which update the R2 matrix is:

1. A beam (`BEAM`) element.
2. An R1 update (`UPDATE, R1`).
3. An R2 update (`UPDATE, R2`).
4. An unknown misalignment element (`ALIGN`).
5. A stray field type code `21.0` entry. (This element is not currently operative. There will be no description of it elsewhere in this manual.)

The dimensions of the initial ellipse are assumed to represent the moments of a Gaussian distribution. †

In calculating the effect of the beam line on the phase ellipse, the sensitivity to assumptions about the initial distribution becomes greater with each additional order. The

*See SLAC-75 [4] for definitions of subscripts in the second order T_{ijk} matrix elements.

†For exact details the reader should consult the Appendix (available separately or as part of CERN Report 80-04)

dimensions of a third-order beam ellipsoid should not be taken as being physically accurate. Rather they should be regarded as an order-of-magnitude estimate of the combined effect of all the aberrations. The dimensions of the beam ellipsoid can be used to assess whether a third-order correction scheme is necessary. Details of the phase-space distribution should be determined with a ray-tracing computer program such as TURTLE (see refs).

RANDOM – Random Errors on Physical Parameters

Using a random number generator, random errors may be placed on the physical parameters used to describe a beam line. Any type of physical parameter can be given an error which falls within a specified range. The type of physical parameter and the magnitude of the error are both specified on the `RANDOM` element. There is only keyword notation for this element.

This element differs from most others in that the keywords must be in a certain order. The element begins with the mnemonic `RANDOM`. The first keyword indicates the type of element on which the random error is imposed. It is an element type, such as `QUAD` or `BEND`.

The second keyword indicates the physical parameter to which the error is applied. The final keyword is the word `VARIATION`, which may be shortened to `VARI`. It is used to indicate the size of the random error on the indicated parameter.

For example, a random error in quadrupole excitation might be represented by the element

```
RANDOM,  QUAD,  B,  VARI = .001  ;
```

Such an element will cause the placement of a random error of maximum magnitude .001 kilogauss on the pole tip field of the quadrupoles in the system. All quadrupoles will be affected, and the random errors will be independent.

For an error to be applied to an element, the element specification must be written in terms of the parameters for which an error magnitude is given. If a quadrupole is specified as

```
QUAD,  L = 10.,  GRAD = 5.   ;
```

then the parameter `B` does not appear. The error specification given above in terms of `B` then introduces no error to the gradient of this quadrupole. An error on the gradient of a quadrupole where the gradient is given directly, as above, can be specified by

```
RANDOM,  QUAD,  GRAD,  VARI = .001  ;
```

At present the list of elements and their parameters to which random errors can be applied is as follows.

Element	Parameter Keywords
ROTAT	ANGLE, HGAP, FINT, H
DRIFT	L
BEND	L, B, RADIUS, ANGLE, N, K1, RMPS, RNMS, VR, NP, K1P, EPS, K2, EPSP, K2P, EPS3, K3
QUAD	L, B, APERTURE, GRADIENT, K1
CENTROID	X, XP, Y, YP, L, DEL
ACCELERA	L, VOLT, LAG, FREQ
SEXTUPOL	L, B, APERTURE, K2
SOLENOID	L, B, KS
SROT	ANGLE
OCTUPOLE	L, B, APERTURE, K3
RBEND, SBEND	L, B, RADIUS, ANGLE, N, K1, RMPS, RNMS, VR, NP, K1P, E1, E2, EPS, K2, EPSP, K2P, H1, H2, EPS3, K3, HGAP, FINT
PLASMA	L, B, APERTURE, GRADIENT, K1

If any physical parameter is to receive a random error, then random numbers for all physical parameters are generated. If a second type of error is introduced, the random numbers used for the first type will then be unchanged. This procedure produces the type of results that a user would be likely to expect. If two types of errors are present, and the magnitude of the second is reduced to zero, the result should be the same as if only the first type of error were present. An example might be an investigation of the errors of focusing elements in a system. If both quadrupoles and bending magnets are to be considered then the two error specifications

```
RANDOM,  QUAD,  B,  VARI = .001  ;
RANDOM,  BEND,  N,  VARI = .0005  ;
```

could be used.

The procedure described has the disadvantage that the sequence of random numbers used for any given type of error will be changed if the beam line is changed in any way. This will be true even if the change is to an element for which no random errors are specified. An example might be the splitting up of a drift space to make two shorter drift spaces. If this is done, the beam line is considered changed. The error specification given above, where a random error is given to the magnetic fields of the quadrupoles in the system will then give a different result. The correspondence of random numbers to magnetic field values will be altered due to the generation of an additional random number for the additional drift space. It will also be possible that different versions of TRANSPORT will give different results if

additional physical parameters can be given random errors. The special parameters element does not use random numbers, since it simply gives default values for the parameters of other elements.

In most cases the mathematical procedure of giving a certain error to a physical parameter is the same as the physical procedure. By the mathematical procedure, we mean simply changing the physical parameter and considering the beam line to be whatever results. By the physical procedure, we mean imposing certain constraints whereby the change in one parameter will cause a change in others resulting in a more physically realistic situation.

There are two cases where the two procedures are significantly different. The first is in the imposition of an error on the length of a magnet. An error may be specified as

```
RANDOM,  BEND,  L,  VARI = 0.02 ;
```

The mathematical error is the procedure currently in TRANSPORT, Here the length of a beam line will increase when the length of magnet is increased. The physical error will correspondingly shorten the adjacent drifts so that the length of the total beam line is unchanged. It is planned to change TRANSPORT in the future so that the physical error is calculated.

The second exception is the case of an error on the field of a bending magnet. An error may be specified by

```
RANDOM,  BEND,  B,  VARI = 0.005  ;
```

The angle of bend is calculated from the magnet length and field and the reference momentum. If the field is changed, the remainder of the beam line is moved. However, in this case, there is provision for introducing an excess field without altering the remainder of the beam line. The excess field is specified using the keyword `RMPS` either in the `BEND` element or with a special parameter. The parameter `RMPS` is the fractional error in the magnetic field. It may be considered as a mispowering of the magnet without affecting the beam layout. An error on this parameter may then by given by

```
RANDOM,  BEND,  RMPS,  VARI = 0.0001 ;
```

The initial value of the random number generator may be reset using the special parameter element `SPECIAL` with the keyword `RANNO`. If the appropriate `SPECIAL` element is included in the data, but the space for the initial value of the random number generator is left blank, then the initial value will be reset by the computer clock. Detailed description is given under the `SPECIAL` element.

LIMIT – Limits on the Range of Varied Physical Parameters

Limits can be placed on the range over which a physical parameter may vary in a fitting procedure. The type of physical parameter, the nature of the limit, and the numerical value of the limit are all specified on the `LIMIT` element. By "the nature of the element" we mean whether the limit is lower or upper or on the absolute value of the parameter. This element is expressed in keyword notation, but unlike physical elements like `QUAD`, the order of the keywords is significant.

For example, a limit on the strength of the pole-tip magnetic field of a quadrupole might be represented by the element

```
LIMIT,  QUAD,  B,  UPPER = 15.0  ;
```

An upper limit of 15 kG is then placed on the pole-tip magnetic field of all quadrupoles whose fields are varied. It should be noted that this limit does not apply to the magnitude of the field. A negative field can still attain an arbitrarily large magnitude. To bound fields both negatively and positively, both lower and upper limit specifications are required. If one replaces the keyword `UPPER` by `ABSOLUTE`, then the upper limit will be retained and a lower limit of -15.0 will also be imposed.

The first keyword, in this case `QUAD`, gives the type of element on which the limited parameter occurs. The second gives the physical parameter to which the limit is to be applied. The third entry is either of the two words `LOWER`, `UPPER`, or `ABSOLUTE` giving the value of the limit.

For a limit to be applied to an element, the element specification must be written in terms of the parameter for which the limit is given. If a quadruopole is specified as

```
QUAD,  L = 10.,  B = 12.,  APER = 5.0  ;
```

and the parameter `B` is varied, the limit specified above will apply. However, if a quadrupole is specified as

```
QUAD,  L = 10.,  GRAD = 5.   ;
```

then the parameter `B` does not appear, and a limit given in terms of `B` will have no relevance. The limit will also not apply if the physical parameter is evaluated in terms of other parameters which, in turn, are varied.

At present the list of elements and their parameters to which limits can be applied in the fitting process is as follows.

Element	Parameter Keywords
BEAM	all parameters
ROTAT	ANGLE
DRIFT	L
BEND	L, B, RADIUS, ANGLE, N, K1
QUAD	L, B, GRADIENT, K1
CORRELATION	all parameters
SEXTUPOLE	L, B, K2
SOLENOID	L, B, KS
SROT	ANGLE
OCTUPOLE	L, B, K3
RBEND, SBEND	L, B, RADIUS, ANGLE, N, K1, E1, E2
PLASMA	L, B, APERTURE, GRADIENT, K1
HKICK, VKICK	L, B, ANGLE

If no limitations are specified on the ranges of any parameters, then TRANSPORT will impose limits only on a very small number of possibly varied parameters. All parameters on the BEAM element are required to be positive. There is no internally specified upper limit. Limits may be placed on the different beam dimensions only if the beam is expressed in σ matrix notation.

The pole-face-rotation angle on the ROTATE element is internally constrained to have a magnitude no greater than 60 degrees. This is simply a practical consideration of magnet geometry. This internal limit may be overridden by the LIMIT element. The user is reminded that upper and lower limits must be specified separately.

The lengths of all physical magnets and drift spaces are internally constrained never to become negative. A fixed length may be negative, but a negative length can never result from a fitting procedure. Elements to which the negative length restriction applies are the DRIFT, BEND, QUAD, SOLENOID, OCTUPOLE, RBEND, SBEND, PLASMALENS, HKICK, VKICK, and KICKER. In the case of a drift space one way around this obstacle is to place a fixed drift of negative length next to a varied drift of positive length.

The zero minimum length on any physical element may also be overridden by the LIMIT element. Thus the fitting procedure can be allowed to produce negative lengths by the insertion of the element:

```
LIMIT,  DRIFT,  L,  LOWER = -1000.0 ;
```

The lower limit to the lengths of any varied drift now becomes -1000 of the prevailing length unit.

The coordinate rotation angle on the `SROT` element is constrained to lie between -360 and 360 degrees. Here nothing is gained physically by expanding the range.

MAGNET – External Magnet Dimensions

The external dimensions of a magnet may be specified for plotting purposes. The external dimensions are used in a floor layout plot where either the floor x (horizontal transverse) coordinate or floor y (vertical transverse) coordinate is plotted against the floor z (longitudinal) coordinate. If both magnet width and magnet height are specified the magnet is drawn into the plot. If only one or neither transverse magnet dimension is specified, then only the reference trajectory through the magnet is drawn into the plot. The dimensions specified on a magnet element apply to a particular `TYPE` of magnet. By `TYPE` we mean the designation specified under the keyword `TYPE` on the description of the individual magnet, be it `RBEND`, `SBEND`, `QUAD`, or other magnet.

Notation

The `MAGNET` element is specified by three keywords.

Keyword	Description
`WIDTH`	The magnet width (normally meters, default: 0 m)
`HEIGHT`	The magnet height (normally meters, default: 0 m)
`TYPE`	The magnet `TYPE` to which these dimensions pertain

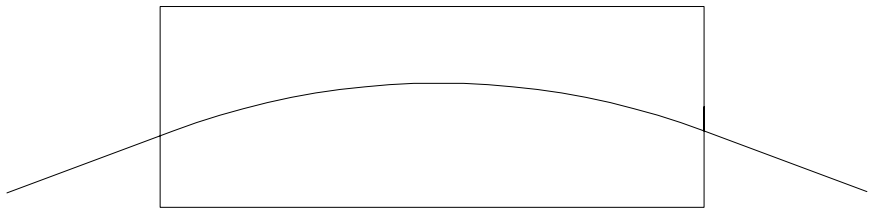
The external dimensions of any magnets of type "`JUMBO`" may be declared by the following `MAGNET` element.

```
MAGNET, WIDTH = 2.0, HEIGHT = 1.5, TYPE = JUMBO ;
```

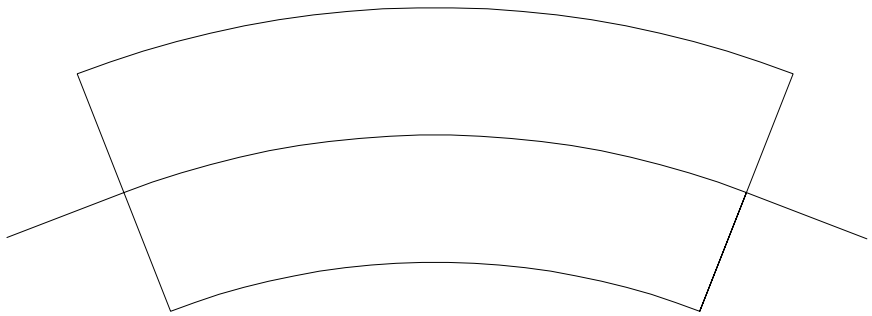
The magnet dimensions will be used to draw the magnet according to the magnet type as specified on the element, `RBEND`, `SBEND`, `QUAD`, or other magnet. For example, a set of `RBEND` magnets may all be declared to be of the general type "`JUMBO`". Several such magnets may appear in a beam line.

```
B1:   RBEND,  L = 10.,  ANGLE = 0.010,  TYPE = JUMBO ;
DRIFT,  L = 1.0 ;
B2:   RBEND,  L = 10.,  ANGLE = 0.010,  TYPE = JUMBO ;
DRIFT,  L = 1.0 ;
B3:   RBEND,  L = 10.,  ANGLE = 0.010,  TYPE = JUMBO ;
DRIFT,  L = 1.0 ;
B4:   RBEND,  L = 10.,  ANGLE = 0.010,  TYPE = JUMBO ;
```

An `RBEND` element as seen from the top will appear as follows:



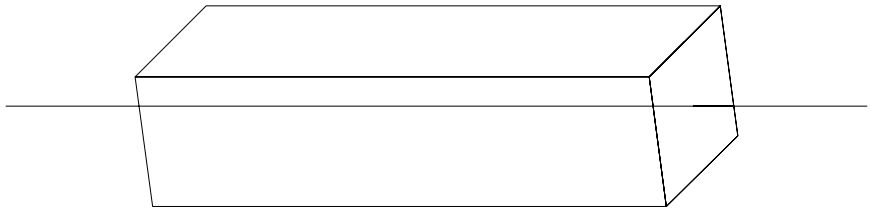
An `SBEND` element from the top will appear as follows:



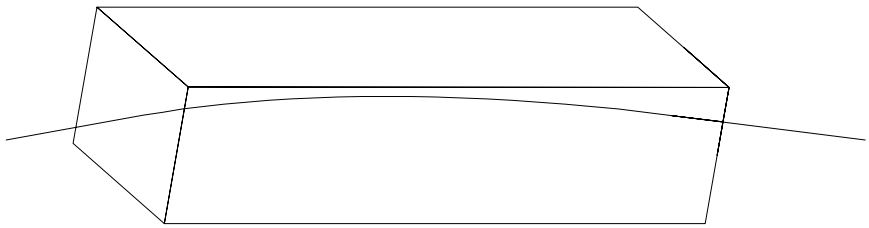
A `QUAD` or other rectilinear magnetic element will appear as follows:

The `RBEND`, `QUAD`, and other rectilinear magnetic elements are represented by rectangular solids. They then appear as rectangles when seen from the side. The `SBEND` is curved to follow the reference trajectory. If the observer is located inside the curvature, the two ends of the magnet, as well as one side, may be visible simultaneously. The `SBEND` element seen from the side will then appear as follows:

A roll (`SROT` element or a `TILT`) in the beam line or an initial slope or azimuthal angle may cause more than one surface of a magnet to be visible from either the side or the top. A `QUAD` or other rectilinear element as seen from an angle might appear as:



An `RBEND` as seen from an angle might appear as:



Description of the Beam

BEAM — Input Beam

The phase space and the reference momentum of the input beam for a TRANSPORT calculation are specified by this element. Usually the `BEAM` card is the fourth card in the deck. It will be immediately preceded by the title card, the indicator card, and the unit set specification. If any individual units changes are to be made, the units elements (`UNIT`) must also precede the `BEAM` celement

If a beam line coordinate layout is desired, the element specifying that a layout is to be made (`PRINT, FLOOR ;`), and any initial coordinates (see page 73) all precede the `BEAM` element. Any `ORDER`, `LIMIT`, `RANDOM` or `MAGNET` elements also precede the `BEAM` element.

For any other than strict MAD notation, a beam element is required. In strict MAD notation a beam element is unnecessary. The magnetic elements in MAD notation are specified in a manner that incorporates the momentum. However the beam element must be present if any of the calculations involves the beam phase space.

The mnemonic used in specifying the input phase space and reference momentum is `BEAM`. The input phase space may be given by specifying its half widths or in accelerator notation. The two may not be mixed in the input. The units associated with the entries depend on the unit set specified.

Sigma Matrix Notation

The input is given in terms of the semi-axes of a six-dimensional erect* beam ellipsoid. The coordinates in the six-dimensional space represent the phase space variables x, x', y, y', ℓ, and δ. Each of these six parameters is entered as a positive quantity, but should be thought of as $\pm x$, $\pm x'$, etc.; hence, the total beam width is $2x$, the total horizontal beam divergence is $2x'$ and so forth.

In keyword notation the `BEAM` element might look like:

```
BEAM,  X=.1,  XP=1.,  Y=.1,  YP=1.,  DL=0.,  DEL=0.2,  P0 = 100.   ;
```

Any of these parameters except `P0` may be omitted and the default value of zero will be used.

*For a rotated (non-erect) phase ellipsoid input, see the `CORR` element description on page 109.

The keywords which can be used are:

Symbol	Keyword	Description
x	`X`	One-half the horizontal beam extent (millimeters with `UMM`, meters with `UMETER`).
x'	`XP`	One-half the horizontal beam divergence (milliradians with `UMM`, radians with `UMETER`).
y	`Y`	One-half the vertical beam extent (mm or meters).
y'	`YP`	One-half the vertical beam divergence (mr or radians).
ℓ	`DL`	One-half the longitudinal beam extent (mm or meters).
δ	`DEL`	One-half the momentum spread (permil with `UMM`, fraction with `UMETER`).
p_o	`P0`	The momentum of the central trajectory (GeV/c).

All quantities not specified on the `BEAM` element will be taken to be zero. The same set of keywords may be used for an rms addition to the beam phase space. Global unit set specifications other than `UMM` or `UMETER` may also be selected and are described on page 64. Individual units changes are described on page 68.

Strictly speaking x' and y' are not angles and are therefore not measurable in either milliradians or radians. They are the tangents of the horizontal and vertical angles θ and ϕ between the given trajectory and the reference trajectory. However, up to and including second order, they are equal to the angles themselves.

In third order, the tangent of an angle is different from the angle. However, the principal effects in third order arise from aberrations which are nonlinear terms in the differential equations of motion. The difference between an angle and its tangent as a measure of beam divergence is still inconsequential. Milliradians as a measure of x' and y' may still be interpretated as parts in a thousand.

Courant-Snyder Notation

The input phase space may alternatively be specified in accelerator notation using the Courant-Snyder parameters α, β, and ϵ. Here the parameters α and β are used only to describe an ellipse. They have no necessary relationship to the transfer matrix of either the entire optical system or to any portion thereof. This means that the beam may or may not be matched to the lattice through which it passes. The accelerator function η is specified on the separate `ETA` element described on page 112.

If the input beam is specified in terms of the Courant-Snyder parameters α and β, then the keywords which can be used are:

Symbol	Keyword	Description
β_x	BETAX	The accelerator function β in the horizontal plane (meters with UMM). The use of meters for a transverse quantity with the unit set UMM may seem odd. However, the function β is really in mm/mr, which is meters.
α_x	ALPHAX	The accelerator function α in the horizontal plane (unitless).
ϵ_x	EPSX	The horizontal emittance (mm-mr with UMM).
β_y	BETAY	The accelerator function β in the vertical plane (meters with UMM).
α_y	ALPHAY	The accelerator function α in the vertical plane (unitless).
ϵ_y	EPSY	The vertical emittance (mm-mr with UMM).
p_o	P0	The momentum of the central trajectory (normally GeV/c).

The default value for the β and α parameters in both planes is zero. The default value for the ϵ parameters in both planes is 1.0. Here the units are given for only the global unit set UMM. It is imperative that the unit set used with accelerator notation be such that the ratio of the transverse linear unit to the transverse angular unit be meters. The global unit set UTRANS does not satisfy this requirement and should not be used with accelerator notation. The global unit sets UMAD, UMETER, UMM, and UMICR are acceptable in this respect. The units for the horizontal and vertical emittance are the transverse linear unit times the transverse angular unit. Thus in UMM the units will be mm-mrad, while in UMICR they will be micron-microradians. The various options for global unit set specifications are described on page 64. Individual units changes are described on page 68.

If accelerator notation is used, the BEAM element might appear as:

```
BEAM,  BETAX=1.0,  ALPHAX=.15,  BETAY=2.0,  ALPHAY=-.20,  P0 = 100.   ;
```

Specification of the emittances also gives a beam element which might be:

```
BEAM,BETAX=1.0,ALPHAX=.15,EPSX=16.,BETAY=2.0,ALPHAY=-.2,EPSY=3., P0=100.;
```

Normally, the units of the horizontal and vertical beta functions are in meters. The accelerator function α is unitless. In TRANSPORT, β is expressed as the ratio of the transverse distance unit over the transverse angular unit. If the transverse distance is meters and the transverse angle is in radians, the the β functions will be in meters. TRANSPORT requires an explicit unit set specification for transverse units of meters and radians. Either of the global units specifications UMAD or UMETER will produce transverse units of meters and radians. The unit sets UMM and UMICR do not produce transverse units of meters and radians, but will

produce β functions in meters.

The transverse units resulting from the specification `UTRANS` are cm and mr. These units would therefore interpret the beta functions as being expressed in cm/mr or dekameters. The desired units of meters will be produced for the beta functions if the transverse units are mm and mr.

Output

The units of the tabulated matrix elements in either the first-order R or sigma matrix, second-order T matrix, or third-order U matrix of a TRANSPORT print-out will correspond to the units chosen for the `BEAM` element. For the unit set `UMM`, the $R_{12} = (x|x')$ matrix element will have the dimensions of mm/mr; and the $T_{236} = (x'|y\delta)$ matrix element will have the dimensions mr/(mm $\cdot$ permil) and so forth.

The longitudinal extent ℓ is useful for pulsed beams. *It indicates the spread in length of particles in a pulse.* For a static magnetic system it does not affect any other component. Only for the accelerator element does the longitudinal position affect the transverse dimensions of the beam. It may be set to zero if the pulse length is not important.

The beam phase-space can be printed at any later location in the beam line. The beam matrix in terms of half widths and correlations may be printed at a single location by the element

```
PRINT,  BEAM  ;
```

The phase ellipse (sigma matrix) beam parameters may be printed as output after every physical element if activated by a

```
PRINT,  BEAM,  ON  ;
```

element. The maximum extent of the ellipsoid in each of the six coordinates axes is printed in a vertical array. The correlations among these components indicating the phase ellipse orientations are printed in a triangular array (see the following pages).

The beam phase space may be printed in terms of the parameters β and α by use of the command

```
PRINT,  TWISS  ;
```

The printing of these parameters after every physical element is activated by the element

```
PRINT,  TWISS,  ON  ;
```

Either or both types of representation of the beam phase space (half widths or β and α) may be used in the same run. The choice of representation for the printed phase space

parameters can be made independently of the choice for the input. In other words, the beam phase space can be input in terms of β, α, and ϵ and then be printed in terms of half widths and correlations.

The Phase Ellipse Beam Matrix

The beam matrix σ carried in the computer has the following construction:

	x	x'	y	y'	ℓ	δ
x	σ_{11}					
x'	σ_{21}	σ_{22}				
y	σ_{31}	σ_{32}	σ_{33}			
y'	σ_{41}	σ_{42}	σ_{43}	σ_{44}		
ℓ	σ_{51}	σ_{52}	σ_{53}	σ_{54}	σ_{55}	
δ	σ_{61}	σ_{62}	σ_{63}	σ_{64}	σ_{65}	σ_{66}

The matrix is symmetric so that only a triangle of elements is needed. In the printed output this matrix has a somewhat different format for ease of interpretation:

			x	x'	y	y'	ℓ
x	$\sqrt{\sigma_{11}}$	MM					
x'	$\sqrt{\sigma_{22}}$	MR	r_{21}				
y	$\sqrt{\sigma_{33}}$	MM	r_{31}	r_{32}			
y'	$\sqrt{\sigma_{44}}$	MR	r_{41}	r_{42}	r_{43}		
ℓ	$\sqrt{\sigma_{55}}$	MM	r_{51}	r_{52}	r_{53}	r_{54}	
δ	$\sqrt{\sigma_{66}}$	PM	r_{61}	r_{62}	r_{63}	r_{64}	r_{65}

where

$$r_{ij} = \frac{\sigma_{ij}}{[\sigma_{ii}\ \sigma_{jj}]^{1/2}} \quad .$$

The quantities r_{ij} are known in statistics as the correlations between the ith and jth coordinates. The units shown above are from the global unit set `UMM`. Those actually shown will be from whatever units are selected.

As a result of the fact that the σ matrix is positive definite, the r_{ij} satisfy the relation

$$|r_{ij}| \leq 1 \quad .$$

The full significance of the σ_{ij} and the r_{ij} is discussed in detail in the appendix ("Description of Beam Matrix"). The units are always printed with the matrix.

In brief, the meaning of the $\sqrt{\sigma_{ii}}$ is as follows:

$\sqrt{\sigma_{11}}$	$= x_{max}$	= the maximum (half)-width of the beam envelope in the x(bend)-plane at the point of the print-out.
$\sqrt{\sigma_{22}}$	$= x'_{max}$	= the maximum (half)-angular divergence of the beam envelope in the x(bend)-plane.
$\sqrt{\sigma_{33}}$	$= y_{max}$	= the maximum (half)-height of the beam envelope.
$\sqrt{\sigma_{44}}$	$= y'_{max}$	= the maximum (half)-angular divergence of the beam envelope in the y(non-bend)-plane.
$\sqrt{\sigma_{55}}$	$= \ell_{max}$	= one-half the longitudinal extent of the bunch of particles.
$\sqrt{\sigma_{66}}$	$= \delta_{max}$	= the half-width ($\Delta p/p_o$) of the momentum interval being transmitted by the system.

The units appearing next to the $\sqrt{\sigma_{ii}}$ in the TRANSPORT print-out are the units chosen for coordinates $x,\ x',\ y,\ y',\ \ell$ and $\delta = \Delta p/p$, respectively.

Beam Center of Gravity

To the immediate left of the listing of the beam envelope size in a TRANSPORT print-out, there appears a column of numbers whose values will normally be zero. They are here indicated with a subscript "g".

			x	x'	y	y'	ℓ
x_g	$\sqrt{\sigma_{11}}$	MM					
x'_g	$\sqrt{\sigma_{22}}$	MR	r_{21}				
y_g	$\sqrt{\sigma_{33}}$	MM	r_{31}	r_{32}			
y'_g	$\sqrt{\sigma_{44}}$	MR	r_{41}	r_{42}	r_{43}		
ℓ_g	$\sqrt{\sigma_{55}}$	MM	r_{51}	r_{52}	r_{53}	r_{54}	
δ_g	$\sqrt{\sigma_{66}}$	PM	r_{61}	r_{62}	r_{63}	r_{64}	r_{65}

These numbers are the coordinates of the center of gravity of the beam phase ellipse (with respect to the initially assumed reference trajectory of the system). They may become nonzero under any of five circumstances:

1. when a steering magnet is used (`KICKER`, `HKICK`, or `VKICK`),
2. when a bending magnet (`RBEND`, `SBEND`, or `BEND`) is mistuned,
3. when the misalignment (`ALIGN`) is used,
4. when a beam centroid shift (`CENTROID`) or coordinate shift (`SHIFT`) is used, or
5. when a second- or higher-order calculation (`ORDER`) is used.

Great care must be taken in the interpretation of these numbers in a second- or higher-order run with nonzero initial phase space. As the beam envelope progresses through the

beam line, it may become distorted by the higher-order terms and may no longer be an ellipsoid. The number printed is the center of gravity of the envelope and may no longer coincide with the image of the original centroid, transformed from the beginning of the beam line. The numbers printed with the transfer matrix <u>are</u> the coordinates of the image of the original centroid. They appear only when the centroid deviates from the reference trajectory and are given the subscript "c" in the following diagram.

$$\begin{array}{cccccccc}
R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} & & x_c \\
R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} & & x'_c \\
R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} & & y_c \\
R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} & & y'_c \\
R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} & & \ell_c \\
R_{61} & R_{62} & R_{63} & R_{64} & R_{65} & R_{66} & & \delta_c
\end{array}$$

The Courant-Snyder Parameters

If the two transverse planes are uncoupled, and the momentum spread of the beam ($\Delta p/p$ or δ) is zero, then the beam phase space can be expressed in terms of the Courant-Snyder Parameters β_x, β_y, α_x, α_y, ϵ_x, and ϵ_y. Two additional parameters γ_x and γ_y are used to complete the phase-space matrix. The γ parameters are not independent of β and α, but in each plane are given by:

$$\beta\gamma - \alpha^2 = 1$$

The phase space area of the beam in the horizontal plane is $\pi\epsilon_x$, and that in the vertical plane is $\pi\epsilon_y$. The maximum extent of the phase ellipse in transverse position and angle is given by:

$$x_{max} = \sqrt{\beta_x \epsilon_x} \quad , \quad x'_{max} = \sqrt{\gamma_x \epsilon_x}$$
$$y_{max} = \sqrt{\beta_y \epsilon_y} \quad , \quad y'_{max} = \sqrt{\gamma_y \epsilon_y}$$

The correlation between transverse position and angle is given by the α parameters. The relation between the α parameters and the σ matrix is then:

$$\sigma_{21} = \sqrt{\alpha_x \epsilon_x} \quad , \quad \sigma_{43} = \sqrt{\alpha_y \epsilon_y}$$

The phase advance ψ in each transverse plane is given by

$$\psi = \int \frac{ds}{\beta}$$

If the Courant-Snyder parameters are printed in the TRANSPORT after each element, then a heading is printed at the beginning of the beam line. The heading identifies the columns in which the Courant-Snyder parameters are printed and looks like:

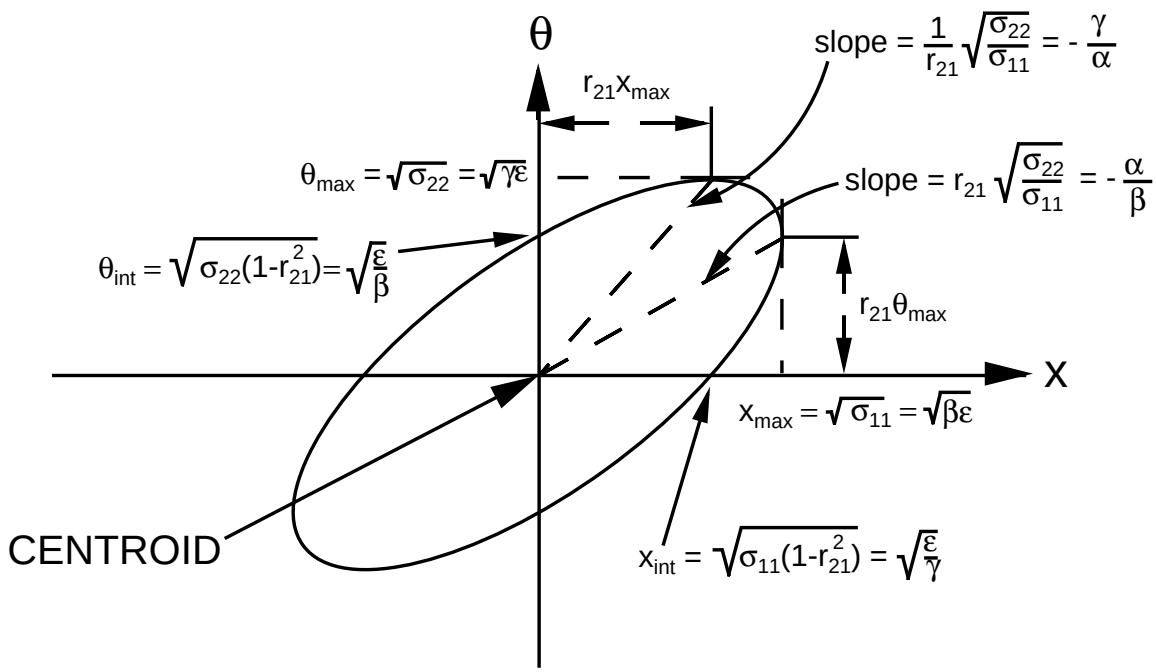
PSIX PSIY BETAX BETAY ALPHAX ALPHAY ETAX ETAY DETAX DETAY

The columns themselves consist of a single row after each physical element, containing the following quantities:

ψ_x ψ_y β_x β_y α_x α_y η_x η_y η'_x η'_y

Example

To aid in the interpretation of the phase ellipse parameters listed above, an example of an (x, x') plane ellipse is illustrated below. †



A Two-Dimensional Beam Phase Ellipse

The area of the ellipse is given by:

$$A = \pi(\det \sigma)^{1/2} = \pi x_{max}\, x'_{int} = \pi x_{int}\, x'_{max} = \pi\epsilon \quad .$$

The equation of the ellipse is:

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \epsilon$$

where

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \epsilon \begin{bmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{bmatrix}$$

and

$$\beta\gamma - \alpha^2 = 1 \quad , \quad r_{21} = r_{12} = -\frac{\alpha}{\sqrt{1+\alpha^2}} = -\frac{\alpha}{\sqrt{\beta\gamma}} \quad .$$

†For further details the reader should refer to the appendix of this report or SLAC-PUB-3381 [11].

R.M.S. Addition to the Beam

To allow for physical phenomena such as multiple scattering and synchrotron radiation emittance growth, provision has been made in the program to permit an r.m.s. addition to the beam envelope.

Notation

The mnemonic used in specifying the r.m.s. addition to the beam is `BEAM`. The r.m.s. addition to the beam is specified in terms of its phase space half-widths. Then the keywords which can be used are:

Symbol	Keyword	Description
x	`X`	One-half the horizontal beam extent (mm in `UMM`, meters with `UMETER`).
x'	`XP`	One-half the horizontal beam divergence (mr with `UMM`, radians with `UMETER`).
y	`Y`	One-half the vertical beam extent (mm or meters).
y'	`YP`	One-half the vertical beam divergence (mr or radians).
ℓ	`DL`	One-half the longitudinal beam extent (mm or meters).
δ	`DEL`	One-half the momentum spread (permil with `UMM`, fractional with `UMETER`).
p_o	`P0`	The momentum change $[\Delta p(0)]$ of the central trajectory (GeV/c).
	`RMS`	Indicates an r.m.s. addition to the beam.

The `BEAM` element might then look like:

```
BEAM,  X=0.,  XP=.2,  Y=0.,  YP=.2,  DL=0.,  DEL=0.2,  P0 = -0.1,  RMS  ;
```

where the keyword (`RMS`) preceding the semicolon signifies an r.m.s. addition to the `BEAM` and the next to the last entry indicates a central momentum change of -0.1 GeV/c.

All quantities not specified on the `BEAM` element will be taken to be zero. Global unit set specifications other than `UMM` or `UMETER` may also be selected and are described on page 64. Individual units changes are described on page 68.

CORR(ELATION) — Beam (Rotated Ellipse)

To allow the output beam from some point in a system to become the input beam of some succeeding system, provision has been made for re-entering the correlation matrix which appears as a triangular matrix in the beam output. (See page 103 for definitions.)

`CORR` is not a MAD element. However, it can be expressed in keyword notation.

Notation

The keywords which can be used in specifying the beam matrix correlations are

`C21`, `C31`, `C32`, `C41`, `C42`, `C43`, `C51`, `C52`, `C53`, `C54`, `C61`, `C62`, `C63`, `C64`, `C65`.

The keywords `Cij` correspond in an obvious manner to the l5 correlations r_{ij} among the 6 beam components. The value assigned to `Cij` is just r_{ij}. The correlations are all dimensionless. In terms of the elements of the σ matrix, the correlations are given by:

$$r_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii}}\sqrt{\sigma_{jj}}}$$

Only the nonzero correlations need be typed. It is often the case that the only nonzero correlations are r_{21} and r_{43}. To specify the values of these correlations, the `CORR` element might look like

```
CORR,  C21 = 0.6,  C43 = -0.8  ;
```

The correlation element often requires several lines. For compatability with the MAD program an implicit semicolon is now assumed to be in column 81 at the end of each line. A line must now be explicitly marked to be continued. This is done by placing an ampersand (&) after the data on a given line.

Usage

Since this element is solely an extension of the beam input, a `CORR` element must immediately be preceded by a `BEAM` element. Correlations must not be specified if the beam phase space is specified in terms of Courant-Snyder parameters. The Courant-Snyder α parameters are already an expression of the correlations.

The effect of this element in the printed output is shown only in the beam matrix. If the beam matrix is printed automatically, it is not printed directly after the `BEAM` element but only after the correlation matrix has been inserted.

CENTROID — Shift in the Beam Centroid

The initial beam centroid is the geometric center of the initial beam ellipsoid. The beam centroid is then an individual trajectory whose progress can be followed through the beam line. In first order, the beam ellipsoid retains an ellipsoidal shape when transformed to a downstream location in the beam line. The centroid then continues to be the geometric center of the beam ellipsoid.

In higher orders the transformed beam ellipsoid becomes distorted and may no longer retain an ellipsoidal shape. The beam centroid is still defined as the transform of the center of the initial beam ellipsoid. Because of the higher-order distortions, the beam centroid may no longer represent the mean position of the particles in the transformed ellipsoid. For that purpose, the beam centroid is replaced by the beam center of gravity, described on page 43.

In TRANSPORT the beam centroid is used as a reference point for the Taylor series expansion of the mapping from the beginning of the beam line to any later point. The coefficients of the Taylor series expansion are known as the transfer matrices. If the beam centroid coincides with the reference trajectory of the optical system, then the expansion about the reference trajectory and that about the beam centroid are the same.

Sometimes it is useful to redefine the beam centroid such that it does not coincide with the TRANSPORT reference trajectory. Provision has been made for this possibility via the `CENTROID` element.

`CENTROID` is not a MAD element, but it may be expressed in keyword notation.

Notation

The mnemonic used in specifying a shift in the beam centroid is `CENTROID`. The keywords which can be used are

Symbol	Keyword	Description
x	`X`	The shift in the x coordinate
x'	`XP`	The shift in x'
y	`Y`	The shift in y
y'	`YP`	The shift in y'
ℓ	`DL`	The shift in ℓ.
δ	`DEL`	The shift in δ.

The units are the same as that used for the beam ellipse. (See page 99.) In the `UMM` unit set, the units are mm, mr, and permil (parts in a thousand). In the `UMETER` set, they are meters, radians, and fractional measure. Other global unit sets are described on page 64.

An example of an increase in y' by 1.0 milliradians might be

```
CENTROID,  YP = 1.0   ; Here the global unit set is assumed to be UMM.
```

Any or all of the six beam centroid shift parameters may be varied in first-order fitting. The centroid position may then be constrained at any later point in the beam line by this procedure.

Usage

The presence of a centroid shift element in the data will automatically cause the transfer matrix to be expanded about the newly defined beam centroid. Both the order used in calculating the transfer matrix, and the order to which it is expanded about the beam centroid are specified by the `ORDER` element.

The representation of the transfer matrices with respect to the beam centroid then involves a transformation of coordinates from the representation with respect to the reference trajectory. Mathematical details can be found on page 11. A discussion of how higher orders in the expansion about the reference trajectory affect lower orders in the expansion about the beam centroid can be found on page 83 in the section on the `ORDER` element.

The `CENTROID` element is intended as a means of directing a particle beam into an optical system in a manner not coincident with the design axis of that optical system. It would occur normally only at the beginning of a beam line. However, the `CENTROID` element has frequently been used to simulate other phenomena. Such uses include the effect of misalignments or steering magnets. For such purposes, it would be better to use directly either the misalignment element (`ALIGN`), or one of the steering magnet elements (`HKICK`, `VKICK`), or `KICKER`.

The `CENTROID` element does not affect the printed floor coordinates. The printed floor coordinates represent the reference trajectory, which is unaffected by the `CENTROID` element.

The centroid shift is considered as being done by an unspecified cause within a fixed optical system. A shift in the centroid coordinates can also occur without a physical centroid deflection. This latter coordinate shift occurs when there is a change in the coordinate system to which the centroid is referred. A change in coordinate systems often occurs when a particle beam is transferred from one optical system to another. Examples are the transference of a beam from an accelerator lattice to a transfer line or from a transfer line to an accelerator lattice. The `SHIFT` element should be used for such purposes. The `SHIFT` element does affect the floor coordinates.

ETA – Accelerator η Function

The accelerator function η is a new element and is specified in a manner similar to the `CENTROID` element. In some previous versions of TRANSPORT the `CENTROID` element was taken to be the η function. However this identification precluded expressing the accelerator functions β and α about an off-axis orbit. The centroid and the η function are now separate.

`ETA` is not a MAD element. but can be expressed in keyword notation.

Notation

The mnemonic for the accelerator η function is `ETA`. The keywords which can be used in specifying the accelerator η function are

Symbol	Keyword	Description
η_x	`ETAX`	The x component of the accelerator η function.
η'_x	`DETAX`	The derivative of the x component of η.
η_y	`ETAY`	The y component of the accelerator η function.
η'_y	`DETAY`	The derivative of the y component of η.
η_ℓ	`ETAL`	The longitudinal displacement of η.
η_δ	`ETAP`	The fractional momentum deviation for η (normally 1.0).

Units are normally the same as those for the beam phase space (see page 99). With the unit set `UMM`, the units are mm, mr, and permil $\Delta p/p$ (parts in a thousand). The other unit sets (see page 68) `UMETER`, `UMICR`, or `UMICR` may also be used. (See page 64.) The use of `UTRANS` is strongly discouraged as it will require that η_x and η_y be in dekameters.

An η function which, in the unit set `UMM` has 1.0 mr of horizontal displacement and no horizontal slope or nonzero vertical coordinates might be represented as

```
ETA,  ETAX = 1.0  ;
```

Any undefined components will be set to zero except for the δ component, where the default value is equal to one of whatever unit is being used for δ.

Any or all of the six beam η function coordinates may be varied in first-order fitting. The η function may then be constrained at any later point in the beam line.

Usage

The printing of the beam matrix in terms of the parameters β and α after each element is effected by the command

```
PRINT,  TWISS,  ON  ;
```

A single line will be printed which will also contain the four transverse components of the η function. The β, α, and η functions will be labelled in the output so there will be no difficulty in identifying them.

The accelerator parameters are typically expressed in meters and radians. To ensure that the η function is expressed in such units, appropriate units changes should be made at the beginning of the data. The single commands `UMETER` or `UMAD` will change all units to those used most often for accelerator physics. These single commands are described on page 64.

The global unit sets `UMM` and `UMICR` will have respective defaults for δ of one part in a thousand and one part in a million. However, the first four components of η will still be in meters and radians, as these two measures are respectively equal to millimeters per part-in-a-thousand and microns per part-in-a-million.

Physical Elements

Symbolic Constants

TRANSPORT recognizes the mathematical and physical constants listed in the table below. The use of recognizably named constants is included for compatability with the MAD program. The list of constants is essentially the same as is found in the MAD program. Users should avoid using the names of such symbolic constants for any other purpose. The use of a symbolic constant name for a user-defined label will cause the user definition to supersede the symbolic constant meaning. Further use as a symbolic constant may then be inconsistent with the new meaning and cause unpredictable results. However, if a symbolic constant is redefined in the data as a parameter with the same value (next page), there will be no problem. Decks of data based on versions of the program before the inclusion of symbolic constants may have such defined parameters.

The symbolic constants recognized are:

Symbol	Name	Value Used	Unit
π	`PI`	3.14159265358979323	
2π	`TWOPI`	2.0*π	
$180/\pi$	`DEGRAD`	$180.0/\pi$	
$\pi/180$	`RADDEG`	$\pi/180.0$	
e	`E`	2.718281828459045	
m_e	`EMASS`	$.510099906 * 10^{-3}$	GeV
m_p	`PMASS`	.93827231	GeV
c	`CLIGHT`	$2.99792458 * 10^{8}$	m/s
	`MTOIN`	.0254	
	`INTOM`	1.0/`MTOIN`	

Further constants can be defined using the parameter statement defined below.

PARAMETERS

The `PARAMETER` element is taken from the MAD program.

A numerical value to be used many times over in the definition of elements can be specified a single time by the use of a parameter statement. The parameter is given a proper name via the parameter element. It is then referred to by that proper name on all other elements. For example, a drift length `LD1` which is to be .5 meters can be defined by

```
LD1:  = .5 ;
```

or alternatively

```
PARAM, LD1 = 0.5 ;
```

In defining a drift space `D1` whose length is `LD1`, one can write

```
D1:  DRIFT, L = LD1 ;
```

It is also possible to define a parameter by its use on a given element. On a later element it is represented by the label of the first element followed by the keyword of the parameter in brackets. The length of the drift space `D1` may now be given directly on the drift specification as:

```
D1:  DRIFT, L = 0.5 ;
```

A second drift space, whose length is always to be the same as that of the first can be given as

```
D2:  DRIFT, L = D1[L] ;
```

The symbol `D1` after the equals sign indicates that the length of element `D2` is to be taken from element `D1`. The `L` in the brackets indicates that the parameter taken from `D1` is the length.

A parameter may be varied as part of a fitting problem. Details are given on page 246.

Algebraic Expressions

Any algebraic expression made up from previously defined parameters can also serve as a parameter. This algebraic expression can be evaluated either on another `PARAMETER` element, or as one of the physical parameters of a physical element. In the example above, a second drift space can be defined in terms of the parameter `LD1`

```
D2:  DRIFT, L = 10.0 - LD1 ;
```

The value of the parameter `LD1` may be varied, but the sum of the two drift lengths will be held fixed at 10.0 meters. The operations which may be used in the algebraic expression are:

`+`	Addition
`-`	Subtraction
*	Multiplication
`/`	Division
`SQRT`	Square root
`ALOG`	Natural logarithm (written Ln in engineering notation).
`EXP`	Exponential function
`SIN`	Sine function
`COS`	Cosine function
`SINH`	Hyperbolic sine function
`COSH`	Hyperbolic cosine function
`ASIN`	Inverse sine function
`ACOS`	Inverse cosine function

For example, the lengths of three drift spaces in a system can be made to increase in geometric progression. The first two drift spaces `D1` and `D2` can be made free parameters and given as:

```
D1:  DRIFT, L = LD1 ;
D2:  DRIFT, L = LD2 ;
```

A third drift space can be defined as

```
D3:  DRIFT, L = LD2*LD2/LD1
```

DRIFT

A drift space is a field-free region through which the beam passes. It is specified by a single parameter, which is its length. The length may be varied in first-, second-, or third-order fitting. The drift space is a MAD compatible element and is expressed in MAD notation.

MAD Notation

A single keyword is used in specifying a drift space. It is `L` which indicates the length of the drift space. A specification for a drift space `D1` of length 6.0 meters is written as

```
D1:  DRIFT,  L = 6.  ;
```

Drift Space Matrix

The first-order R matrix for a drift space is as follows:

$$\begin{bmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\gamma^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where

L = the length of the drift space.
γ = relativistic factor

The dimensions of L are those chosen for longitudinal length. In any of the global units specifications (`UMM`, `UMETER`, `UMAD`, etc.) this unit is meters. An alternate longitudinal unit may be chosen via the `UNIT` element. For example, a change of the longitudinal unit to feet would be effected by placing the element

```
UNIT,  L,  FT  ;
```

preceding the `BEAM` element. Details about unit changes can be found on page 68.

For a the transverse coordinates of a normal drift space, the first-order transformation is the exact solution of the equations of motion. There are no higher-order terms. For the longitudinal coordinates (ℓ and δ), the transformation is exact only for massless particles.

For the R_{56} matrix element to be nonzero, the particle must be given a nonzero mass.

The particle mass is specified via the Special Parameter element as explained on page 75.

A normal drift space is one which begins and ends at a plane which is perpendicular to the reference trajectory. It is sometimes desirable to have a drift space end at a plane which makes a different angle to the reference trajectory. One example might be the focal plane of a dispersive focusing system. The foci at the different momenta are spread out transversely because of the dispersion and longitudinally because of the chromatic aberration of the focusing devices (quadrupoles or their optical equivalents). A drift space can be specified to end at such a tilted focal plane. For details of implementation, see page 76.

BEND – Sector Bending Magnet

A sector bending magnet implies that the central trajectory of the beam enters and exists perpendicularly to the pole-face boundaries. In previous editions of this manual, the term "wedge" has been used to describe such a bending magnet. There is no intended distinction between the terms "sector" and "wedge". The term "sector" has now been adopted for reasons of compatability with MAD, especially with the MAD element `SBEND`, meaning sector bend.

The `BEND` element is used to represent the interior field of a bending magnet. No fringe field effects are included in the `BEND` element. It is especially useful when segmenting a magnet longitudinally to determine the trajectory transformation at interior points. To include fringing-field effects and non-perpendicular entrance or exit boundaries with the bend element see the `ROTATE` and `SPECIAL` elements. The `ROTATE` element (see page 129) represents the fringe field itself. The `SPECIAL` element (see page 72) may be used to define certain parameters used to describe the fringe field.

First-, second, and third-order matrix elements can be calculated by TRANSPORT for the `BEND` element. The order actually calculated will depend on the `ORDER` command as described on page 83. With no `ORDER` element the first-order transformation will be calculated. The number of parameters necessary to specify a `BEND` element depends on the order of calculation requested.

The element `BEND` is not found in MAD, but can be described by the same set of parameters as is used for the elements `RBEND` and `SBEND` in MAD. It can also be described by several other sets of parameters, including those used in the original TRANSPORT numerical notation.

To represent a bending magnet complete with fringing fields as a single element one should use the `RBEND` or `SBEND` elements described on page 134. A more complete description of the set of parameters used to represent the magnetic field is given on page 2. Some of this description is repeated here for the user's convenience.

MAD-like Notation

The `BEND` element, without fringing fields, is not found in the MAD program. It is, however, a TRANSPORT element which can be described in keyword notation. The parameterization of it, described here as MAD-like, has been implemented for consistency with the `RBEND` and `SBEND` elements. To first order, a `BEND` element is specified by three real attributes.

```
label: BEND,  L = value ,  ANGLE = value ,  K1 = value;
```

A sample `BEND` element might look like:

```
BR:  BEND,  L = 10.,  ANGLE = 0.010,  K1 = 0.5E-4  ;
```

The bend angle shown is 0.010 radians or 0.574 degrees. The value in radians is shown since radians are the standard MAD units for bending angle. The use of MAD units is indicated by the choice of the global unit set `UMAD`. The same element, but with the bending angle expressed in degrees would be:

```
BR:  BEND,  L = 10.,  ANGLE = 0.574,  K1 = 0.5E-4  ;
```

The bend angle is expressed in milliradians in the `UMM` data set. Other global unit sets are described on page 64

The MAD-like keywords which can be used in specifying a sector bending magnet are

Symbol	Keyword	Description
L	`L`	The magnet length (normally meters)
α	`ANGLE`	The angle through which the reference trajectory is bent (milliradians with unit set `UMM`, radians with `UMETER`, default 0.0).
K_1	`K1`	The quadrupole coefficient (normally meters^{-2}, default 0.0), defined by $K_1 = \frac{1}{B\rho}\frac{\partial B_y}{\partial x}$
K_2	`K2`	The sextupole coefficient (normally meters^{-3}, default 0.0), defined as $K_2 = \frac{1}{2B\rho}\frac{\partial^2 B_y}{\partial x^2}$
K_3	`K3`	The octupole coefficient (normally meters^{-4}, default 0.0), defined as $K_3 = \frac{1}{6B\rho}\frac{\partial^3 B_y}{\partial x^3}$

In terms of these parameters, the field expansion on the magnetic midplane is given by

$$B_y = (B_o\rho)\sum K_n x^n$$

$$B_x = 0$$

The definition of the multipole components α, K_1, K_2, and K_3 allows the orbit or a particle to be computed without knowledge of its momentum. The normalization of K_1,

K_2, and K_3 is such that a multipole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. In the MAD program itself the values of K_1, K_2, and K_3 are defined as the x derivatives of the field divided by $B_0\rho$. The value of K_2 in MAD is then a factor of 2 greater than the value defined above. The MAD value of K_3 is a factor of 6 (= 3!) greater than that used here. When the global unit set `UMAD` is used, the MAD definition of the multipoles is also used.

The units for any of the parameters may be changed as desired. The units of the parameters K_1, K_2, and K_3 are in inverse powers of the unit of longitudinal length. Global unit sets are described on page 64. Individual units changes are described on page 68.

The parameters L, α, and K_1 may be varied in first-order fitting. The parameter K_2 may be varied in second-order fitting. The parameter K_3 may be varied in third-order fitting. The `VARY` command is described on page 246 in the section on the `FIT` element.

Other Keyword Options

To allow keyword representation of data in the original TRANSPORT variables and to permit other options for specifying a bending magnet, the following keywords are also recognized. They may be used instead of or combined with the MAD-like keywords.

Symbol	Keyword	Description
B	`B`	The magnetic field (normally kilogauss)
ρ	`RADIUS`	The radius of curvature of the reference trajectory (normally meters).
n	`N`	The normalized field derivative (dimensionless).
ϵ	`EPS`	The quadratic field dependence (in inverse squared units of horizontal beam width – mm in `UMM`, meters in `UMETER`.
ϵ_3	`EPS3`	The cubic field dependence (in inverse cubed units of horizontal beam width).

The complete set of variables available for defining a bending magnet contains a great deal of redundancy. The bending magnet, used as an example earlier, may be described to first order in any of ten ways. We assume the beam momentum to be 600 GeV/c. The following five lines are then all equivalent.

```
BEND,  L = 10.,  B = 20.,  N = 0.5 ;

BEND,  L = 10.,  ANGLE = 10.,  N = 0.5 ;

BEND,  L = 10.,  RADIUS = 1000.,  N = 0.5 ;

BEND,  B = 20.,  ANGLE = 10.,  N = 0.5 ;
```

```
BEND,  RADIUS = 1000.,  ANGLE = 10.,  N = 0.5 ;
```

Here the length is assumed to be in meters, the angle in milliradians, the field in kilo-Gauss, and the bend radius in meters. The angular unit specified by the global unit set `UMM` is milliradians. The angular unit specified by `UMETER`, is radians. Other units may be specified by use of an alternate global units specification (`UMAD`, `UTRANS`, etc.) or a `UNITS` element.

In addition to the above five options, the field gradient may be specified by K_1 instead of n. The two are related by

$$K_1 = -\frac{n}{\rho^2} = \frac{1}{B\rho}\frac{\partial B_y}{\partial x}$$

The expansion of the magnetic field on the magnetic midplane is given by

$$B_y = B_o(1 + r_s)(1 - nhx + \beta\ h^2x^2 + \gamma\ h^3x^3 + \ldots)$$

$$B_x = 0$$

The sextupole component β is related to the coefficients ϵ and K_2 by

$$K_2 = \frac{\epsilon}{\rho} = \frac{\beta}{\rho^3} = \frac{1}{2B\rho}\frac{\partial^2 B_y}{\partial x^2}$$

Only one of the two quantities ϵ and K_2 should be specified. If neither is specified, the sextupole component is set equal to the default value given by the special parameters (`SPEC` element). If the default value is also unspecified, it is taken to be zero.

The octupole component γ is related to the cubic variation of the magnetic field by.

$$K_3 = \frac{\epsilon_3}{\rho} = \frac{\gamma}{\rho^4} = \frac{1}{6B\rho}\frac{\partial^3 B_y}{\partial x^3}$$

The default value for the octupole component ϵ_3 is also set by the special parameters (`SPEC` element). Otherwise it is taken to be zero.

Any of the first-order parameters L, B, ρ, α, and n can be varied in first-order fitting. The parameter ϵ can be varied in second-order fitting. The parameter ϵ_3 can be varied in third-order fitting.

Mispowering and Violations of Midplane Symmetry

The vertical field in a bending magnet along the reference trajectory may be mispowered so that the reference particle will be bent away from the reference trajectory. This can happen simply by increasing the actual field in the magnet without changing the reference

field. A physical bending magnet may also have a horizontal field component on what is taken as the magnetic midplane. A set of parameters to describe both the effects of excess bend field and violation of midplane symmetry follows. The multipole components which violate midplane symmetry are also known as skew multipoles. For a discussion of the magnetic field components, see page 2.

Some of these parameters are MAD-like, and should be used if the midplane-symmetric field is given a MAD-like parameterization. Specifically the parameters K_1' and K_2' are MAD-like. The parameters n' and ϵ' are original- TRANSPORT-like. The two scaling parameters r_s and r_a can be used with either representation.

Symbol	Keyword	Description
r_s	`RMPS`	The fractional excess magnetic field (unitless).
r_a	`RNMS`	Scaling factor for the skew field components (unitless).
v_r	`VR`	The normalized skew dipole magnetic field (unitless).
K_1'	`K1P`	The skew quadrupole coefficient (normally meters^{-2}).
K_2'	`K2P`	The skew sextupole coefficient (normally meters^{-3}).
n'	`NP`	The midplane-symmetry-violating normalized field derivative (dimensionless).
ϵ'	`EPSP`	The midplane-symmetry-violating quadratic field dependence (same units as `EPS`).

In terms of these new parameters, the field expansion on the nominal magnetic midplane is given by

$$B_y = (B_o\rho)(1 + r_s) \sum K_n x^n$$

$$B_x = (B_o\rho) r_a \sum K_n' x^n$$

where

$$B_0 v_r = (B_0\rho) K_0'$$

or

$$B_y = B_o(1 + r_s)(1 - nhx + \beta\ h^2x^2 + \gamma\ h^3x^3 + \ldots)$$

$$B_x = B_o r_a(v_r - n'hx + \beta' h^2x^2 + ...)$$

The quantity r_s represents the fractional excess field. The reference trajectory is defined as if r_s equalled zero. The field of the magnet may then be increased or decreased to steer the beam about the reference trajectory. The quantity r_a exists only to facilitate simultaneous scaling of all midplane symmetry violating multipoles.

If r_s and r_a are not specified for a particular BEND element, they are set equal to the default values given on the special parameter element (SPEC element – see page 82). If the default values are also unspecified, they are taken to be zero.

If the magnet in the example just above is mispowered so that the field is excessively strong by 1%, it might be specified as:

```
BR:  BEND,  L = 10.,  ANGLE = 10.,  K1 = 0.5E-4,  RMPS = 0.01  ;
```

If there is a skew quadrupole component which is whose strength is 1% that of the midplane symmetric quadrupole component, the bend magnet could be described as:

```
BR: BEND, L = 10., ANGLE = 10., K1 = 0.5E-4, RNMS = 0.01, K1P = 0.5E-4 ;
```

or as:

```
BR: BEND, L = 10., ANGLE = 10., K1 = 0.5E-4, RNMS = 1.0, K1P = 0.5E-6  ;
```

Any of the parameters, r_s, r_a, v_r, K_1', K_2', n', and β' may be set on the individual bend element, or left to equal the default value. The default values may be set on the special parameter element. Otherwise they are all zero. The parameters K_1' and K_2' are related to n', β', and ϵ' respectively in the same way that K_1 and K_2 are related to n, β, and ϵ.

Any of the first-order parameters r_s, r_a, v_r, K_1', and n' may be varied in first-order fitting. The parameters K_2' and ϵ' can be varied in second-order fitting. There are no third-order midplane-symmetry-violating transfer matrix elements in TRANSPORT.

Fringing Fields

If fringing field effects are to be included, a ROTAT element must immediately precede and follow the pertinent BEND element (even if there are no pole-face rotations). Thus a typical TRANSPORT input for a bending magnet including fringing fields might be:

```
ROTAT,  ANGLE = 0.   ;
BEND,  L = 10.0,  B = 20.,  N = 0.5  ;
ROTAT,  ANGLE = 0.   ;
```

For non-zero pole-face rotations a typical data input might be

```
ROTAT,  ANGLE = 10.   ;
BEND,  L = 10.0,  B = 20.0,  N = 0.5  ;
ROTAT,  ANGLE = 20.  / ;
```

Direction of Bend

The sign conventions for bending magnet entries are illustrated in the following figure. For TRANSPORT a positive bend is to the right looking in the direction of particle travel. To represent a bend in another sense, the coordinate rotation matrix (`SROT` element) should be used as follows:

A bend up is represented by a rotation of the (x,y) coordinates by -90.0 degrees about the (z) beam axis as follows:

```
SROT,  ANGLE = -90.   ;
ROTATE,  ANGLE = 10.0  ;
BEND,  L = 10.0,  B = 20.0,  N = 0.5  ;
ROTATE,  ANGLE = 20.0  ;
SROT,  ANGLE = +90.   ; (returns coordinates to their initial orientation)
```

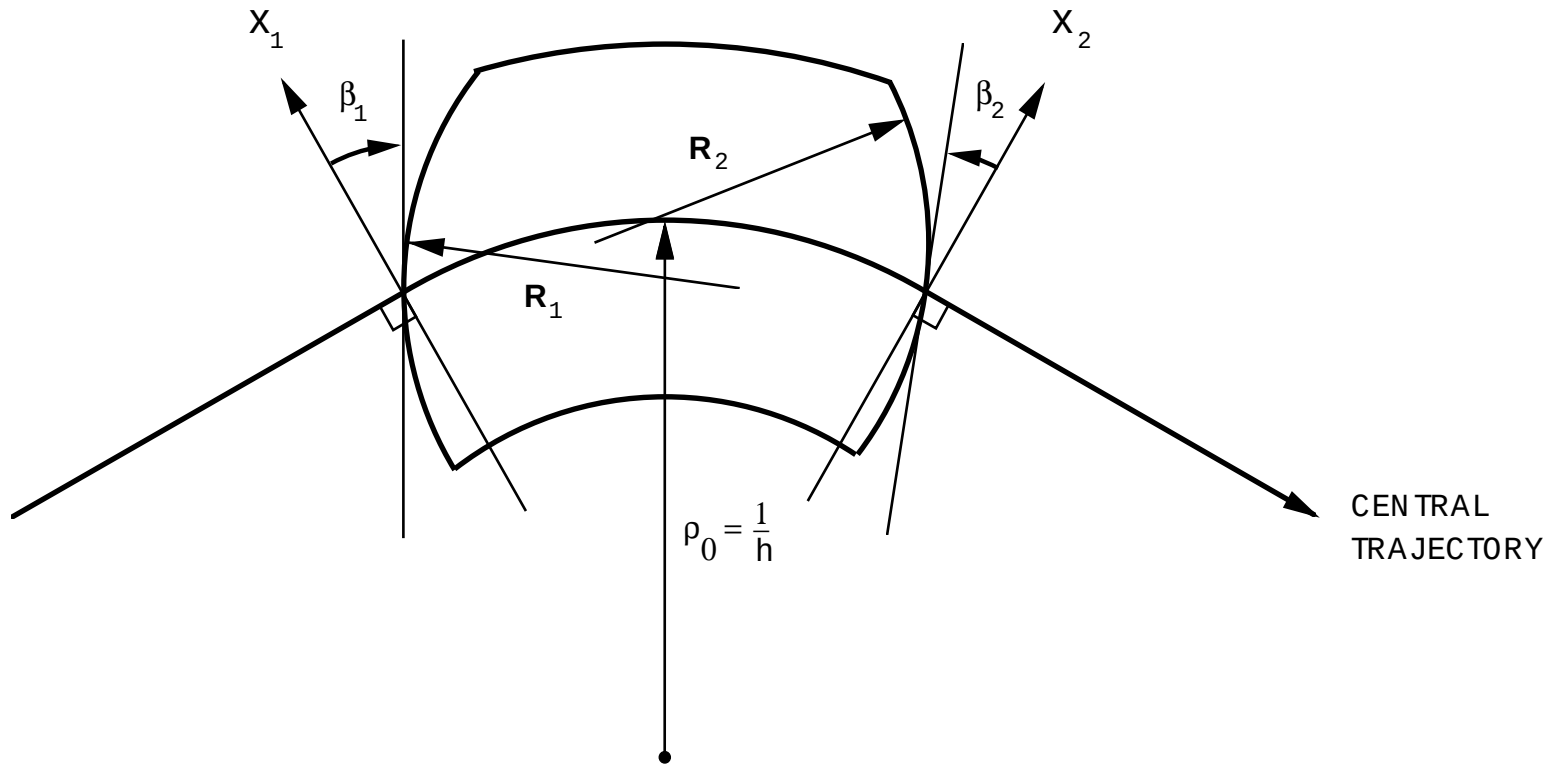
A bend down is accomplished via:

```
SROT,  ANGLE = +90.   ;
ROTATE
BEND
ROTATE
SROT,  ANGLE = -90.   ;
```

A bend to the left (looking in the direction of beam travel) is accomplished by rotating the x,y coordinates by 180 degrees, e.g.

```
SROT,  ANGLE = +180.   ;
ROTATE
BEND
ROTATE
SROT,  ANGLE = -180.   ;
```

The `SROT` need be used only with the `BEND` element. For the `RBEND` and `SBEND` elements the coordinate rotation may be specified on the element itself as the `ROLL` parameter.



Field Boundaries for Bending Magnets

The TRANSPORT sign conventions for x, β, R and h are all positive as shown in the figure. The positive y direction is out of the paper. Positive β's imply transverse focusing. Positive R's (convex curvatures) represent negative sextupole components of strength $S = (-h/2R)\ sec^3\beta$. (See SLAC-75 [4], page 71 or SLAC-PUB-3381 [11], page 22.)

First-Order Sector Bending Magnet Matrix

$$\begin{bmatrix} \cos k_x L & \frac{1}{k_x}\sin k_x L & 0 & 0 & 0 & \frac{h}{k_x^2}(1-\cos k_x L) \\ -k_q \sin k_x L & \cos k_x L & 0 & 0 & 0 & \frac{h}{k_x}\sin k_x L \\ 0 & 0 & \cos k_y L & \frac{1}{k_y}\sin k_y L & 0 & 0 \\ 0 & 0 & -k_y \sin k_y L & \cos k_y L & 0 & 0 \\ \frac{h}{k_x}\sin k_x L & \frac{h}{k_x^2}(1-\cos k_x L) & 0 & 0 & 1 & -\frac{h^2}{k_x^3}(k_x L - sin k_x L) + \frac{L}{\gamma^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Definitions: $h = 1/\rho$, $k_x^2 = (1-n)h^2$, $k_y^2 = nh^2$
$\alpha = hL =$ the angle of bend
$L =$ path length of the central trajectory
$\gamma =$ relativistic factor

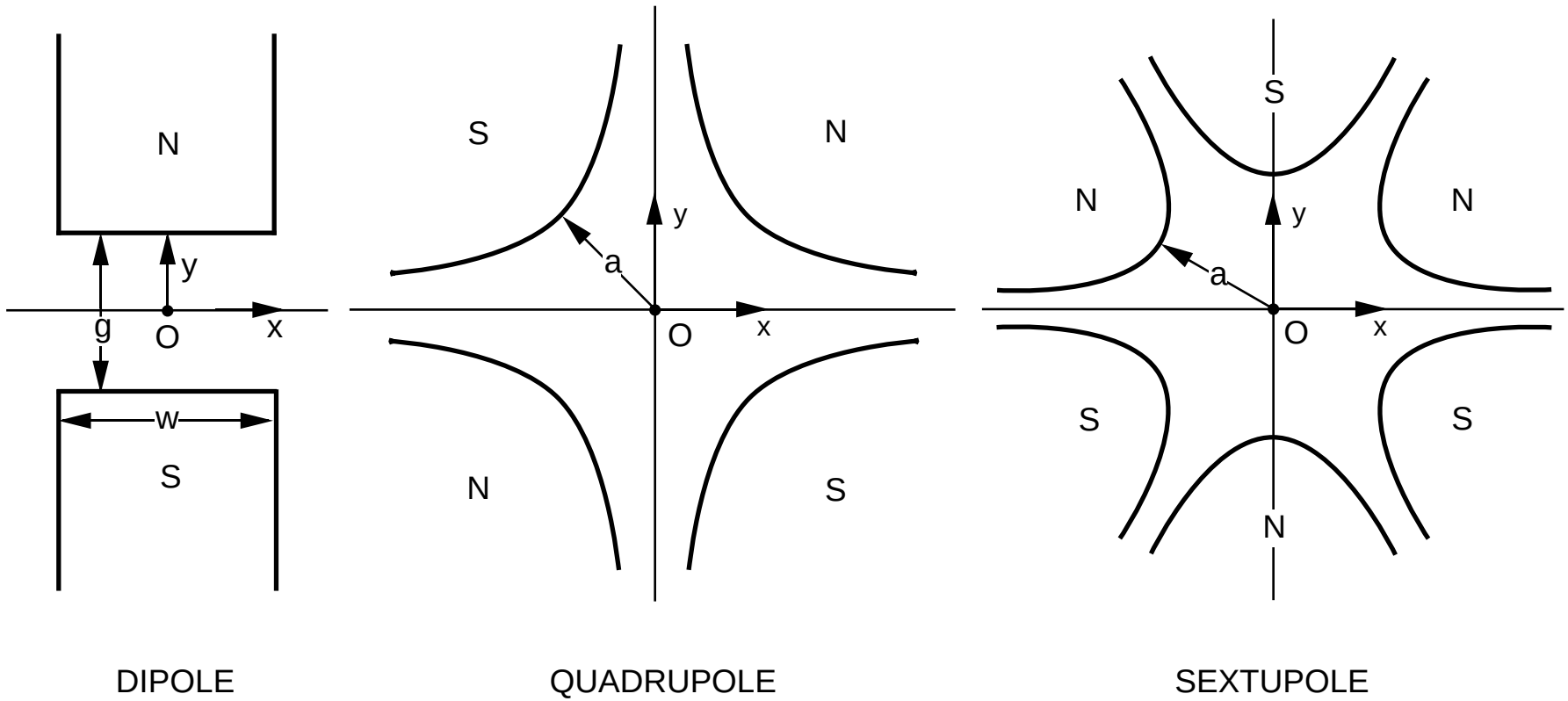


Illustration of the magnetic midplane (x axis) for dipole, quadrupole and sextupole elements. The magnet polarities indicate multipole elements that are positive with respect to each other. The coordinate system is right-handed, so that the beam particles will be emerging from the paper and moving into the face of the reader.

ROTATION — Fringing Fields and Pole-Face Rotations for Bending Magnets

To provide for fringing fields and/or pole-face rotations on bending magnets specified by the `BEND` element, the `ROTATION` element is used. The combined use of the `BEND` and the `ROTATION` elements allows the user to separate the effects of the fringing fields from those of the central body of a bending magnet.

First-, second-, and third-order matrix elements are included in TRANSPORT for the `ROTATION` element. In a third-order calculation, an extended fringe field (not an abrupt beginning or end) is required for the matrix elements to be finite (not infinite). For the fringe field to be extended, it is necessary to give a nonzero value to the magnet half gap (explained below).

Like the `BEND` element, the `ROTATION` element is not found in MAD. The combination of `BEND` and `ROTATION` elements may be replaced in MAD with the single `RBEND` or `SBEND` element. The `ROTATION` element is expressed in keyword notation, similar to MAD input.

Notation

A first-order specification of a `ROTATION` element will typically have a value for just the rotation angle.

```
label: ROTAT,  ANGLE = value ;
```

A pole face rotation of 5 mr might be represented as:

```
ROTAT,  ANGLE = 5.0  ;
```

The units for the pole face rotation angle will be milliradians in the global unit set `UMM` and radians with `UMETER`. Other global unit sets are described on page 64. Other individual unit changes are listed on page 68.

To represent the entrance face rotation the `ROTATION` element should be placed immediately <u>before</u> the `BEND` element. To represent the exit face rotation the `ROTATION` element should be placed immediately <u>after</u> the `BEND` element. Additional parameters (see below) are available for higher-order calculations or in situations where greater precision is required is describing the configuration of the fringing field.

In the above example we omit the optional label. `ROTATION` is not a MAD element and is typically not used when a beam line is assembled from previously defined elements. Labels used for other purposes, such as simple identification, are generally attached to the accompanying `BEND` element.

The mnemonic used in specifying a pole face rotation is `ROTATION`. The MAD-like keywords which can be used are

Symbol	Keyword	Description
β	`ANGLE`	Angle of pole-face rotation (mr in `UMM`, radians with `UMETER`).
$g/2$	`HGAP`	Half gap (mm with `UMM`, meters with `UMETER`, default 0.0).
κ_1	`FINT`	Fringe field integral (unitless, default 0.5).
$1/R_{1,2}$	`H`	Pole face curvature (normally meters^{-1}, default 0.0).

None of these parameters need be specified. The default value of the pole-face rotation angle is zero. The keywords `HGAP`, `FINT`, and `H` allow the user to override temporarily the default values. The values specified by these three keywords on any `ROTAT` element will apply only to that element. The default values of these three parameters may be changed by the special parameter element described on page 72.

The pole-face rotation angle β can be varied in first order. The pole face curvature $1/R_{1,2}$ can be varied in second order. The procedure for indicating that a parameter is to be varied is given on page 246 in the section on the `VARY` command.

A symmetrically oriented rectangular bending magnet whose total bend is 10 mr would be represented by the three entries:

```
ROTAT,  ANGLE = 5.   ;
BEND,   - - -   ;
ROTAT,  ANGLE = 5.   ;
```

Originally in TRANSPORT notation, only the pole-face rotation angle is given in the `ROTAT` element. The other three parameters are given by the special parameter element. The keywords `HGAP`, `FINT`, and `H` now allow the user to override temporarily the default values given by the special parameter cards. The values specified by these three keywords on any `ROTAT` element will apply only to that element. Default values which assumed if not specified through the special parameter cards. These default values are given on page 72 and following.

The use of keywords to override the defaults for special parameters allows the user to specify the characteristics of a pole face rotation on the card for that element alone. A complete bending magnet with fringe fields and pole face rotations and curvatures might be written as

```
ROTAT,  ANGLE = 5.0,  H = 0.01  ;
BM:  BEND,  L = 10.,  B = 20.   ;
ROTAT,  ANGLE = 5.0,  H = -0.1  ;
```

Here a bending magnet 10 meters long with a field of 20 kG has the entrance and exit pole face rotations specified also. Both pole face rotation angles are 5.0 mr. The entrance pole face is convex and has a curvature of 0.01 inverse meters, or a radius of curvature of 100

meters. The exit pole face is concave, and the radius of curvature is 10 meters. The units are based on the assumption that the global unit set `UMM` is employed.

The angle of rotation may be varied. For example, the element `ROTAT, ANGLE = 5. ;` would allow the angle to vary from an initial guess of 5 degrees to a final value. This final value might satisfy a vertical focus constraint imposed upon the system.

Usage

The `ROTATION` element must either immediately precede a bending magnet (`BEND` element) element (in which case it indicates an entrance fringing field and pole-face rotation) or immediately follow a `BEND` element (exit fringing field and pole-face rotation) with no other data entries between.* A positive sign of the angle on either entrance or exit pole-faces corresponds to a non-bend plane focusing action and bend plane defocusing action.

Even if the pole-face rotation angle is zero, (`ROTAT, ANGLE = 0. ;`) entries must be included in the data set before and after a `BEND` element entry if fringing-field effects are to be calculated.

A single `ROTAT` element entry that follows one bending magnet and precedes another will be associated with the latter.

Should it be desired to misalign such a magnet, an update element or alignment marker must be inserted immediately before the first `ROTATION` element. Then the convention appropriate to misalignment of a set of elements should be applied, since, indeed, three separate transformations are involved. See page 180 for a discussion of misalignment calculations and page 212 element for a discussion of updates or page 216 for the alignment marker.

Pole-Face Rotation Matrix

The first-order R matrix for a pole-face rotation used in a TRANSPORT calculation is as follows:

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{tan\beta}{\rho_0} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{tan(\beta-\psi)}{\rho_0} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

**It is extremely important that no data entries be made between a* `ROTAT` *element and a* `BEND` *element* entry. If this occurs, it may result in an incorrect matrix multiplication in the program and hence an incorrect physical answer. If this rule is violated, an error message will be printed.

where [†] [‡]

$$\psi = \kappa_1 \left(\frac{g}{\rho_0}\right) \left(\frac{1 + sin2\beta}{cos\ \beta}\right) \left[1 - \kappa_1\kappa_2 \left(\frac{g}{\rho_0}\right)\ tan\ \beta\right]$$

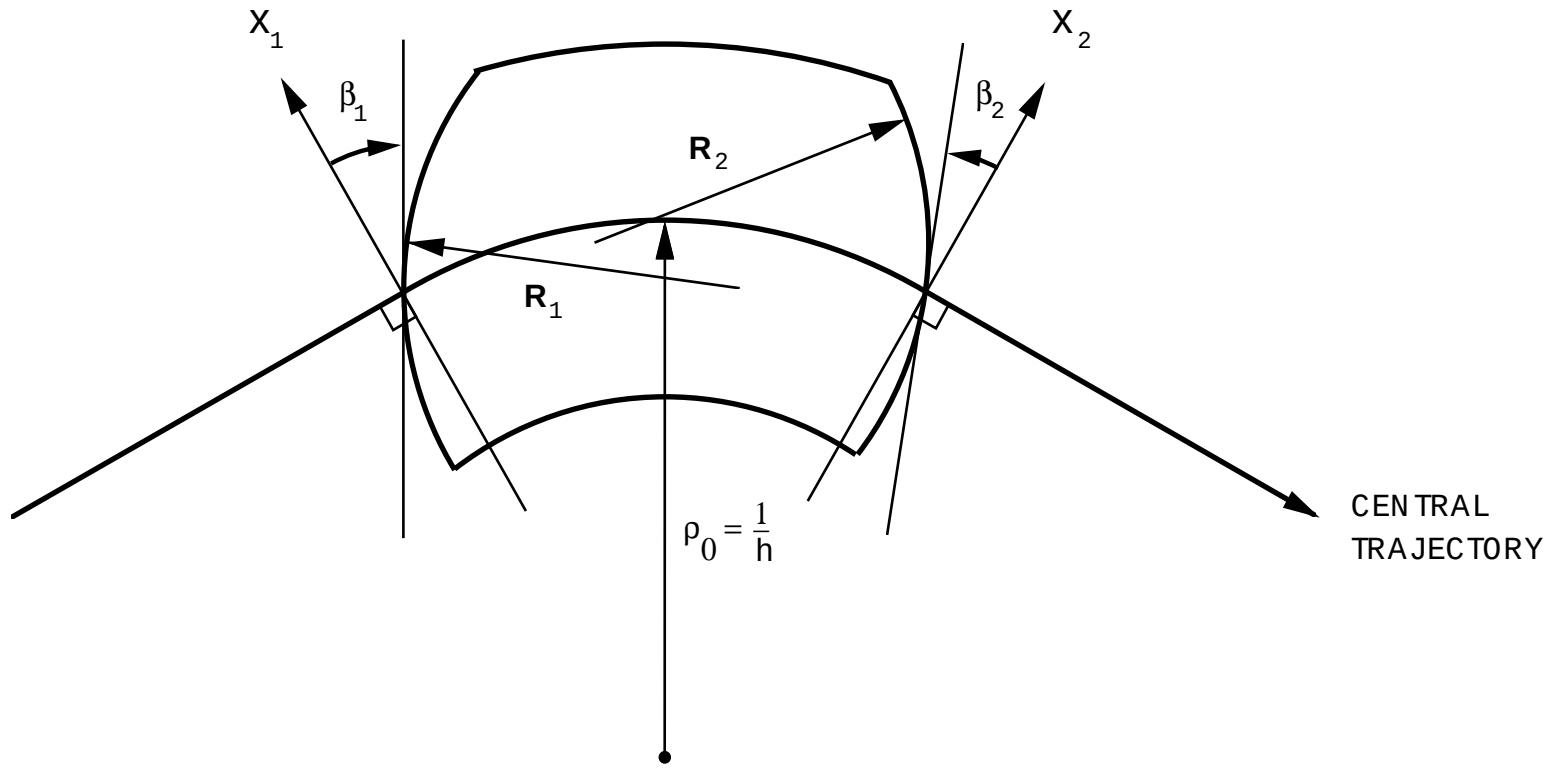
Definitions:

β	=	Angle of rotation of pole-face (see figure on following page for sign convention of β).
ρ_0	=	Bending radius of central trajectory.
g	=	Total gap of magnet.
ψ	=	Correction term resulting from spatial extent of fringing fields.

The transverse shift in the reference trajectory is given by

$$\Delta x = \frac{1}{cos^2\beta}\ \frac{g^2}{\rho}\ \kappa_0$$

[†] See `SPECIAL` element on page 72 for input formats for g, κ_1, κ_2 TRANSPORT entries.
[‡] See SLAC-75 [4] (page 74) for a discussion of ψ.



Field Boundaries for Bending Magnets

The TRANSPORT sign conventions for x, β, R and h are all positive as shown in the figure. The positive y direction is out of the paper. Positive β's imply transverse focusing. Positive R's (convex curvatures) represent negative sextupole components of strength $S = (-h/2R)\ sec^3\beta$. (See SLAC-75 [4], page 71 or SLAC-PUB-3381 [11], page 22.)

RBEND and SBEND – Bending Magnets with Fringing Fields

The elements RBEND and SBEND indicate bending magnets including the fringing fields. The mnemonic RBEND indicates a rectangular bending magnet. The rectangular shape pertains to the entire magnet, as seen from above (plan view). The default value for both the entrance and exit pole face rotation angles is half the bend angle.

The mnemonic SBEND indicates a sector bending magnet. Here the reference trajectory enters and exits from the magnet perpendicular to the magnet pole faces. Illustrations of both the RBEND and the SBEND magnets can be found at the end of this section. These mnemonics are the ones used to describe a bending magnet in the MAD program.

The term "wedge magnet" has also been used to describe an SBEND element. We intend no distinction between the terms "sector" and "wedge". The default values for the entrance and exit pole face rotation angles are therefore both zero.

For both RBEND and SBEND the default values for both the entrance and exit pole face curvatures are zero. The default values for the quadrupole, sextupole, and octupole components of the field are also zero. Any of these parameters can be set to whatever value is desired. A more complete description of the set of parameters used to represent the magnetic field is given on page 2.

First-, second-, and third-order matrix elements are included in TRANSPORT for the RBEND and SBEND elements. In a third-order calculation, an extended fringe field (not an abrupt beginning or end) is required for the matrix elements to be finite (not infinite). For the fringe field to be extended, it is necessary to specify a nonzero value to the magnet half gap (explained below).

An extended fringe field also results in a transverse displacement of the floor coordinates in comparison to a sharply cut off field. Details, including a description of when this effect is important, can be found in the ROTATE section starting on page 129.

RBEND and SBEND are MAD elements. They may be represented in MAD notation or other keyword notation. Strict MAD notation requires the use of the UMAD unit set. In TRANSPORT other unit sets, such as UMM can also be used. In the immediately following example, we first use the UMM unit set but also mention the UMETER unit set. The unit sets are described on page 68.

MAD Notation

To first order, a BEND element might be specified by three real attributes.

```
label: RBEND,  L = value ,  ANGLE = value ,  K1 = value;
```

A sample SBEND element might look like:

```
BR:  SBEND,  L = 10.,  ANGLE = 0.010,  K1 = 0.5E-4  ;
```

The bend angle shown is 0.010 radians or .574 degrees. The value in radians is shown since radians are the standard MAD units for bending angle. The same element, but with the bending angle expressed in degrees would be

```
BR:  SBEND,  L = 10.,  ANGLE = 0.574,  K1 = 0.5E-4  ;
```

The bend angle is expressed in degrees in the `UTRANS` unit set.

The keywords used in the MAD program for RBEND and SBEND are

Symbol	Keyword	Description
L	`L`	The magnet length (normally meters).
α	`ANGLE`	The angle through which the reference trajectory is bent (mr with the unit set `UMM`, radians with `UMETER`).
K_1	`K1`	The quadrupole coefficient (normally meters^{-2}, default 0.0), defined by $$K_1 = \frac{1}{B\rho}\frac{\partial B_y}{\partial x}$$
β_1	`E1`	The rotation angle for the entrance pole face (mr in `UMM`, radians in `UMETER`, default 0.0).
β_2	`E2`	The rotation angle for the exit pole face (mr in `UMM`, radians in `UMETER`, default 0.0).
K_2	`K2`	The sextupole coefficient (normally meters^{-3}, default 0.0), defined as $$K_2 = \frac{1}{2B\rho}\frac{\partial^2 B_y}{\partial x^2}$$
	`H1`	The curvature of the entrance pole face (normally meters^{-1}, default 0.0).
	`H2`	The curvature of the exit pole face (normally meters^{-1}, default 0.0).
K_3	`K3`	The octupole coefficient (normally meters^{-4}, default 0.0), defined as $$K_3 = \frac{1}{6B\rho}\frac{\partial^3 B_y}{\partial x^3}$$
$g/2$	`HGAP`	The half gap of the magnet (mm in `UMM`, meters in `UMETER`, default 0.0).

`FINT`	The field integral (given the symbol κ_1 in TRANSPORT).
`TILT`	The rotation angle about the entrance reference trajectory. (degrees in `UMM` and `UMETER`, default 0.0).

In terms of these parameters, the field expansion on the magnetic midplane is given by

$$B_y = (B_o\rho)\sum K_n x^n$$

$$B_x = 0$$

The definition of the multipole components α, K_1, K_2, and K_3 allows the orbit or a particle to be computed without knowledge of the reference momentum. The normalization of K_1, K_2, and K_3 is such that a multipole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. In the MAD program itself the values of K_1, K_2, and K_3 are defined as the x derivatives of the field divided by $B_0\rho$. The value of K_2 in MAD is then a factor of 2 greater than the value defined above. The MAD value of K_3 is a factor of 6 (= 3!) greater than that used here. When the global unit set `UMAD` is used, the MAD definition of the multipoles is also used.

The units of any of the parameters may be changed as described on page 68. The units of the parameters K_1, K_2, and K_3 are in inverse powers of the unit of longitudinal length.

The keyword `TILT` indicates that the bending magnet is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. The value given is the angle of rotation in degrees. If the word `TILT` appears alone without a value, the angle of rotation is taken to be 90 degrees, corresponding to a downward bend. A bend upward can be obtained simply by reversing the sign of the bend angle.

An example of the same bend as above, but vertically bending in the downward direction would be:

```
BR:  SBEND,  L = 10.,  ANGLE = 10.0,  K1 = 0.5E-4,  TILT ;
```

An example of the same bend but rotated by only 10° to provide a slight downward bend would be:

```
BR:  BEND,  L = 10.,  ANGLE = 10.0,  K1 = 0.5E-4,  TILT = 10.0 ;
```

Here both bend angle and rotation angle about the beam axis are assumed to be in degrees. The units for the rotation about the beam axis may be changed as described on page 68. Rotations about the longitudinal axis are in degrees for all globally specified units sets (`UMM`, `UMETER`, etc.) except `UMAD`, for which it is in radians.

The parameters L, α, and K_1, β_1, β_2, and the `TILT` may be varied in first-order fitting. The parameters K_2 and $1/R_{1,2}$ may be varied in second-order fitting. The parameter K_3

may be varied in third-order fitting. The `VARY` command is described on page 246 in the section on the `FIT` element.

Other Keyword Options

To allow keyword representation of data in original TRANSPORT variables and to permit other options for specifying a bending magnet, the following keywords are also recognized. Some may be used instead of the MAD variables, others in addition to a MAD specification.

Symbol	Keyword	Description
B	`B`	The magnetic field (normally kilogauss)
ρ	`RADIUS`	The radius of curvature of the reference trajectory (normally meters).
n	`N`	The normalized field derivative (unitless).
ϵ	`EPS`	The quadratic field dependence (in inverse squared units of horizontal beam width – mm in `UMM`, meters in `UMETER`.
ϵ_3	`EPS3`	The cubic field dependence (in inverse cubed units of horizontal beam width).

This set of variables contains a great deal of redundacy. The bending magnet, used as an example earlier in this section, may be described to first order in any of ten ways. For a beam momentum of 600 GeV/c, the following five items are then all equivalent.

```
SBEND,  L = 10.,  B = 20.,  N = 0.5  ;

SBEND,  L = 10.,  ANGLE = 10.0,  N = 0.5  ;

SBEND,  L = 10.,  RADIUS = 1000.,  N = 0.5  ;

SBEND,  B = 20.,  ANGLE = 10.0,  N = 0.5  ;

SBEND,  RADIUS = 1000.,  ANGLE = 10.0,  N = 0.5  ;
```

Here the length is assumed to be in meters, the angle in milliradians, the field in kilo-Gauss, and the bend radius in meters.. The angular unit specified by the global unit set `UMM` is milliradians. The angular unit specified by `UMETER` is radians. Other units may be specified by use of an alternate global units specification (`UMAD`, `UTRANS`, etc.) or a `UNITS` element.

In addition to the above five options, the field gradient may be specified by K_1 instead of n. The two are related by

$$K_1 = -\frac{n}{\rho^2} = \frac{1}{B\rho}\frac{\partial B_y}{\partial x}$$

The expansion of the magnetic field on the magnetic midplane is given by

$$B_y = B_o(1 + r_s)(1 - nhx + \beta\, h^2x^2 + \gamma\, h^3x^3 + \ldots)$$

$$B_x = 0$$

The sextupole component β is related to the coefficients ϵ and K_2 by

$$K_2 = \frac{\epsilon}{\rho} = \frac{\beta}{\rho^3} = \frac{1}{2B\rho}\frac{\partial^2 B_y}{\partial x^2}$$

The octupole component γ is related to the cubic variation of the magnetic field by.

$$K_3 = \frac{\epsilon_3}{\rho} = \frac{\gamma}{\rho^4} = \frac{1}{6B\rho}\frac{\partial^3 B_y}{\partial x^3}$$

The default value for the octupole component ϵ_3 is also set by the special parameters (`SPEC` element). Otherwise it is taken to be zero.

The multipole structure of a magnet should be specified in a consistent manner. One should use either K_1, K_2, and K_3 and the mid-plane symmetry violating counterparts or use n, β, and ϵ and their counterparts. One should not attempt to mix the two forms of field expansion. The two normalization parameters r_s, and r_a can be used with either type of specification.

Any of the first-order parameters L, B, ρ, α, and n can be varied in first-order fitting. The parameter ϵ can be varied in second-order fitting. The parameter ϵ_3 can be varied in third-order fitting.

Mispowering and Violations of Midplane Symmetry

The vertical field in a bending magnet along the reference trajectory may be mispowered so that the reference particle will be bent away from the reference trajectory. This can happen simply by increasing the field in the magnet without redefining the reference momentum. A physical bending magnet may also have a horizontal field component on what is taken as the magnetic midplane. A set of parameters to describe both the effects of excess bend field and violation of midplane symmetry follows. The multipole components which violate midplane symmetry are also known as skew multipoles. For a discussion of the magnetic field components, see page 2.

Some of these parameters are MAD-like, and should be used if the midplane-symmetric

field is given a MAD-like parameterization. Specifically the parameters K_1' and K_2' are MAD-like. The parameters n' and ϵ' are original- TRANSPORT-like. The two scaling parameters r_s and r_a can be used with either representation.

Symbol	Keyword	Description
r_s	`RMPS`	The fractional excess magnetic field (unitless).
r_a	`RNMS`	Scaling factor for skew field components (unitless).
v_r	`VR`	The normalized skew dipole magnetic field (unitless).
K_1'	`K1P`	The skew quadrupole coefficient (normally meters^{-2}).
K_2'	`K2P`	The skew sextupole coefficient (normally meters^{-3}).
n'	`NP`	The skew normalized field derivative (unitless).
ϵ'	`EPSP`	The skew quadratic field dependence (same units as `EPS`).

In terms of these new parameters, the field expansion on the nominal magnetic midplane is given by:

$$B_y = (B_o\rho)(1 + r_s) \sum K_n x^n$$

$$B_x = (B_o\rho) r_a \sum K_n' x^n$$

where

$$B_0 v_r = (B_0\rho) K_0'$$

or

$$B_y = B_o(1 + r_s)(1 - nhx + \beta\ h^2x^2 + \gamma\ h^3x^3 + \ldots)$$

$$B_x = B_o r_a(v_r - n'hx + \beta' h^2x^2 + ...)$$

The quantity r_s represents the fractional excess field. The reference trajectory is defined as if r_s equalled zero. The field of the magnet may then be increased or decreased to steer the beam about the reference trajectory. The quantity r_a exists only to facilitate simultaneous scaling of all midplane symmetry violating multipoles.

The four parameters, r_a, v_r, n', and β' may also be set on the individual bend element, or left to equal the default value. The default values may be set on the special parameter element. Otherwise they are all zero. The parameters K_1' and K_2' are related to n', β', and ϵ' respectively in the same way that K_1 and K_2 are related to n, β, and ϵ.

If any of this set of parameters is not specified it is set equal to the default value given on the special parameter element (`SPEC` element). If the default value is also unspecified, it is taken to be zero.

If the magnet in the example above is mispowered so that the field is excessively strong by 1%, it might be specified as:

```
BR:  SBEND,  L = 10.,  ANGLE = 10.0,  K1 = 0.5E-4,  RMPS = 0.01  ;
```

If there is a skew quadrupole component which is whose strength is 1% that of the midplane symmetric quadrupole component, the bend magnet could be described as:

```
BR:  SBEND,  L = 10.,  ANGLE = 10.0,  K1 = 0.5E-4,  RNMS = 0.01,  K1P = 0.4E-4  ;
```

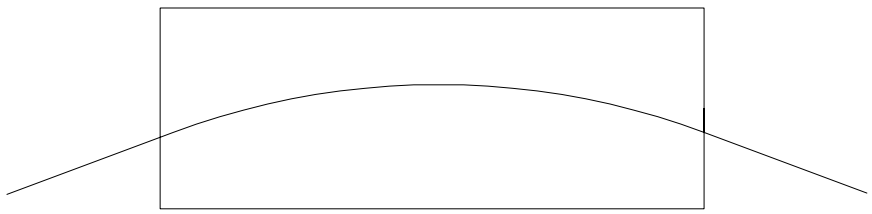
or as:

```
BR:  SBEND,  L = 10.,  ANGLE = 10.0,  K1 = 0.5E-4,  RNMS = 1.0,  K1P = 0.4E-6  ;
```

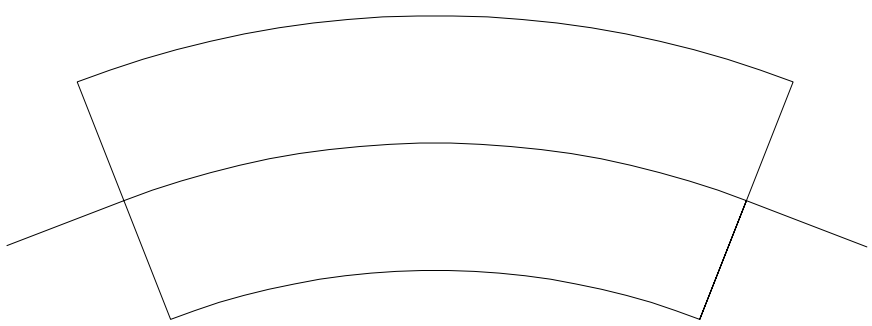
In the use of these keywords, the elements `RBEND` and `SBEND` are identical.

Any of the first-order parameters r_s, r_a, v_r, K_1', and n' may be varied in first-order fitting. The parameters K_2' and ϵ' can be varied in second-order fitting. There are no third-order midplane-symmetry-violating transfer matrix elements in TRANSPORT.

The sign conventions for bending magnet entries are illustrated in the following figure. For TRANSPORT a positive bend is to the right looking in the direction of particle travel. To represent a bend in another sense, the `TILT` keyword should be used to rotate the bend about the entrance reference trajectory.



An RBEND Type of Bending Magnet



An SBEND Type of Bending Magnet

HKICK or VKICK – Horizontal and Vertical Steering Magnets

The difference between a steering magnet and a bending magnet is that the steering magnet does not affect the reference trajectory. If the beam line is perfectly aligned with no errors, and the beam centroid follows the reference trajectory, the field of the steering magnets is zero. The steering magnets are used to compensate for errors and misalignments in the beam line. If they are given a nonzero field, the beam line reference trajectory is not affected. However, the beam is steered either toward or away from the reference trajectory.

In TRANSPORT the elements `HKICK` and `VKICK` should always be used for steering magnets. The centroid shift element `CENTROID` should never be used for that purpose. The steering magnet elements contain a complete set of first-, second-, and third-order transfer matrix elements. This includes the effect of having the angle of deflection vary inversely with the momentum of the particle. The centroid shift element contains none of these effects, but simply deflects the beam by a fixed distance or angle.

A positive angle bends to the right for the `HKICK` element or down for the `VKICK` element. This convention is consistent with that used for bending magnets. In accelerator usage, a steering magnet is often described by its "kick". A kick is considered to be positive if it increases p_x or p_y, or x' or y'. It is then opposite in sense but equal in magnitude to the angle.

There is no provision for non-uniform field in this element. The fringe fields are included automatically. The pole-face rotation angle is automatically set so that the magnet is rectangular. The entrance and exit pole faces are still both perpendicular to the reference trajectory, since the reference trajectory is considered to be undeflected by the kicking magnet. There is also no provision for pole face curvature.

Both `HKICK` and `VKICK` are MAD elements. They may also be expressed in other keyword notation.

MAD Notation

The `HKICK` and `VKICK` elements are specified by two real attributes:

```
label:   HKICK,  L = value,  KICK = value  ;
```

or

```
label:   VKICK,  L = value,  KICK = value  ;
```

An example of a horizontal steering magnet element would then be:

```
HKICK,  L = 1.0,  KICK = -0.1  ;
```

Here the units for the kick are assumed to be in mr. Strict MAD compatability would require the use of `UMAD` units. The kick would then be measured in radians. The angle is the same as in subsequent examples in this section.

Symbol	Keyword	Description
L	`L`	The effective length of the magnet (meters).
$\Delta x'$ or $\Delta y'$	`KICK`	The kick angle (mr in `UMM`, radians in `UMETER`).

The units illustrated are those specified by one of the global unit sets `UMM` or `UMETER`. Other global unit sets are described on page 64. The kick angle may be varied in first-order fitting.

Other Keyword Options

For consistency with bending magnet specifications, it is also possible to parameterize a `KICK` in terms of its length and bend angle. Recall that for a `BEND`, `RBEND`, or `SBEND` a positive bend is one which deflects the reference trajectory toward negative x. The bend angle is then equal in magnitude and opposite in sense from the kick value described above.

```
label: HKICK, L = value, ANGLE = value ;
```

The same `KICK` element as above can then be expressed in terms of the `ANGLE` keyword as

```
HKICK,  L = 1.0,  ANGLE = 0.1  ;
```

The keyword format can also be used to specify a steering magnet in terms of parameters which are more like the original TRANSPORT parameters for a bending magnet.

```
label: HKICK, L = value, B = value ;
```

The keywords available are:

Symbol	Keyword	Description
L	`L`	The effective length of the magnet (meters).
α	`ANGLE`	The deflection angle (mr in `UMM`, radians in `UMETER`).
B	`B`	The magnetic field (kilogauss).

The units for either the length, the bend angle, or the magnetic field may be changed by use of the `UNIT` element.

Assuming a 600 GeV beam, the following element is also equivalent to the previous examples.

```
HKICK,  L = 1.0,  B = 2.0  ;
```

The deflection angle α or the magnetic field B may be varied in first-order fitting.

KICKER – A Steering Magnet in Both Planes

The KICKER element is a steering magnet that steers in both planes simultaneously. The MAD standard does not have a TILT parameter on either HKICK or VKICK. Instead steering in both planes simultaneously is done using the KICKER element. All angle conventions applying to HKICK or VKICK apply to KICKER also.

The KICKER element exists only in MAD notation It also contains first-, second-, and third-order matrix elements. The fringe fields are included automatically.

MAD Notation

The KICKER element is specified by three real attributes:

```
label: KICKER, L = value , HKICK = value, VKICK = value ;
```

A simultaneous kick in the horizontal plane of $-.2$ mr and in the vertical plane of 0.12 mr could be represented as:

```
KICKER, L = 1.0, HKICK = -0.2, VKICK = 0.12 ;
```

Symbol	Keyword	Description
L	L	The effective length of the magnet (meters).
$\Delta x'$	HKICK	The horizontal kick angle (radians in UMAD, degrees in UTRANS).
$\Delta y'$	VKICK	The vertical kick angle (radians in UMAD, degrees in UTRANS).

Here the kicks are assumed to be measured in mr, as in the unit set UMM. Strict MAD compatability would require the use of the unit set UMAD and therefore the angular unit of radians. Other unit sets are also allowable and are described on page 64. Both the horizontal and vertical kick angles are variable in first-order fitting.

Output

A physical steering magnet which bends in both planes is a rotated dipole magnet. To facilitate installation of such a magnet in a beam line, the TRANSPORT output describes a KICKER as a rotated dipole. The rotation angle and the net magnetic field are calculated and printed.

QUADRUPOLE

A quadrupole provides focusing in one transverse plane and defocusing in the other. The reference trajectory through a quadrupole is a straight line. If a quadrupole is misaligned (via an `ALIGN` element) a dipole component will be introduced, steering the beam away from the aligned reference trajectory.

A quadrupole has no second-order geometric aberrations. The only second-order aberrations are chromatic, representing the variation in focusing strength of the quad with momentum.

A quadrupole has geometric aberrations of third and higher orders. When TRANSPORT performs a third-order calculation, the quadrupole fringe fields* are included automatically. The longitudinal field profile is assumed to be rectangular, meaning that the field begins and ends abruptly and is otherwise uniform. No current provision exists for extended fringing fields.

The `QUAD` is a MAD element. It may be expressed in MAD notation or other keyword notation.

MAD Notation

For the MAD program, a magnetic quadrupole is specified by three real attributes.

label: `QUAD(RUPOLE), L =` value, `K1 =` value, `TILT =` value ;

A typical quadrupole specification might then look like:

```
Q1:  QUAD,  L = 5.,  K1 = .01  ;
```

Symbol	Keyword	Description
L	`L`	The quadrupole length (normally in meters, default = 0.0)
K_1	`K1`	The quadrupole coefficient (defined below). A positive quadruole strength indicates horizontal focussing of positively charged particles. (The units are normally meters^{-2})
	`TILT`	The roll angle about the longitudinal axis (degrees with `UMM` and `UMETER`, radians with `UMAD`).

Strict MAD compatability requires the use of `UMAD` units, including radians for the `TILT` parameter. The units of any of the physical parameters can be changed to any desired. Other unit set specifications are described on page 64. Individual unit changes are described on page 68.

*G.E.Lee-Whiting, Nuclear Instruments and Methods 83, 232 (1970).

The keyword `TILT` indicates that the quadrupole is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. *If the word* `TILT` *appears alone without a value, the angle of rotation is taken to be 45 degrees.* This configuration is known to accelerator designers as a skew quadrupole.

The length L, the quadrupole coefficient K_1, and the `TILT` parameter may be varied in first- or higher-order fitting.

Other Keyword Options

The same quadrupole can also be specified by other sets of variables. The first of the two following options corresponds to the original TRANSPORT variables expressed in keyword notation.

label: `QUADRUPO, L =` value `, B =` value, `APER =` value `;`

label: `QUADRUPO, L =` value `, GRAD =` value `;`

The `TILT` parameter may also be used with either of these two options.

Symbol	Keyword	Description
L	`L`	The magnet length (normally meters).
B_o	`B`	The magnetic field (normally kilogauss).
a	`APER(TURE)`	The magnetic half aperture (mm with `UMM`, meters with `UMETER`).
g	`GRAD(IENT)`	The gradient of the magnetic field (kG/mm with `UMM`, kG/m with `UMETER`).

The `UMM` units are meters, kG, and mm respectively. The `UMETER` units for L, B and a are meters, kG, and meters, respectively. Global unit set specifications other than `UMM` or `UMETER` may also be selected and are described on page 64. Individual units changes are described on page 68.

The following three elements are all equivalent.

```
Q1:  QUAD,  L = 5.,  B = 10.,  APER = 50.   ;

Q1:  QUAD,  L = 5.,  GRAD = 0.2  ;

Q1:  QUAD,  L = 5.,  K1 = .01  ;
```

Here we have assumed a beam momentum of 600 GeV/c. We have also assumed that the

aperture is measured in the `UMM` unit of mm. The parameter K_1 is measured in the inverse square of the unit of longitudinal distance (usually meters). The description in terms of K_1 has the advantage that it does not require a separate specification of the beam momentum. The quantities K_1 and g are related to the pole tip field B_o and the aperture a by:

$$K_1 = \frac{g}{(B\rho)} = \frac{1}{(B\rho)} \frac{B_o}{a}$$

The length L, pole-tip magnetic field B, aperture a, and gradient g of a quadrupole may be varied in first- or higher-order fitting.

Output

The horizontal focal length is printed in parentheses as output. A positive focal length indicates horizontal focusing and a negative focal length indicates horizontal defocusing. The quantity actually printed is the reciprocal of the $(x'|x)$ transfer matrix element $(1/R_{21})$ for the quadrupoles. Thus two identical quadrupoles of opposite polarity will have different horizontal focal lengths due to the difference between the sine and the hyperbolic sine.

First-Order Quadrupole Matrix

$$\begin{bmatrix} \cos k_q L & \frac{1}{k_q} \sin k_q L & 0 & 0 & 0 & 0 \\ -k_q \sin k_q L & \cos k_q L & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh k_q L & \frac{1}{k_q} \sinh k_q L & 0 & 0 \\ 0 & 0 & k_q \sinh k_q L & \cosh k_q L & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\gamma^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

These elements are for a quadrupole which focuses in the horizontal (x) plane (B positive). A vertically (y-plane) focusing quadrupole (B negative) has the first two diagonal submatrices interchanged.

<u>Definitions</u>:

L	=	the effective length of the quadrupole.
a	=	the radius of the aperture.
B_0	=	the field at the radius a.
k_q^2	=	$(B_0/a)(1/B_0\rho_0)$, where $(B\rho_0)$ = the magnetic rigidity (momentum) of the central trajectory.
γ	=	relativistic factor.

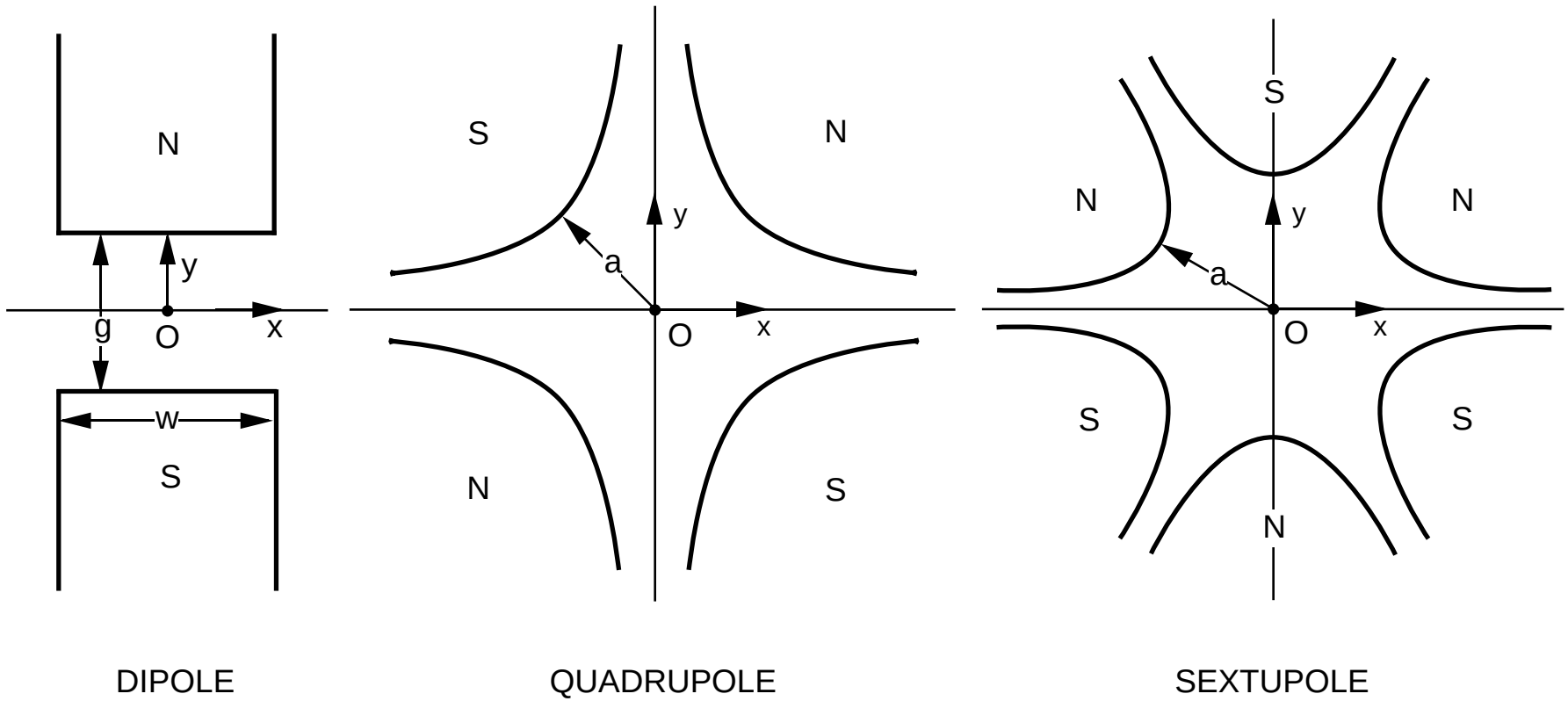


Illustration of the magnetic midplane (x axis) for dipole, quadrupole and sextupole elements. The magnet polarities indicate multipole elements that are positive with respect to each other. The coordinate system is right-handed, so that the beam particles will be emerging from the paper and moving into the face of the reader.

SEXTUPOLE

Sextupole magnets are used to modify second-order aberrations in beam transport systems. The reference trajectory through a sextupole is a straight line. The action of a sextupole on beam particles is a second- and higher-order effect. In first order runs (absence of the `ORDER` element) a correctly aligned sextupole will act as a drift space. See SLAC-75 [4] for a tabulation of sextupole matrix elements.

A misaligned sextupole will produce effective quadrupole and dipole components in addition to the sextupole field. For a more complete description of the effects of misalignments on sextupoles, see the `ALIGN` element on page 180.

First-, second-, and third-order matrix elements for a sextupole are included in TRANSPORT. Third-order terms arise from the momentum dependence of the second-order terms and from the coupling between second-order terms at different longitudinal locations within the same sextupole. For the second- or third-order terms to have any effect, a higher-order calculation must be specified with the `ORDER` command described on page 83.

The `SEXTUPOLE` is a MAD element. It may be expressed in MAD notation or other keyword notation.

MAD Notation

For the MAD program, a magnetic sextupole is specified by three real attributes.

```
label: SEXTUPOLE,  L = value,  K2 = value ,  TILT = value ;
```

A typical sextupole specification might be:

```
S1:   SEXT,  L = 5.,  K2 = 0.2  ;
```

Symbol	Keyword	Description
L	`L`	The magnet length (normally meters).
K_2	`K2`	The normalized sextupole strength (in meters^{-3}).
	`TILT`	The angle of rotation about the optic axis (degrees with `UMM` and `UMETER`, radians with `UMAD`).

Strict MAD compatability requires the use of `UMAD` units, including radians for the `TILT` parameter. The units of any of the physical parameters can be changed to any desired. Other unit set specifications are described on page 64. Individual unit changes are described on page 68.

The quantity K_2 is related to the pole tip field B_o and the aperture a by:

$$K_2 = \frac{1}{(B\rho)} \frac{B_o}{a^2}$$

The normalization of K_2 is such that an sextupole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. It should be noted that in the MAD program itself, the parameterK_2 is defined to be the third derivative of the field divided by $B_0\rho$. The MAD value of K_2 is then greater by a factor of 2 (= 2!) than the value defined here. Specification of the global unit set `UMAD` will cause TRANSPORT to use the MAD definition of multipole strength.

The keyword `TILT` indicates that the sextupole is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. The value given is the angle of rotation (degrees in `UTRANS`, radians in `UMAD`). *If the word* `TILT` *appears alone without a value, the angle of rotation is taken to be 30 degrees.*

The normalized sextupole strength K_2 can be varied in second- or third-order fitting. The variation of a physical parameter is indicated by the `VARY` command, described on page 246.

Other Keyword Options

The same sextupole can be specified by another set of variables. The following parameterization corresponds to the original TRANSPORT variables expressed in keyword notation.

label: `SEXTUPOL, L =` value `, B =` value `, APER =` value `;`

Symbol	Keyword	Description
L	`L`	The magnet length (normally meters).
B_o	`B`	The magnetic field (normally kilogauss).
a	`APER(TURE)`	The magnetic half-aperture or pole-tip radius (mm with `UMM`, meters with `UMETER`).

The `TILT` parameter may also be used with this option.

The `UMM` units are meters, kG, and mm respectively. The `UMETER` units for L, B and a are meters, kG, and meters, respectively. Global unit set specifications other than `UMAD` or `UTRANS` may also be selected and are described on page 64. Individual units changes are described on page 68.

For a beam momentum of 600 GeV/c, the following two elements are equivalent.

```
S1:   SEXT,  L = 5.,  B = 10.,  APER = 50.   ;
```

```
S1:   SEXT,  L = 5.,  K2 = 0.2  ;
```

Here we assume that the aperture is measured in mm. The parameter K_2 is measured in the inverse cube of the unit of longitudinal distance (usually meters). The description in terms of K_2 has the advantage that it does not require a separate specification of the beam momentum. The quantity K_2 is related to the pole tip field B_o and the half aperture a by:

$$K_2 = \frac{1}{(B\rho)} \frac{B_o}{a^2}$$

The normalization of K_2 is such that a sextupole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. It should be noted that in the MAD program itself, the parameter K_2 is defined to be the second derivative of the field divided by $B_0\rho$. The MAD value of K_2 is then greater by a factor of 2 than the value defined here.

The pole-tip magnetic field B can be varied in second- or third-order fitting. It may also be constrained not to exceed a certain specified maximum field. The aperture may not be varied. The variation of a physical parameter is indicated by the `VARY` command, described on page 246.

The normal orientation of a sextupole is shown in the following figure. Other orientations of the sextupole may be obtained using the beam rotation element (`SROT`).

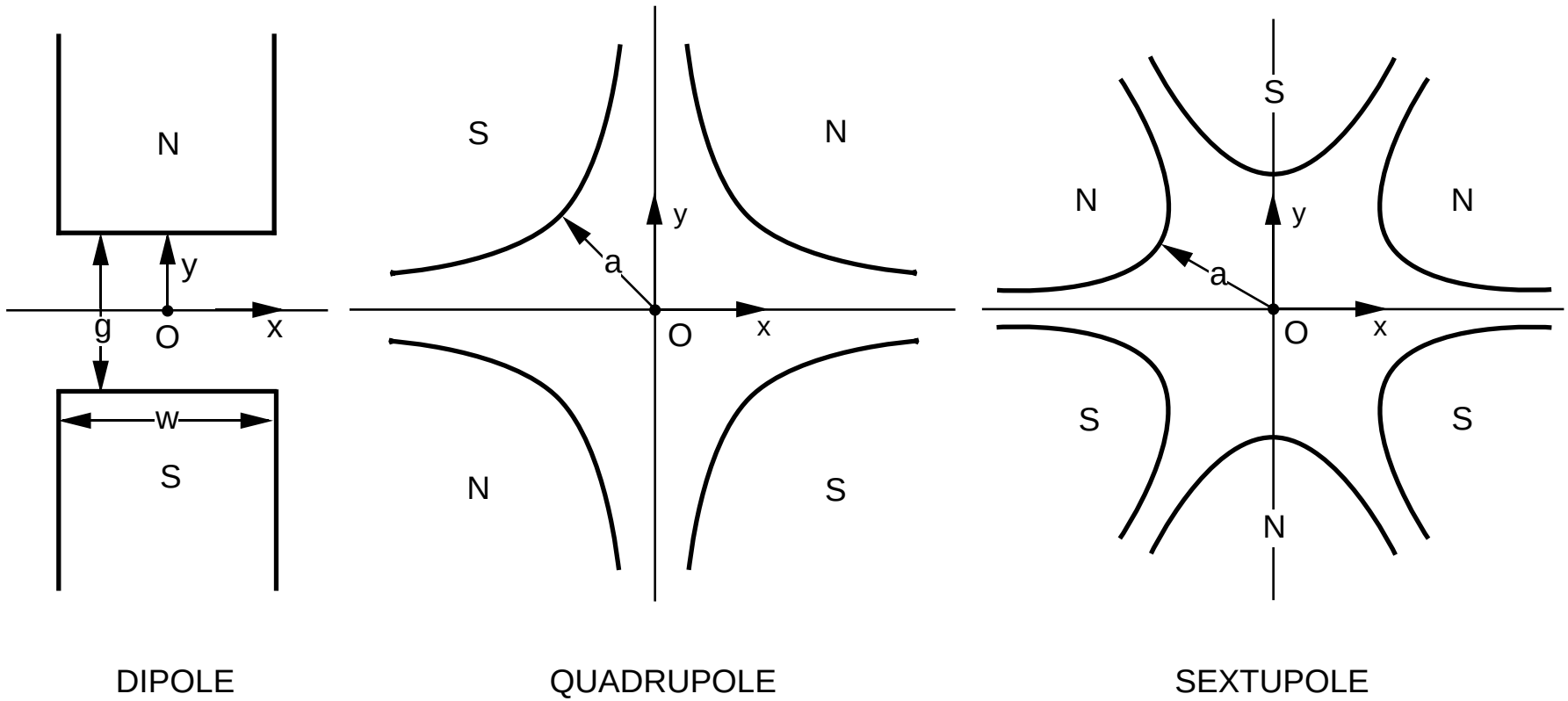


Illustration of the magnetic midplane (x axis) for dipole, quadrupole and sextupole elements. The magnet polarities indicate multipole elements that are positive with respect to each other. The coordinate system is right-handed, so that the beam particles will be emerging from the paper and moving into the face of the reader.

OCTUPOLE

Octupole magnets are used to affect third-order aberrations in charged particle optical systems. The effect of an octupole is limited to third and higher orders. In a first- or second-order calculation, a correctly aligned octupole will act as a drift space. A misaligned octupole will have effective sextupole, quadrupole, and dipole components also.

Matrix elements up to and including third order can be calculated by TRANSPORT for an octupole. The second-order terms for an octupole are all zero. The difference between an octupole and a drift space begins at third order. In order to activate the third-order matrix elements for an octupole a third-order calculation must be specified via the `ORDER` element, described on page 83.

The `OCTUPOLE` is a MAD element. It may be expressed in MAD notation or other keyword notation.

MAD Notation

For the MAD program, a magnetic octupole is specified by three real attributes.

label: `OCTUPOLE, L =` value , `K3 =` value , `TILT =` value ;

A typical octupole specification might then look like:

```
O1:   OCTUPOLE,  L = 5.,  K1 = 4.0  ;
```

Symbol	Keyword	Description
L	`L`	The magnet length (normally meters).
K_3	`K3`	The normalized octupole component (normally meters^{-4}, default 0.0).
	`TILT`	The angle of rotation about the optic axis (degrees with `UMM` and `UMETER`, radians in `UMAD`).

Strict MAD compatability requires the use of `UMAD` units, including radians for the `TILT` parameter. The units of any of the physical parameters can be changed to any desired. Other unit set specifications are described on page 64. Individual unit changes are described on page 68.

The quantity K_3 is related to the pole tip field B_o and the aperture a by:

$$K_3 = \frac{1}{(B\rho)} \frac{B_o}{a^3}$$

The normalization of K_3 is such that an octupole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory.

It should be noted that in the MAD program itself, the parameterK_3 is defined to be the third derivative of the field divided by $B_0\rho$. The MAD value of K_3 is then greater by a factor of 6 (= 3!) than the value defined here. Specification of the global unit set `UMAD` will cause TRANSPORT to use the MAD definition of multipole strength. The description in terms of K_3 has the advantage that it does not require a separate specification of the beam momentum.

The keyword `TILT` indicates that the octupole is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. The value given is the angle of rotation in degrees. *If the word* `TILT` *appears alone without a value, the angle of rotation is taken to be 22.5 degrees.*

The normalized octupole strength K_3 can be varied in third-order fitting. The variation of a physical parameter is indicated by the `VARY` command, described on page 246.

Other Keyword Options

The same octupole can also be specified by another set of variables. The following option exists for consistency with the original TRANSPORT set of variables

```
label: OCTUPOLE,  L = value ,  B = value ,  APER = value ;
```

Symbol	Keyword	Description
L	`L`	The magnet length (normally meters).
B_o	`B`	The magnetic field (normally kilogauss).
a	`APER(TURE)`	The magnetic aperture (mm in `UMM`, meters in `UMETER`).

The `TILT` parameter may also be used with this option.

The `UMM` units are meters, kG, and mm respectively. The `UMETER` units for L, B, and a are meters, kG, and meters respectively. Global unit set specifications other than `UMAD` or `UTRANS` may also be selected and are described on page 64. Individual units changes are described on page 68.

For a beam momentum of 600 GeV/c, the following two elements are equivalent.

```
O1:   OCTUPOLE,  L = 5.,  B = 10.,  APER = 50.   ;

O1:   OCTUPOLE,  L = 5.,  K1 = 4.0  ;
```

Here we assume that the aperture is specified in mm. The parameter K_3 is measured in the inverse fourth power of the longitudinal distance (usually meters). The description in terms of K_3 has the advantage that it does not require a separate specification of the beam momentum. The quantity K_3 is related to the pole tip field B_o and the half aperture a by:

$$K_3 = \frac{1}{(B\rho)} \frac{B_o}{a^3}$$

The normalization of K_3 is such that a octupole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. It should be noted that in the MAD program itself, the parameter K_3 is defined to be the second derivative of the field divided by $B_0\rho$. The MAD value of K_3 is then greater by a factor of 6 than the value defined here.

The pole-tip magnetic field B or the quantity K_3 can be varied in third-order fitting. The aperture a may not be varied. The variation of a physical parameter is indicated by the `VARY` command, described on page 246.

PLASMALENS – Plasma (Lithium) Lens

A plasma lens has focusing characteristics similar to those of a quadrupole, except that the plasma lens can focus in both transverse planes simultaneously. The specification for a plasma lens is the same as that for a quadrupole.

First-, and second-order matrix elements for a plasma lens are included in TRANSPORT. The second-order terms arise from the momentum variation of the first order terms (chromatic aberration). Third-order terms for a plasma lens are not in TRANSPORT at this time. A higher-order calculation is requested with the `ORDER` element, described on page 83.

The plasma lens is not a MAD element. However, it can be described using the same parameters as a quadrupole. It therefore can be expressed in MAD-like notation or other keyword notation.

MAD-like Notation

The only reason that the notation is described as MAD-like is that this element is not in the MAD program. The keywords which can be used in specifying a plasma lens are the same as for a quadrupole. They are here repeated for convenience. A plasma lens is specified by two real attributes.

```
label: PLASMA,  L = value,  K1 = value  ;
```

A typical plasma lens specification might then look like:

```
P1:   PLAS,  L = 5.,  K1 = .01  ;
```

Symbol	Keyword	Description
L	`L`	The magnet length (normally meters).
K_1	`K1`	The normalized quadrupole strength (normally in meters^{-2}).

The units of any of the physical parameters can be changed to any desired. Other unit set specifications are described on page 64. Individual unit changes are described on page 68. The length and normalized quadrupole strength K_1 of the plasma lens can be varied in first-order fitting.

Other Keyword Options

The same plasma lens can also be specified by other sets of variables. The first of the two following options corresponds to the original TRANSPORT variables.

```
label: PLASMA,  L = value ,  B = value ,  APER = value ;

label: PLASMA,  L = value ,  GRAD = value ;
```

Symbol	Keyword	Description
L	`L`	The magnet length (normally meters).
B_o	`B`	The magnetic field (normally kilogauss).
a	`APER(TURE)`	The magnetic aperture (mm with `UMM`, meters with `UMETER`).
g	`GRAD(IENT)`	The gradient of the magnetic field (kg/mm with `UMM`, kg/m with `UMETER`).

If the unit set `UMM` is specified, the corresponding units are meters, kG, and mm respectively. If the global unit set `UMETER` is specified, the units for L, B and a are meters, kG, and meters, respectively. Global unit set specifications other than `UMM` or `UMETER` may also be selected and are described on page 64. Individual units changes are described on page 68.

For a beam momentum of 600 GeV/c, the following three elements are all equivalent.

```
P1:   PLAS,  L = 5.,  B = 10.,  APER = 50.   ;

P1:   PLAS,  L = 5.,  GRAD = 0.2  ;

P1:   PLAS,  L = 5.,  K1 = .01  ;
```

Here we assume that the aperture is measured in mm.

The description in terms of K_1 has the advantage that it does not require a separate specification of the beam momentum. The quantities K_1 and g are related to the pole tip field B_o and the aperture a by:

$$K_1 = \frac{g}{(B\rho)} = \frac{1}{(B\rho)} \frac{B_o}{a}$$

The length L, magnetic field B, and gradient g of a plasma lens may all be varied in first-order fitting. The aperture a may not be. The variation of a physical parameter is indicated by the `VARY` command, described on page 246.

SOLENOID

The solenoid is most often used as a focusing element in systems passing low-energy particles. The solenoid transformation is also used in conjunction with particle detectors in interaction regions of particle colliders.

Particles in a solenoidal field travel along helical trajectories. The solenoid fringing field effects necessary to produce the focusing are included.

First- and second-order matrix elements for a solenoid are included in TRANSPORT. The second-order matrix elements arise from the momentum variation of the first-order elements. Third-order matrix elements for a solenoid have yet to be included in TRANSPORT. The order of the calculation is indicated by the `ORDER` element described on page 83.

The `SOLENOID` is a MAD element. It can be expressed in MAD notation or other keyword notation.

MAD Notation

For the MAD program, a solenoid is specified by two real attributes.

```
label: SOLENOID,  L = value ,  KS = value  ;
```

A typical solenoid specification might then look like:

```
SOL1:   SOLEN,  L = 1.0,  KS = .00025  ;
```

Symbol	Keyword	Description
L	`L`	Effective length of the solenoid (meters).
θ	`KS`	The solenoid strength in terms of the reference momentum rotation angle (degrees with `UMM` or `UMETER`, radians with `UMAD`).

Other units may be selected either by use of a different global unit specification (page 64) or individual unit changes (page 68).

The length L and solenoid strength θ may be varied in first-order fitting.

Other Keyword Options

A solenoid may alse be specified by its length and field.

```
label: SOLENOID,  L = value ,  B = value  ;
```

The same example as above would then be written as:

```
SOL1:   SOLEN,  L = 1.0,  B = 10.0  ;
```

Here we have assumed a beam momentum of 600 GeV/c. This is a higher value than than the momentum used in most solenoid applications. However, we continue to use it here for consistency with the other sections.

Symbol	Keyword	Description
L	`L`	Effective length of the solenoid (normally meters).
B	`B`	The field (normally kiloGauss).

The units are meters and kG in the three global unit sets `UMM`, `UMETER`, and `UMAD`. For strict compatability to the MAD program, the unit set `UMAD` is required.

If other units are desired they can be chosen via other global units specifications or the appropriate `UNIT` element preceding the `BEAM` element. The units of either physical parameter can be changed with a `UNIT` command as described on page 68.

The relation among its length and field and its rotation angle is

$$\frac{BL}{(B\rho)} = 2\theta$$

If the reference coordinate system is rotated by the angle θ after the solenoid, then the transfer matrix will take the block diagonalized form similar to that of a quadrupole.

The solenoid length L and magnetic field B may be varied in first-order fitting

First-Order Solenoid Matrix *

Definitions:

L = effective length of solenoid.
$K = B_0/(2B\rho_0)$, where B_0 is the field inside the solenoid and $(B\rho_0)$ is the magnetic rigidity (momentum) of the central trajectory.
$C = \cos KL$
$S = \sin KL$
γ = relativistic factor

$$R(\text{Solenoid}) = \begin{pmatrix} C^2 & \frac{1}{K}\,SC & SC & \frac{1}{K}\,S^2 & 0 & 0 \\ -KSC & C^2 & -KS^2 & SC & 0 & 0 \\ -SC & -\frac{1}{K}\,S^2 & C^2 & \frac{1}{K}\,SC & 0 & 0 \\ KS^2 & -SC & -KSC & C^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\gamma^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Alternate Forms of Matrix Representation of the Solenoid:

Rotating the transverse coordinates about the z axis by an angle $= -KL$ decouples the x and y first-order terms, i.e.

$$R(-KL)\cdot R(\text{Solenoid}) = \begin{pmatrix} C & \frac{1}{K}\,S & 0 & 0 & 0 & 0 \\ -KS & C & 0 & 0 & 0 & 0 \\ 0 & 0 & C & \frac{1}{K}\,S & 0 & 0 \\ 0 & 0 & -KS & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\gamma^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where $KS = \dfrac{1}{F}$, and F = focal length of solenoid.

*For a derivation of this transformation see report SLAC-4 by R. Helm [14].

ACCELERATION

An energy gain is reflected in both the divergence and the width of the beam. This element provides a simulation of a travelling wave linear accelerator energy gain over a field free drift length (i.e. no externally applied magnetic field).

The energy of the reference trajectory is assumed to increase linearly over the entire accelerator length. If this is not the case, an appropriate model may be constructed by combining separate `ACCEL` elements. An `ACCEL` element with a zero energy gain is identical to a drift length.

First-order matrix elements are included in TRANSPORT for the accelerator section. Second- or third-order matrix elements have not been incorporated in the program.

The `ACCEL` element is MAD compatible. It can be expressed in MAD notation or other keyword notation.

MAD Notation

For the MAD program, a travelling wave linear accelerator element is specified by four real attributes.

label: `ACCEL, L` = value , `VOLT` = value , `LAG` = value , `FREQ` = value ;

A typical accelerator element might then look like:

```
ACCEL,  L = 10.0,  VOLT = 0.1,  LAG = 30.0,  FREQ = 30.   ;
```

Symbol	Keyword	Description
L	`L`	Accelerator length (meters).
	`VOLT`	Energy gain (GeV) at crest of wave.
ϕ	`LAG`	Phase lag (degrees).
ν	`FREQ`	The RF frequency in MHz (no default value).

The units of only the length and the energy gain may be changed. The different unit set specificiations are described on page 64. Individual unit changes are described on page 68. None of the parameters can be varied.

Other Keyword Notation

The original TRANSPORT set of parameters can also be expressed in keyword notation by four attributes.

label: `ACCEL, L` = value , `VOLT` = value , `LAG` = value , `WAVE` = value ;

The following two parameterizations of the accelerator element are equivalent.

```
ACCEL,  L = 10.0,  VOLT = 0.1,  LAG = 30.0,  FREQ = 30.   ;

ACCEL,  L = 10.0,  VOLT = 0.1,  LAG = 30.0,  WAVEL = 1000.0  ;
```

Here we have taken the units of wavelength to be mm.

Symbol	Keyword	Description
L	`L`	Accelerator length (meters).
	`VOLT`	Energy gain (GeV) at crest of wave.
ϕ	`LAG`	Phase lag (degrees).
λ	`WAVELENGTH`	Wavelength (mm in `UMM`, meters in `UMETER`).

The units for the length, the energy gain, and the wavelength may be changed. The beam unit of longitudinal separation is used for the wavelength and may be changed by the `UNIT` command described on page 68. None of the parameters can be varied.

The new beam energy is taken as the new energy of the reference particle and is printed as output. The energy gain of the reference particle is $\Delta E \cos \phi$.

Accelerator Section Matrix for Effectively Massless Particles

This matrix assumes that $E_0 \gg m_0 c^2$.

$$\begin{bmatrix} 1 & \left[L\,\dfrac{E_o}{\Delta E\cos\phi}\,\log\left(1+\dfrac{\Delta E\cos\phi}{E_o}\right)\right] & 0 & 0 & 0 & 0 \\ 0 & \dfrac{E_o}{E_o+\Delta E\cos\phi} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \left[L\,\dfrac{E_o}{\Delta E\cos\phi}\,\log\left(1+\dfrac{\Delta E\cos\phi}{E_o}\right)\right] & 0 & 0 \\ 0 & 0 & 0 & \dfrac{E_o}{E_o+\Delta E\cos\phi} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \left(\dfrac{\Delta E\sin\phi}{E_o+\Delta E\cos\phi}\right)\left(\dfrac{2\pi}{\lambda}\right) & \dfrac{E_o}{E_o+\Delta E\cos\phi} \end{bmatrix}$$

<u>Definitions</u>:

L = effective length of accelerator sector.

E_0 = particle energy at start of sector.

ΔE = Energy gain over sector length at crest of accelerating wave.

ϕ = phase lag of the reference particle <u>behind</u> the crest of the accelerating wave, i.e. if ϕ is positive then for some $\ell > 0$ the particles having this value are riding the crest of the wave; the units of ϕ are degrees.

λ = Wavelength of accelerating wave; the units of λ are those of ℓ (normally cm).

Accelerator Section Matrix for Massive Particles

The nonrelativistic case involves more complicated matrix elements, so the expressions for the elements must be given individually: The case for massless particles can be solved exactly. The massive particle case can be solved only approximately. The solution given here is based on the WKB approximation of quantum mechanics.

$$R_{11} = R_{33} = \left(\frac{\eta_f}{\eta_i}\right)^{\frac{1}{4}} \cosh\alpha_1 - \frac{1}{4}\,\frac{\eta_i'}{\eta_i}\,\frac{\eta_i^{\frac{3}{2}}}{\sqrt{Q}}\left(\frac{\eta_f}{\eta_i}\right)^{\frac{1}{4}} \sinh\alpha_1$$

$$R_{12} = R_{34} = \frac{\eta_i^{\frac{3}{2}}}{\sqrt{Q}}\left(\frac{\eta_f}{\eta_i}\right)^{\frac{1}{4}} \sinh\alpha_1$$

$$R_{21} = R_{43} = \frac{1}{4}\left[\frac{\eta_f'}{\eta_f^{\frac{3}{4}}\eta_i^{\frac{1}{4}}} - \frac{\eta_i'\eta_i^{\frac{1}{4}}}{\eta_f^{\frac{5}{4}}}\right]\cosh\alpha_1 + \left[\left(\frac{\eta_f}{\eta_i}\right)^{\frac{1}{4}}\frac{\sqrt{Q}}{\eta_f^{\frac{3}{2}}} - \frac{1}{16}\,\frac{\eta_f'\eta_i'\eta_i^{\frac{1}{4}}}{\eta_f^{\frac{3}{4}}\sqrt{Q}}\right]\sinh\alpha_1$$

$$R_{22} = R_{44} = \frac{1}{4}\,\frac{\eta_i^{\frac{3}{2}}}{\sqrt{Q}}\,\frac{\eta_f'}{\eta_f^{\frac{3}{4}}\eta_i^{\frac{1}{4}}}\sinh\alpha_1 + \left(\frac{\eta_i}{\eta_f}\right)^{\frac{5}{4}}\cosh\alpha_1$$

$$R_{55} = \frac{\beta_f}{\beta_i}\left(\frac{\eta_i}{\eta_f}\right)^{\frac{3}{4}}\left[\cos\alpha_2 + \frac{3}{4}\,\frac{\eta_f'\eta_f^{\frac{1}{2}}}{\sqrt{Q_L}}\sin\alpha_2\right]$$

$$R_{56} = -\frac{\beta_f\lambda}{2\pi}\,\frac{L}{\Delta E\sin\phi_s}\left\{\frac{3}{4}\left[\left(\frac{\eta_i}{\eta_f}\right)^{\frac{1}{4}}\eta_f' - \left(\frac{\eta_i}{\eta_f}\right)^{\frac{3}{4}}\eta_i'\right]\beta_i\cos\alpha_2\right.$$

$$\left. - \left[\frac{9}{16}\,\frac{\eta_i'\eta_f'}{\eta_f^{\frac{1}{4}}\eta_i^{\frac{1}{4}}}\,\frac{\eta_i}{\sqrt{Q_L}} + \left(\frac{\eta_i}{\eta_f}\right)^{\frac{1}{4}}\frac{\sqrt{Q_L}}{\eta_f^{\frac{1}{2}}}\right]\beta_i\sin\alpha_2\right\}$$

$$R_{65} = \frac{\eta_i^{\frac{3}{4}}}{\eta_f^{\frac{1}{4}}\beta_i\beta_f}\sqrt{Q_L}\sin\alpha_2$$

$$R_{66} = \left(\frac{\eta_i}{\eta_f}\right)^{\frac{1}{4}}\frac{\beta_i}{\beta_f}\left[\cos\alpha_2 - \frac{3}{4}\,\frac{\eta_i^{\frac{1}{2}}\eta_i'}{\sqrt{Q_L}}\sin\alpha_2\right]$$

<u>Definitions</u>:

$$L = \text{effective length of accelerator sector.}$$

$$E_0 = \text{particle energy at start of sector.}$$

$$\Delta E = \text{energy gain over sector length at crest of accelerating wave.}$$

ϕ = phase lag of the reference particle <u>behind</u> the crest of the accelerating wave, i.e. if ϕ is positive then for some $\ell > 0$ the particles having this value are riding the crest of the wave; the units of ϕ are degrees.

λ = wavelength of accelerating wave; the units of λ are those of ℓ (normally cm).

$$\alpha_1 = \sqrt{Q} I_\eta$$

$$\alpha_2 = \sqrt{Q_L} I_\eta$$

$$Q = \frac{\Delta E \sin\phi}{L} \; \frac{\pi}{\lambda} \; \frac{1}{m_p c^2} \quad , \quad Q_L = 2Q$$

$$I_\eta = \int_0^L \frac{dz}{\eta^{\frac{3}{2}}}$$

$$\approx \frac{1}{\eta_i^{\frac{3}{2}} \gamma_f'} \left[\Delta\gamma - \frac{3}{4} \frac{\gamma_i}{\eta_i^2} (\Delta\gamma)^2 - \frac{1}{4\eta_i^2} (\Delta\gamma)^3 + \frac{7}{8} \frac{\gamma_i^2}{\eta_i^4} (\Delta\gamma)^3 \right]$$

$$\Delta\gamma = \gamma_f - \gamma_i \; , \quad \gamma' = \frac{\Delta\gamma}{L}$$

$$\eta = \beta\gamma \; , \quad \eta' = \frac{\gamma}{\eta} \gamma'$$

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The index i indicates initial values. The index f indicates final values, so that

$$\gamma_i = \frac{E_0}{m_p}$$

$$\gamma_f = \frac{E_0 + \Delta E \cos\phi}{m_p}$$

SEPTUM

An electrostatic septum is used to split a particle beam into two portions differently directed. Wires are strung transversely on the element $(x - z)$ midplane. The electric field above the wires is in the positive y direction and deflects the particles upward. The electric field below the wires is in the negative y direction and deflects the particles downward. There is therefore a sudden discontinuity in field direction at the plane of the wires. Such a discontinuity is not consistent with the formalism of TRANSPORT.

The `SEPTUM` element is included in TRANSPORT for compatability with the program TURTLE. In TRANSPORT this element is treated by assuming that the field above the wires defines the beam we wish to follow. The field above the wires is then considered to fill the entire space through which the beam passes. The discontinuity in the field at the wire plane is ignored.

The septum is assumed to be aligned along the incoming coordinate axes. The septum element is treated like a kick. The reference trajectory is defined as if the electric field is not present. It is a straight line continuation of the entrance reference trajectory. The beam centroid will be displaced from the reference trajectory in the direction of the electric field. The septum element will also produce a vertical dispersion. Unlike the case with magnetic elements, this dispersion will depend on the particle mass in addition to its central momentum.

Only first-order transfer matrix elements are included in TRANSPORT for the septum element.

The `SEPTUM` element is not a MAD element.

Keyword Notation

An electrostatic septum is specified by the following real attributes.

Symbol	Keyword	Description
L	`L`	The septum length (meters, default: 0.0 m)
V	`VOLTAGE`	The voltage across the septum gap (megavolts, default: 0.0 MV)
a	`APERTURE`	The septum gap (mm in `UMM`, meters in `UMETER`)
E	`E`	The electric field (Mv/mm in `UMM`, MV/m in `UMETER`)
Δy	`OFFSET`	The vertical offset of the wire plane (mm in `UMM`, meters in `UMETER`)
d	`WIRE`	The wire diameter (mm in `UMM`, meters in `UMETER`)
ΔL	`SPACING`	The wire spacing (normally in meters)
r	`RADL`	The radiation length in the wire (used for scattering calculations) (normally in meters)

`TILT` The roll angle about the longitudinal axis (degrees in `UMM` and `UMETER`). The septum is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. If the word `TILT` appears alone without a value, the angle of rotation is taken to be 90 degrees.

Global unit set specifications other than `UMM` or `UMETER` may also be selected and are described on page 64. Individual units changes are described on page 68.

Several of the parameters are redundant, and others are useful only for TURTLE. For TRANSPORT, a complete description of an unrotated septum can be given by:

```
label: SEPTUM,  L = value ,  VOLT = value ,  APER = value ;
```

or by

```
label: SEPTUM,  L = value,  E = value ;
```

Assuming the `UMM` set of units, a typical septum specification might appear as:

```
SEPTUM,  L = 3.0,  VOLT = 0.1,  APER = 10.0  ;
```

or

```
SEPTUM,  L = 3.0,  E = 0.01  ;
```

indicating a septum of length 3 meters and a voltage of 0.1 megavolts on a 10 mm aperture.

None of the parameters of a `SEPTUM` element may be varied.

MARKER – Location markers

In the MAD program, beam lines are defined by explicit commands which assemble the beam lines from previously defined components. Marker elements may be among those components. Their sole purpose is to demarcate a place in the beam line. Operations, which refer to those markers, can be defined after the beam line specification is complete. By the use of markers a beam line can be specified in an uninterrupted manner in MAD notation. The operations, not necessarily in MAD notation, will be all together after the beam line specification.

The `MARKER` is a MAD element. A marker has no additional keywords but is completely given by the word `MARKER` plus a label. Any print statements, constraints, matrix updates or other operations may refer to the markers placed in the beam line. When TRANSPORT runs through the beam line, it executes the operations when it encounters the corresponding marker.

A marker element with the label "`MAR1`" would have the appearance:

```
MAR1:   MARKER ;
```

If the MAD procedure for assembling beam lines is used, these markers may then be inserted into a beamline or subline defined by the `LINE` statements. This procedure is described on page 204.

If the beam line is defined by simply listing the elements, the `MARKER` element can be placed in order with the other physical elements. Reference to the markers can be made by any of the elements specifying operations. These operations include the `UPDATE` element, fitting constraints (the `FIT` element), input-output options (the `PRINT` element), the storage of matrix elements (`STORE` element), the alignment marker (`ALMARK`), and the plot command (`PLOT`).

Normally, markers make their presence known through the operations which refer to them. They are not printed in the output as separate elements. However, if the command

```
PRINT,  MARKER  ;
```

is placed in the data near the beginning, then all subsequent markers will be printed explicitly in the output.

Other Transformations

MATRIX — Arbitrary Transformation Input

To allow for the use of empirically determined fringing fields and other specific (perhaps nonphase-space-conserving) transformations, provision has been made for reading in an arbitrary transformation matrix. First-, second-, and third-order matrix elements may be included. The order of calculation that will be used in the program is determined by the order parameters on the `ORDER` element described on page 83.

The first-order 6×6 matrix is read in row by row.

Notation

A typical `MATRIX` element setting the R_{12} matrix entry to 5.0 would look like:

```
MATRIX,  R12 = 5.0  ;
```

Any or all of the members of a given row of a matrix may be specified in a single TRANSPORT element.

A complete matrix must be read and applied one row at a time. Rows that do not differ from the unit transformation need not be read. If one of the elements on the diagonal of the matrix is unspecified, it will be set equal to 1.0.

The units must be those used to print the transfer matrix; in other words, consistent with the `BEAM` input/output. The `BEAM` element and the units used for the specification of the input phase space are described on page 99.

Usage

Any of the components of a row may be varied; however, there are several restrictions.

`MATRIX` elements that immediately follow one another will all be used to form a single transformation matrix. If distinct matrices are desired, another element must be inserted to separate the `MATRIX` elements. Several do-nothing elements are available; for example, a zero length drift (`DRIFT,  L = 0.  ;`) is a convenient one.

When the last of a sequence of `MATRIX` cards is read, the assembled transformation matrix will be printed in the output. Note that

$$\begin{pmatrix} 1 & 0 \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ 0 & 1 \end{pmatrix} \neq \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

Hence, a matrix formed by successive (`MATRIX  ; DRIFT,  L = 0.  ;  MATRIX ;`) – elements is not always equal to the one formed by leaving out the (`DRIFT,  L = 0.  ;`)

element.

If components of a `MATRIX` element are to be varied, they must be specified on the last `MATRIX` card for the given matrix. Components to be varied can occur in any single row, as the rows do not have to be specified in order (i.e. the second row can be specified before the first, etc.) If more than one row is to have variable components, the user must therefore split the matrix into factors .

Second Order

If it is desired to include some second-order matrix coefficients for the ith row, then the keywords used for such coefficients consist of the letter "T", followed by three integers. If, in the example given above, we also wished to set the T_{126} matrix element to 2.5E-2, then we would write the TRANSPORT element as

```
MATRIX,  R12 = 5.0,  T126 = 2.5E-2  ;
```

As in the first-order case, full rows not different from the identity matrix (i.e., $R_{ii} = 1$, all other $R_{ij} = 0$, and all $T_{ijk} = 0$) need not be read in. The second-order arbitrary matrix elements can not be varied.

Third Order

It is also possible to include any third-order matrix coefficients for the ith row by specifying any non-zero matrix elements.. If the U matrix element U_{1266} is to be set to -2.5E-4, then the example above would be modified to

```
MATRIX,  R12 = 5.0,  T126 = 2.5E-2,  U1266 = -2.5E-4  ;
```

All U matrix elements not specified otherwise will be assumed to be zero. The third-order arbitrary matrix elements can not be varied.

SROT — Coordinate Rotation

The transverse coordinates x and y may be rotated through an angle α about the z axis (the axis tangent to the central trajectory at the point in question.* Thus a rotated bending magnet, quadrupole, or sextupole may be inserted into a beam transport system by preceding and following the element with the appropriate coordinate rotation. (See examples below.) The positive sense of rotation is clockwise looking in the direction of the positive z axis, i.e. the direction of beam travel.

The coordinate rotation is an exact first-order transformation. There are no higher order terms. However, the coordinate rotation will affect the higher-order terms from the elements that precede it.

The `SROT` element is used in MAD. Since there is only parameter to be specified, there is no other keyword notation.

Notation

For the MAD program, a rotation about the longitudinal axis is specified by a single physical attribute:

```
label: SROT,  ANGLE=value ;
```

A rotation of 90° expressed in radians (using the global unit set `UMAD`) would then be:

```
SROT,  ANGLE = 1.570796327  ;
```

and in degrees (using the global unit set `UMM` or `UMETER`) would be

```
SROT,  ANGLE = 90.   ;
```

Symbol	Keyword	Description
α	`ANGLE`	The angle of rotation (degrees in `UMM` or `UMETER`, radians in `UMAD`, default: 0.0 rad)

The units of the rotation angle can be changed to any desired. Other unit set specifications are described on page 64. Individual unit changes are described on page 68.

The angle of rotation may be varied in first-order fitting. The variation of physical parameters is specified with the `VARY` command. (For details, see page 246.)

*See SLAC-75 [4], page 45 and page 12, fig. 4, for definitions of x, y, and z coordinates.

Printing Transfer and Beam Matrices

In early versions of TRANSPORT the transfer and beam matrices and the floor coordinates were always printed in the rotated system. The transfer matrix elements would have mysterious gaps or regions of value zero when traversing a vertically bending magnet. These gaps occurred because the transfer matrix temporarily became block off-diagonalized when the coordinate system was rotated by 90 degrees.

It is now possible to have all these quantities printed in the unrotated system. Then the directions of vertical and horizontal will remain fixed even if the beam line contains many magnets rotated about the beam axis at various angles. The transfer matrix elements and the accelerator functions β, α, and η will now have the physical meaning one would normally expect them to have. The command

```
PRINT,  REFER ;
```

placed at the beginning of the data, will cause the various matrices and coordinates to be printed in the unrotated system. The constraints will also pertain to the unrotated system.

Examples:

For a bending magnet, the beam rotation matrix may be used to specify a rotated magnet. In all these examples, original TRANSPORT notation is either used or implied. With keyword notation, the rotation of certain magnetic elements can also be done by the use of the `TILT` parameter on the element specification. Bending magnets, including fringing fields, can be represented as a single `RBEND` or `SBEND` element. (See page 134) These two elements, along with the `QUAD`, `SEXT`, `OCT`, and `SEPTUM` elements, all can be rotated with a `TILT` parameter.

Example No. 1

A bend up is represented by rotating the x,y coordinates by -90.0 degrees as follows:

```
SROT,  ANGLE = -90.   ;
ROTAT,  -  ;
BEND,  -  -  -  ;
ROTAT,  -  ;
SROT,  ANGLE = +90.   ;   (returns coordinates to their initial orientation)
```

A bend down is accomplished via a +90 degree rotation.

```
SROT,  ANGLE = +90.   ;
ROTAT,  -  ;
BEND,  -  -  - ;
ROTAT,  -  ;
SROT,  ANGLE = -90.   ;
```

A bend to the left (looking in the direction of beam travel) is accomplished by rotating the x,y coordinates by 180 degrees, e.g.

```
SROT,  ANGLE = +180.  ;
ROTAT,
BEND,
ROTAT,
SROT,  ANGLE = -180.  ;
```

Example No. 2

A quadrupole rotated clockwise by 60 degrees about the positive z axis would be specified as follows:

```
SROT,  ANGLE = +60.   ;
QUAD,  -  -  -  ;
SROT  ANGLE = -60.   ;
```

Beam rotation matrix

$$R = \begin{pmatrix} C & 0 & S & 0 & 0 & 0 \\ 0 & C & 0 & S & 0 & 0 \\ -S & 0 & C & 0 & 0 & 0 \\ 0 & -S & 0 & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where $C = \cos\alpha$,
$S = \sin\alpha$,
α = angle of coordinate rotation about the beam axis.

e.g. for $\alpha = +90$ degrees, this matrix interchanges rows 1 and 2 with 3 and 4 of the accumulated R matrix as follows:

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} R(11) & R(12) & R(13) & R(14) \\ R(21) & R(22) & R(23) & R(24) \\ R(31) & R(32) & R(33) & R(34) \\ R(41) & R(42) & R(43) & R(44) \end{bmatrix} = \begin{bmatrix} R(31) & R(32) & R(33) & R(34) \\ R(41) & R(42) & R(43) & R(44) \\ -R(11) & -R(12) & -R(13) & -R(14) \\ -R(21) & -R(22) & -R(23) & -R(24) \end{bmatrix}$$

(The rest of the matrix is unchanged.)

SHIFT — Shift in the Reference Coordinate System

If a particle beam is transferred from one optical system to another, there may be a discontinuity in the reference coordinate systems used. The centroid coordinates may change at a given point due to this discontinuity. The effect may be parameterized by the change in the centroid coordinates. The `SHIFT` element is specified in exactly the same manner as the `CENTROID` element, except that the keyword here is `SHIFT` instead of `CENTROID`.

The `SHIFT` element does affect the floor coordinates. This property is the main difference between the `CENTROID` and the `SHIFT` elements.

The `SHIFT` element is not in MAD, but may be expressed in keyword notation.

Notation

The mnemonic used in specifying a shift in the reference coordinate system is `SHIFT`. The keywords which can be used are

Symbol	Keyword	Description
x	`X`	The shift in the x coordinate (mm with `UMM`, meters with `UMETER`)
x'	`XP`	The shift in x' (mr with `UMM`, radians with `UMETER`)
y	`Y`	The shift in y (mm or meters)
y'	`YP`	The shift in y' (mr or radians)
ℓ	`L`	The shift in ℓ. (mm or meters)
δ	`DEL`	The shift in δ (permil with `UMM`, fractional with `UMETER`).

The units are the same as for the `BEAM` element (page 99. The unit sets for which the units are given above are `UMM` and `UMETER`. Other unit sets in addition to those mentioned above can be used and are described on page 64. Individual units changes are described on page 68.

An example of a downward kink in the reference trajectory increasing the vertical angle by 1.0 milliradians might be

```
SHIFT,  YP = 1.0  ;
```

Any of the parameters can be varied in first-order fitting. The floor coordinates or centroid position at any subsequent point in the beam line can be constrained.

Usage

As with a centroid shift, the presence of a reference coordinate system shift in the data will automatically cause the transfer matrix to be expanded about the new beam centroid position. Both the order used in calculating the transfer matrix, and the order to which it is expanded about the new reference trajectory are specified by the `ORDER` element.

REALIGN — Realignment of the Reference Trajectory

(`REALIGN ;`): At the location of this element, the reference trajectory is shifted to line up with the first-order image of the initial beam centroid. Thus, the reference trajectory of the beam line, as followed by the program, will not be continuous. The beam centroid location will be continuous, but will seem to be discontinuous if expressed relative to the reference trajectory.

If the centroid does not coincide with the reference trajectory, then quadrupoles, sextupoles, and octupoles will show dispersive effects. All the fitting options connected with this element operate as expected. Thus the original beam centroid displacement parameters can be varied to fit values of floor coordinates specified after the shift.

Part of a TRANSPORT set of data might then have the following appearance:

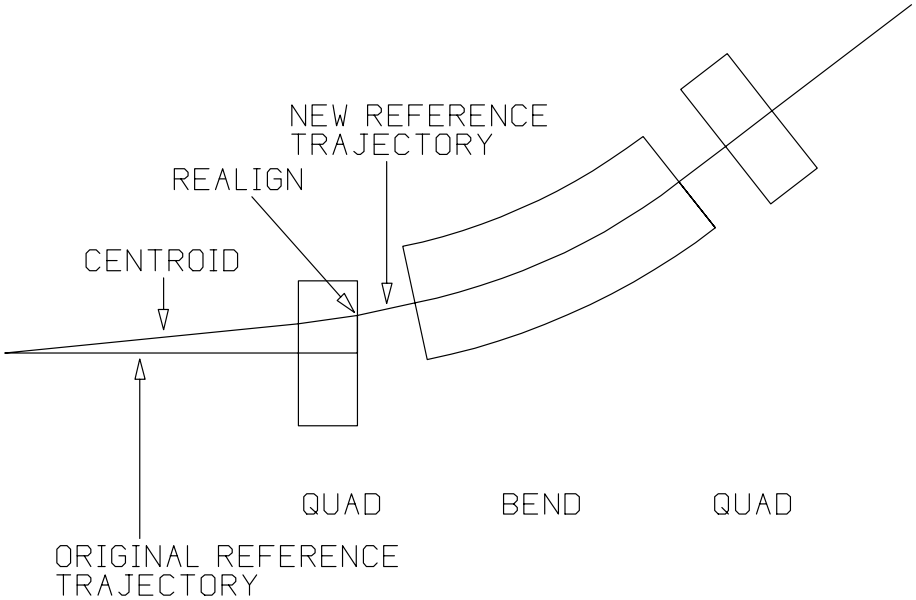
```
BEAM  .  .  .  .  .  .  .  ;
CENTROID  .  .  .  ;
DRIFT  .  .  .  ;
QUAD  .  .  .  ;
REALIGN  ;
DRIFT  .  .  .  ;
BEND  .  .  .  ;
DRIFT  .  .  .  ;
QUAD  .  .  .  ;
DRIFT  .  .  .  ;
```

The `CENTROID` element indicates an initial centroid displacement which is then propagated through the system. The centroid is focused by the quadrupoles. After traversing a quadrupole the transfer matrix will show dispersive elements (R_{16} and R_{26}), since the transfer matrix is a linear expansion about the centroid and the centroid does not follow the axis of the quadrupoles. A diagram of this simple system is shown on the following page.

When the `REALIGN` element is encountered, the centroid dispacement is set to zero. The reference trajectory is redefined to be a continuation of the previous centroid. The transfer matrix, being a linear expansion about the centroid, is unchanged. TRANSPORT continues to accumulate the transfer matrix after the `REALIGN` element. The floor coordinates will be shifted immediately after the realign element. The amount of shift will be that necessary to align the beam line components on the off-axis centroid.

An example of the use of the `REALIGN` command occurs when the first optical element in a beam line is a quadrupole, but because of geometric constraints, the optic axis of the quadrupole cannot be placed so that it is aligned with the center of the accepted phase space. The centroid may be placed at the center of the accepted phase space. The beam line can be laid out by allowing the centroid to be focused by the quadrupole and then redefining the reference trajectory to be lined up with the centroid as it exits the quadrupole.

The effect of the **REALIGN** command in the data above can be shown by the following figure:



The Effect of the **REALIGN** element

ALIGN — Magnet Alignment Tolerances

The first-order effects of the misalignment of a magnet or group of magnets are a shift in the centroid of the beam and a change in the beam focusing characteristics. The transverse displacement of any focusing device, such as a quadrupole, will cause a deflection of the beam centroid, and therefore a downstream transverse shift in the position of the beam ellipse. A longitudinal shift of a quadrupole, or a rotation of a quadrupole about its optic axis, or a transverse displacement of a sextupole will cause a change in the beam focusing characteristics. This change will appear as a change in the transfer matrix or in the dimensions of the beam ellipse. More on this subject will be found later in this section under "The Effects of Misalignments."

Three varieties of misalignment simulation are commonly needed:

(1) The physical position of the magnet is uncertain within a given tolerance.
or (2) The magnet is displaced and/or rotated by a known amount;
or (3) The magnet is displaced and/or rotated by a definite amount chosen from a user-specified range by computer-generated random numbers.

The results may be displayed in either of two ways:

(1) In the printed output of the beam (σ) matrix, including the beam centroid.
or (2) Tabulated in a special misalignment table (described below).

The object(s) to be misaligned may be designated in three different ways:

(1) A single bending magnet, quadrupole, sextupole, or octupole
or (2) A section of a beam line
or (3) All bending magnets and/or quadrupoles)

In addition, there is some choice among coordinate systems to which the misalignment is to be referred. There are three possible positions for the origin or the coordinate system:

(1) The intersection of the reference trajectory with the entrance face of the magnet
or (2) The longitudinal midpoint of the magnet
or (3) The intersection of the reference trajectory with the exit face of the magnet

For each choice of origin, there are also two possibilities for the orientation of the coordinate misalignment coordinate system.

(1) For a magnet which changes the direction of the reference trajectory, the z axis of the misalignment coordinate system is along the chord of the magnet. If the longitudinal midpoint is chosen as the origin of the misalignment coordinate system, then the origin is taken to be on the midpoint of the chord. If the reference trajectory in the magnetic element is straight, the misalignment coordinate system is the same as the beam coordinate system.

or (2) The misalignment reference coordinates are the same as the beam coordinate system for bending magnets also. If the midpoint of the magnet is chosen as the origin of the misalignment coordinate systen, then that midpoint is the intersection of the projected entrance and exit reference trajectories. The z axis for the misalignments is parallel to the reference trajectory at the logitudinal midpoint.

The element `ALIGN` is a TRANSPORT element but is not a MAD element. However, in TRANSPORT it may be expressed either in keyword notation similar to the MAD format.

Notation

The mnemonic for magnet misalignments is `ALIGN`. The keywords which can be used in specifying the parameters describing the misalignments are given below. The parameters consist of three displacements and three rotations plus additional keywords indicating the type of misalignment. There is one parameter for a displacement along and one parameter for a rotation about each of the coordinate axes x, y, and z.

Here the global unit set specifications described are `UMM` and `UMETER` Other global unit set specifications may also be selected and are described on page 64. Individual units changes are described on page 68.

Symbol	Keyword	Description
Δx	`X`	The magnet displacement in the x direction (mm with `UMM`, meters with `UMETER`).
Δr_x	`RX`	A rotation about the x axis (mr with `UMM`, radians with `UMETER`).
Δy	`Y`	The magnet displacement in the y direction (mm or meters).
Δr_y	`RY`	A rotation about the y axis (mr or radians).
Δz	`Z`	The magnet displacement in the z (beam) direction (mm or meters).
Δr_z	`RZ`	A rotation about the z axis (mr or radians).

Any parameters not specified are taken to equal zero. The meanings of other keywords used to describe the nature of the misalignment or the coordinate system to which it is referred are described below. Any of the physical parameters may be varied to satisfy some constraint later in the beam line. (See page 246 a description of the procedure for varying parameters.)

An example of an element in keyword notation which specifies an uncertainty of 2 mrad in roll of the preceding magnet is

```
MIS1:   ALIGN,  RZ = 2.0,  UNCERTAIN  ;
```

A roll is a rotation about the longitudinal or z axis.

The Effects of Misalignments

The transverse displacement of any focusing device will cause a deflection of the beam centroid. The deflection will then result in a transverse shift in the position of the beam ellipse at all points downstream. Focusing devices include quadrupoles, sextupoles, and octupoles. A bending magnet serves as a focusing device with two exceptions: (1) A uniform field sector magnet (with the pole faces perpendicular to the reference trajectory) does not focus vertically, and (2) A uniform-field rectangular bending magnet does not focus horizontally.

A horizontally displaced quadrupole will induce a horizontally bending dipole component. The beam centroid will suffer a horizontal deflection due to this bending component. A vertically displaced quadrupole will induce a vertically bending dipole component. A displacement of a quadrupole in any direction does not affect the quadrupole component of the magnetic field, nor does it induce field components of order higher than first.

The deflection of the centroid by a transversely displaced sextupole is an effect of second-order in the field expansion of the sextupole. Therefore, if a displaced sextupole is to have a visible effect in the program output, the order of the calculation, as specified by the `ORDER` element, must be two or greater. The order of the calculation is given by the first parameter on the `ORDER` element. (See page 83.)

The displaced sextupole induces both dipole and quadrupole components about the aligned reference trajectory. The effect of the induced field components will be exhibited both as a shift in the beam centroid and as a change in the first-order transfer matrix due to the focusing effect of the induced quadrupole. There will be no change in the second or higher-order transfer matrix.

A displaced octupole will similarly induce dipole, quadrupole, and sextupole effects. For these effects to be displayed in the output, the first parameter on the `ORDER` element must be at least three. For the induced sextupole field component to affect the printed output, the second parameter on the `ORDER` element must be at least two.

The rotation of any magnet about the x axis at the entrance face is equivalent to holding the position of the entrance face fixed and vertically shifting the exit face. Similarly, the entrance face rotation about the y axis is equivalent to fixing the entrance face position and horizontally shifting the exit face. The rotation of any magnet about either axis at the longitudinal midpoint consists of holding the magnet fixed at the longitudinal midpoint and displacing the entrance and exit faces in opposite directions.

The rotation of a bending magnet about either the x or y axis causes a break in the reference trajectory and therefore a downstream displacement of the beam centroid. The rotation of a quadrupole, sextupole, or octupole about the x or y axis at the entrance face causes a transverse displacement of the downstream end of the magnet. The effect is a displacement of the beam centroid which is smaller than if the magnet were transversely displaced without rotation.

The rotation of a quadrupole, sextupole, or octupole about a transverse axis (x or y) at the longitudinal midpoint also causes a deflection of the beam centroid. However, this deflection is typically much smaller than is caused by a rotation about a a transverse axis at the magnet entrance or exit face.

The quadrupole component which is induced by a displaced sextupole causes a change in the first-order transfer matrix and a corresponding alteration in the dimensions of the beam ellipse. The horizontal displacement of a sextupole will induce a midplane-symmetric quadrupole component. The midplane-symmetric quadrupole component may be corrected by the placement of another midplane-symmetric quadrupole elsewhere in the beam line lattice.

A vertically displaced sextupole will induce a skew quadrupole component (a quadrupole rotated by 45 degrees) and a midplane symmetric dipole component. The skew quadrupole component changes the focal characteristics of the beam line by causing a first-order mixing of the horizontal and vertical planes. The effect of the skew induced quadrupole may be correctable by another skew quadrupole placed at the appropriate phase position in the beam line.

The quadrupole component induced by a displaced octupole will cause a change in the first-order transfer matrix. The similarly induced sextupole component will cause a change in the second order transfer matrix. Both induced components will alter the dimensions of the calculated beam ellipse. A horizontally displaced octupole will induce midplane symmetric dipole, quadrupole, and sextupole components. A vertically displaced octupole will induce a vertically bending dipole, a midplane-symmetric quadrupole component, and a skew sextupole component (a sextupole rotated by 30 degrees).

In general, a horizontally shifted midplane-symmetric multipole of any order will induce midplane-symmetric multipoles of all lower orders, but of no higher order. A vertically shifted midplane-symmetric multipole of any order will also induce multipoles of all lowers, but of no higher order. In the vertically shifted case, the lower multipoles will alternate in being midplane symmetric and skew (midplane antisymmetric). This alternation will include the original multipole.

A longitudinal shift or a rotation about the reference trajectory of a bending magnet will again cause a break in the reference trajectory and a corresponding downstream shift in the beam centroid. The bending magnet is a special case in this respect since it is the only element through which the aligned reference trajectory is not a straight line.

The longitudinal shift or a rotation about the axis of a quadrupole, sextupole, or an octupole does not have an effect on a centroid which is initially on the optic axis. It will emerge from the magnetic element still aligned on the optic axis. However, the misaligned element will have an effect on the focal characteristics of the beam. A roll of a quadrupole about its axis will cause a first-order coupling in the two transverse planes. This effect will appear in both the transfer matrix and the phase ellipse.

The effect of a roll in a sextupole will also be to cause a mixing of the two transverse planes, but of second order. Similarly, the roll of a octupole about its axis will cause a mixing of transverse planes in third order. Quadrupoles, sextupoles, and octupoles also have higher-order terms in their individual transfer matrices. Rotation of any of these elements about their longidutinal axes will therefore cause mixing of transverse planes in higher orders also.

Known Misalignments

A "known" misalignment means that the magnets have definite positions which are different from the design position for the beam line. These positions may have been determined by surveying the beam line. They may also be hypothetical, so as to determine the sensitivity of the centroid position or the beam ellipse size to the magnet alignment tolerances.

A definite but random misalignment means that that the parameters on the misalignment element are taken to specify the range of possible positions and angles describing the misalignment. The actual positions and angles are taken to be definite values within the given range. Each value on the `ALIGN` element is multiplied by a random number to determine the actual magnitude of the misalignment. The random numbers are distributed uniformly from -1 to $+1$. The random number generator is the same as the one used for the `RANDOM` element, described on page 89.

The definite but random misalignment feature of TRANSPORT is used to give simultaneous but independent misalignments to the components of the beam line. It obviously makes little sense to use this feature if only one component of the beam line is being misaligned. The definite but random feature should be used either when a number of components are being given separate alignment specifications or when one misalignment element is being used to misalign all subsequent bending magnets and/or quadrupoles. (This last feature is described below under "What Can Be Misaligned.")

A known misalignment, as explained above, will typically affect the beam centroid, and possibly the beam dimensions as well. The phase ellipse will represent the actual phase space volume of the beam particles. Therefore one should place on the `BEAM` card the actual dimensions of the beam entering the system. The definite but random misalignment is interpreted in the same manner as the known misalignment.

If the user specifies that the results of a known misalignment are to be displayed in the beam matrix, then the transfer matrix will be affected also. The effect of the misalignment at the entrance and exit faces of the misaligned magnet or section is treated as a known

element. The resulting centroid displacement and change in the transfer matrix will be calculated and included in the accumulated transfer matrix for the beam line. The transfer matrix may be accumulated to first, second, or third order. If the floor coordinates are printed, the misaligned coordinates will be shown inside the misaligned section.

Uncertain Misalignments

An uncertain misalignment means that the magnet position is given a certain probability distribution. Rather than the magnet displacement or rotation being known, the magnet has uniform probability of being anywhere within a certain range of displacements or rotations. The results of a calculation using this type of misalignment is similarly a range of possible centroid positions.

The interpretation of the set of numbers describing what we call the "beam ellipse" depends on the nature of the misalignment being imposed. For an uncertain alignment the most common use for the beam ellipse is to represent the uncertainty in the position of the beam centroid. The initial beam dimensions should be set to zero, i.e., the beam element at the beginning of the system should appear as follows:

```
BEAM,  P0 = 50.0  ;
```

Here the beam momentum is taken to be 50 GeV. Since the other beam dimensions are unspecified they are all taken to be equal to zero.

At positions downstream of the misaligned magnet, the actual beam centroid will be at an unknown position within the region described by the beam ellipse. The printed centroid will be the average value of the unknown centroid range. The printed centroid will be unaffected by the misalignment. The printed beam ellipse dimensions will give the range of possible positions for the unknown centroid.

The effects of each misalignment of each misaligned element are added in quadrature to the effects of the previous elements. The elements are then represented as having been misaligned independently. If the parameters on the misalignment element are taken as representing the standard deviation of a probabilistic distribution, then the dimensions of the resulting beam ellipse represent the standard deviation of a similar distribution for the position of the beam centroid.

Some types of misalignment do not show an effect on the beam centroid. For example, the roll of a quadrupole about its axis leaves the centroid undisturbed, but affects the focusing of the beam. For the quadrupole roll to have a perceptable effect a nonzero initial phase space must be specified. The effect of the roll of the magnet is then shown by a broadening of the phase ellipse. The effect on the phase ellipse is caused by an $x - y$ coupling of the rotated quadrupole. The beam ellipse at this point becomes a confused mess containing both the transformed initial phase space and the effect of the uncertainty in the roll angle of the

quadrupole.

The effects of an uncertain misalignment are limited to first order in TRANSPORT. The average change in any element of the first-order transfer matrix is zero. Since the beam ellipse is a quadratic function of the transfer matrix, results of an uncertain misalignment may be seen in the beam ellipse. The alterations in the beam ellipse can come from three sources:

1. A direct shift in the beam centroid caused by the transverse displacement of a magnetic element or a rotation about a transverse axis.
2. A change in the first-order transfer matrix caused by the misalignment, such as the mixing of planes resulting from a rotation of a quadrupole about the longitudinal axis.
3. A change in the first-order transfer matrix induced by a higher-order component. An example is the quadrupole component induced by a transversely displaced sextupole magnet.

An uncertain alignment pertaining to a single magnet or section of the beam line updates (reinitializes) the R2 transfer matrix, but not the R1 matrix. The R2 matrix is an auxiliary transfer matrix to supplement R1, the normal transfer matrix. (For a more detailed explanation, see page 214.) The R2 matrix is used to calculate the beam matrix, and is also made available to users to impose constraints on portions of the beam line.

A misalignment element may also indicate that uncertain misalignments are to be given to all subsequent bending magnets and/or quadrupoles. In such cases the R2 matrix will be updated after each such misaligned magnet. If the magnets to be misaligned include bending magnets then the R2 matrix will be updated before each bending magnet for which the presence of fringing fields is indicated. Such bending magnets consist of the `RBEND` element, the `SBEND` element, and the `BEND` element with an adjacent `ROTAT` element.

Parameters Describing the Misalignment

The three displacements and three rotations described above comprise the six degrees of freedom of a rigid body and are used as the six misalignment coordinates. The displacements are taken to be along each of the three coordinate axes x, y, and z in turn. The rotations are taken to be about the same three coordinate axes. For the displacements the interpretation is straightforward. However, the interpretation of the rotations requires some care.

A rotation about the x (horizontal) axis will not affect the x coordinate or the angle x' in the $x - z$ plane of any trajectory. It may affect the y (vertical) coordinate and will affect the (vertical) angle y'. For an element with a positive (counterclockwise) rotation about the x axis, y' will experience a sudden increase at the entrance of the misaligned element. The increase is caused by the transition from the external aligned coordinate system to the internal misaligned system. The internal misaligned system is the coordinate system that is

attached to the misaligned magnet. The coordinates and angles will then be altered by the transition through the misaligned magnet.

The angle y' will also experience a sudden decrease at the exit of the misaligned element. The decrease is caused by the transition from the internal misaligned coordinate system to the external aligned system. The vertical coordinate y may also experience changes at the magnet entrance and exit faces depending on the position of the origin of the coordinate system for the magnet rotations. Similarly, a rotation about the y or vertical axis will cause a sudden decrease in x' at the entrance of the misaligned element, and a sudden increase in x' at the exit.

For an element with a straight reference trajectory, such as a quadrupole, sextupole or octupole, the default coordinate system employed is that to which the beam is referred at the point it enters the magnet. For example, a rotation of a bending magnet about the beam direction (parameter 7 above) is referred to the direction of the beam where it enters the magnet.

For a bending magnet, the origin of the default coordinate system continues to be the point at which the reference trajectory. The z axis is along the chord of the magnet. The x axis is in the bend plane, perpendicular to the z axis. The y plane is perpendicular to the bend plane. It coincides with the y axis for the beam.

Other reference coordinate systems may be specified by use of additional keywords described below. The longitudinal midpoint or the point where the reference trajectory crosses the exit face of the magnet may be used as the origin of the coordinate system. For a bending magnet the intersection of the projected entrance and exit reference trajectories can be used as the origin of the misalignment cooredinate system.

What Can Be Misaligned

The misalignment of any physical element or section of a beam line may be simulated. Misaligned sections of a beam line may be nested. A beam line rotation (`SROT` element) may be included in a misaligned section. Thus, for example, one can simulate the misalignment of magnets that bend vertically. *The arbitrary matrix* (`MATRIX` *element) may not be included in a misaligned section.*

A misalignment element may indicate that a single magnet or section of the beam line is to be misaligned, or it may indicate that all subsequent quadrupoles and/or bending magnets are to be misaligned. The location of the `ALIGN` element depends on the type of misalignment. The type of misalignment is specified by means of additional keywords.

The first option is to treat the `ALIGN` element as part of the beam line description. The misalignment may pertain to a single magnet or a single section of the beam line. In this case the `ALIGN` element must directly follow that magnet or section of the beam line. If the misalignment pertains to a section of the beam line, then the beginning of that section is indicated by an `UPDATE` element or by an alignment marker `ALMARK`.

A second option is to consider the ALIGN element as an operation. It can then be placed toward the end of the data, after the beam line description. The location in the beam line is indicated by means of the MARKER element. The ALIGN element refers to marker by name by means of the LOCATION keyword. If, for example, the marker

```
M1:   MARKER  ;
```

is placed in the beam line, then the following misalignment element can be placed after the beam line description.

```
ALIGN, -  -  -  -  -  -  LOCA = M1  ;
```

The ALIGN element will be treated as if it were at the location of the marker to which it refers. The beginning of a section to be misaligned is indicated by an UPDATE element or an ALMARK element. Either of these elements can also be placed after the beam line description and refer to a MARKER via the LOCA keyword.

The third option is to misalign individual magnets by name. Here the ALIGN element is placed after the beam line description. The label of the magnet to be misaligned is used as a keyword of the ALIGN element. One may want to give a known misalignment to a quadrupole with the label Q1. In that case the ALIGN element would look like:

```
ALIGN,  Q1,  -  -  -  -  -  -  KNOWN  ;
```

The fourth type of misalignment element indicates that all subsequent magnets of a given type (quad or bend) are to be misaligned. Such an ALIGN element must precede the first of such magnets. Further description of the available types of misalignment is given in the table later on page 190..

If a known misalignment is to apply to all subsequent magnets of a given type, then the results should be displayed in the misalignment table (described below). Then the magnets will be misaligned one at a time, and the effect of each will be shown separately in the misalignment table. If all magnets are given a known misalignment and the effects are combined, then the effect is no different from a situation of simply relocating the entire beam line intact. A calculation of this effect is of little use.

Displaying the Results

The results of the misalignment may be displayed in either the beam (σ) matrix or in a misalignment table. If the results are displayed in the beam (σ) matrix, then that matrix is altered by the effects of the misalignment. The effects of additional misalignments cause further alterations, so that at any point along the beam line, the beam (σ) matrix will contain the combined effects of all previous misalignments. If the misalignment is of a known amount, it will also affect the floor coordinates of the magnets within the misaligned section. A known misalignment displayed in the beam sigma matrix also shows results in

the transfer matrix. The transfer matrix may be displayed up to third order, and the effect of the misalignment will be included in the calculated matrix.

The misalignment table represents an alternative way of displaying the effects of misalignments. It can be used to show independently the effect on the beam matrix of a misalignment in each degree of freedom of each misaligned magnet. Each new misalignment to be entered in the table creates a new set of six duplicates of the beam matrix. Printed for each duplicate beam matrix are the centroid displacement and the beam half width in each of the six beam coordinates.

Each of the six misalignment table matrices shows the beam matrix that would result from the misalignment in a single coordinate of a single magnet or section of the beam line. In a single TRANSPORT run the results of misaligning up to ten magnets or sections of the beam line may be included in the misalignment table. Further requests for entry in the misalignment table will be ignored. Examples of such a table and the input which generated it are shown later in this section.

If the misalignment table is selected for the output of a known misalignment, then the centroid displacements will show the effect of the misalignment of any misaligned magnet or section of the beam line in each of the misalignment parameters separately. However, the beam matrix dimensions found in the misalignment table will be calculated from only the first-order terms in the transfer matrices of the elements in the beam line. These first-order terms include those induced from the displacement of higher-order elements, such as a quadrupole component induced by the transverse shift of a sextupole. They will also include those induced from higher-order elements arising from a centroid already off axis. If a misaligned quadrupole causes an off-axis beam centroid, then that off-axis centroid will result in an induced quadrupole component from a downstream aligned sextupole. However, the intrinsic second-order effect from the sextupole itself will not be included in the misalignment table. To include such higher-order effects the effect of the misalignment must be displayed in the beam matrix.

Similarly, if the misalignments are uncertain, then the beam dimensions in the misalignment table will be those resulting from the effect of the uncertainty in position of the individual elements. Only first-order effects, including induced first-order effects, will be shown in the output.

There is no point in directing the output for a known but random misalignment to the misalignment table. In the misalignment table the effect of the misalignment of each magnet of section of the beam line is shown separately. A randomization of the magnitude of the misalignment applied will only cause confusion.

The entries in the centroid column of the misalignment table for a known misalignment are sometimes referred to as the partial derivatives of the centroid position with respect to the misalignment parameters. This description is useful, but not quite correct. The centroid displacements shown are those due to the magnet misalignment. A partial derivative would be a centroid displacement divided by the magnitude of the misalignment causing it.

If a centroid shift is due to a transversely misaligned quadrupole, then the magnitude of the centroid shift will be linear in the displacement of the quadrupole. However, if a centroid shift is due to a transversely misaligned sextupole, then the magnitude of the centroid shift will be quadratic in the displacement of the sextupole. For a quadrupole, transverse displacements in opposite directions will produce centroid shifts in opposite directions. For a sextupole, transverse displacements in opposite directions will produce centroid shifts in the same direction. Care should be taken in the interpretation of the numbers in the misalignment table to insure that the effects observed are of the order expected.

Additional Keywords

Additional keywords can be used to indicate whether the misalignment is an uncertainty in position, a known displacement, or a random error in position. They can also be used to specify the form in which the results are displayed, the magnet or beam line section to be misaligned, and both the origin and the orientation of the axes to which the misalignment is referred. Another option continuously restores the beam centroid to the reference trajectory. The meanings for the additional keywords are as follows:

A. The interpretation of the misalignment parameters and of the resulting beam matrix dimensions is given by the following keyword options.

Keyword	Interpretation
`UNCERTAIN`	The magnet position is uncertain. The alignment parameters give the range of uncertainty for the magnet position and orientation. The beam ellipse does not give the dimensions of a bunch of particles. It describes the region of phase space in which, for all we know, the particles might be found. If the initial phase space dimensions are all zero, then the beam ellipse gives the range of possible positions of the beam centroid. This keyword is the default option. If there is no keyword indicating the interpretation of the misalignment parameters, then the option assumed is `UNCERTAIN`.
`KNOWN`	The magnet or section of beam line is misaligned by a known amount. Both the transfer matrix and beam matrix are affected.
`RANDOM`	A third option is a random displacement. The effect of the misalignment will be calculated in the same manner as if the parameters describing the misalignment were known. However, those parameters will be determined by multiplying the parameters on the misalignment element by random numbers. The random numbers are distributed uniformly from -1 to 1. *The parameters on the misalignment element then represent the maximum magnitude of the misalignment.* The random numbers are drawn from the same sequence as those used for errors on the physical parameters of the elements. For a description of such errors, the user should see page 89.

B. Additional keywords define the mode of display of the results of the misalignment.

Keyword	Interpretation
`BEAM`	The beam matrix contains the results of the misalignment. The beam matrix is printed wherever a (`PRINT BEAM ;`) element is encountered. The beam matrix will then contain contributions from all previous misalignments. If no mode of display is specified, then the default option is `BEAM`
`TABLE`	A table is used to store the results of misalignments. The effect of up to ten independently misaligned magnets may be shown in the table in a single run. The table is printed via a (`PRINT ALIGN ;`) element, and may be compared with the undisturbed beam matrix (printed by a (`PRINT BEAM ;`) command) at any point. An example of such a table is shown later in this section.

C. A third set of keywords is used to specify the magnet(s) or section of the beam line to be misaligned.

Keyword	Interpretation
`SINGLE`	The single magnet (element) immediately preceding the align element is to be misaligned. This option is the default, so no specification of section to be misaligned is required to invoke this option. A bending magnet with fringe fields should be misaligned using one of the options described below.
`R1`	The last R1 matrix update (the start of the beam line or a (`UPDATE R1 ;`) element) or alignment marker marks the beginning of the section to be misaligned. The misalignment element itself marks the end. The section is treated as a unit and misaligned as a whole. The misalignments of quadrupole triplets and other combinations involving two or more quadrupoles may be studied using this keyword.

`R2` The last R2 matrix update (see the `UPDATE` element for a list of elements which update R2) or alignment marker marks the beginning of the misaligned section. The misalignment element marks the end. This option makes use of the fact that R2 matrix updates do not affect the R1 matrix.
A bending magnet with fringing fields or pole face rotations (`ROTAT` element) should be misaligned using this option. See examples 1 and 2 below for an illustration of this. An array of quadrupoles provides another example of the use of this option. By successive application of align elements, the elements of a quadrupole triplet could be misaligned relative to each other, and then the triplet as a whole could be misaligned. See example 3 below for an illustration of this.

label The label on an individual magnetic element may be used as a keyword. This. usage indicates that the misalignment applies to that magnetic element. Misalignment specifications of this type are usually grouped together after the beam line specification.

`MAGNETS` All subsequent bending magnets and quadrupoles are independently misaligned by the amount specified. This option is useful in conjunction with the tabular display of the misalignment results (see below). A `ALIGN` element with this keyword should be placed near the beginning of the data, before any magnets to which it applies. A bending magent, with fringing fields included, is treated as a single unit and misaligned accordingly.

`BENDS` All subsequent bending magnets, including fringing fields, are independently misaligned by the amount specified. See the description for `MAGNETS` above for further comments.

`QUADS` All subsequent quadrupoles are independently misaligned by the amount specified. See example 4 below for an illustration of this. See `MAGNETS` above for further comments.

Any combination of one keyword from each category (A, B, or C) may be used to define the exact circumstances intended. However, only some of the combinations are useful. In the above descriptions and in the following examples, we have tried to outline the more useful combinations. A brief summary follows.

The keywords `UNCERTAIN`, `BEAM`, and `SINGLE` taken together indicate an uncertainty in the position of a single magnet, with the effects contributing to the beam matrix. The three keywords mentioned are all the default, so the results would be the same if any or all of these keywords are omitted. Their explicit presence simply makes the `ALIGN` element easier to read. The keywords `UNCERTAIN` and `BEAM` are typically included. The keyword `SINGLE` is typically omitted as it is a rather obvious default.

The combination of keywords `KNOWN`, `BEAM`, `SINGLE` taken together describe a known

misalignment in an otherwise identical situation. Once again, the keywords `BEAM` and `SINGLE` are unnecessary, but `BEAM` is usually included for clarity.

The keyword `MAGNETS` causes all subsequent quadrupoles and bending magnets to be given an uncertain misalignment, with the results contributing to the beam matrix. In this example, we have not included the keywords `UNCERTAIN` or `BEAM` as they are the default and their presence is optional.

The combination of the two keywords `TABLE`, and `MAGNET` and the combination of three keywords `KNOWN`, `TABLE`, and `MAGNET` impose respectively uncertain and known misalignments on all subsequent quadrupoles and bending magnets, with the results placed in the misalignment table. The keyword `UNCERTAIN` is here omitted since it is the default. However, it is often included for clarity.

The keyword combination `RANDOM`, `BEAM`, and `MAGNET` randomly misaligns all subsequent quadrupoles and bending magnets, with the results tabulated in the beam and transfer matrices. Here none of the keywords used is the default and all must be included.

D. A fourth set of keywords indicates the location of the origin of the coordinate system to which the misalignment is referenced. The origin is the point which is unmoved when any of the purely rotational misalignments are imposed. The default orientation for the misalignment reference coordinate system depends on the location of its origin.

Keyword	Interpretation
`ENTRANCE`	The origin of the misalignment coordinate system is at the point where the reference trajectory crosses the entrance face of the magnet. This option is the default and need not be specified. For an element with a straight reference trajectory, the default orientation for the misalignment coordinate system in this case is that it coincide with the beam coordinate system. In other words, the x axis is transverse and in the bend plane. The y axis is transverse and perpendicular to the bend plane. The z axis is along the reference trajectory at the entry point to the magnet. For a bending magnet the position of the origin is the same, but the z axis of the misalignment coordinate system points along the chord of the magnet. The x axis is in the bend plane, perpendicular to the z axis. The y axis is perpendicular to the bend plane and is the same as the y axis of the beam coordinates.
`MIDPOINT`	The longitudinal midpoint of the magnet serves as the origin of the misalignment reference coordinate system. The longitudinal midpoint is well defined if the reference trajectory in the magnet is straight, as is the case for quadrupoles, sextupoles, octupoles, and solenoids. If the magnet is curved, such as in a `BEND`, `RBEND`, or `SBEND` the default position of the longitudinal midpoint is the midpoint of the chord of the magnet.

`EXIT` The point where the reference trajectory crosses the exit face of the magnet serves as the origin of the misalignment reference coordinate system. The default orientation of the misalignment reference trajectory is such that the z axis points along a continuation of the chord of the magnet. The x axis is perpendicular to the z axis and lies in the bend plane. The y axis is perpendicular to the bend plane and is the same as the y axis for the beam coordinates.

E. The fifth set of keywords indicates the orientation of the axes in the misalignment reference coordinate system, if they are other than the default for the choice of origin.

Keyword	Interpretation
`CHORD`	For a magnet which changes the direction of the reference trajectory (`BEND`, `RBEND`, `SBEND`), the z axis or the misalignment reference coordinate system is along the chord of the magnet. The x axis is in the bend plane, perpendicular to the z axis. The y axis is perpendicular to the bend plane, as before. If the keyword `MIDPOINT` is used to indicate the location of the origin of the alignment coordinate system, then that origin is taken as being the midpoint of the chord.
`TRAJECTORY`	Here the misalignment coordinate system always always has the same orientation as the beam coordinate system. If the origin is specified as being at the entrance or exit face of the magnet, then the origin also coincides with the origin of the coordinate system for the beam. If the origin is specified as being at the `MIDPOINT` of the magnet, then it is at the intersection of the projected entrance and exit reference trajectories.

F. The sixth set of keywords allows the user to explore the effect of correction elements to repair the damage to the beam resulting from the misalignments.

Keyword	Interpretation
`FOCUS`	The beam centroid is restored to the reference trajectory after each element. Only the focusing effect of the misalignment will be considered.
`COMPLETE`	The effect of this keyword is the default, and therefore it need never be included in the data. It does not insert hypothetical correction elements into the beam line. It does not limit the effect of the misalignment to the focusing effects, but instead includes the centroid displacement.

Examples

Example No. 1: A Bending Magnet with a Known Misalignment

A bending magnet (including fringe fields) misaligned by a known amount might be represented as follows:

```
DRIFT,  L = 10.0  ;
UPDATE,  R2  ;
ROTATE,  ANGLE = 0.0  ;
BEND  L = 5.0,  B = 10.0,  N = 0.5  ;
ROTATE,  ANGLE = 0.0  ;
ALIGN,  RZ = 2.0,  KNOWN,  R2  ;
DRIFT,  L = 10.0  ;
```

This represents a known rotation of the bending magnet about the incoming beam direction (z axis) by 2.0 mr. The result of this misalignment will be a definite shift in the beam centroid, and a mixing of the horizontal and vertical coordinates. The use of the (`UPDATE R2 ;`) transform 2 update and the keywords `KNOWN` and `R2` is necessary because the magnetic array (bending magnet + fringing fields) consists of three elements instead of one.

The complete magnet including both central field and fringing fields can be represented by a single element using the `SBEND` element. This single element may then be misaligned using the keyword `KNOWN`. The `UPDATE` is now unnecessary. The above example would then look like:

```
DRIFT,  L = 10.0  ;
SBEND,  L = 5.0,  B = 10.0,  N = 0.5  ;
ALIGN,  RZ = 2.0,  KNOWN  ;
DRIFT,  L = 10.0  ;
```

An `RBEND` would be a rectangular bending magnet, but would be treated in the same manner.

Example No. 2: A Bending Magnet with an Uncertain Position

A bending magnet having an uncertainty of 2 mrad in its angular positioning about the incoming beam (z axis) would be represented as follows:

```
DRIFT,  L = 10.0  ;
UPDATE,  R2  ;
ROTATE  ANGLE = 0.   ;
BEND  L = 5.0, B = 10.0, N = 0.5  ;
ROTATE,  ANGLE = 0.   ;
ALIGN, RZ = 2.0,  R2.   ;
DRIFT, L = 10.0  ;
```

To observe the uncertainty in the location of the outcoming beam centroid, the input `BEAM` card should have zero phase space dimensions as follows:

```
BEAM, P0 = 25.0.    ;
```

The default value for all unspecified beam phase space dimensions is zero. Hence only the reference trajectory momentum need be specified.

If the beam dimensions on the input `BEAM` card are nonzero, the resultant beam (σ) matrix will show the envelope of possible rays, including both the input beam and the effect of the misalignment.

As with example no. 1, the combination of `ROTATE` and `BEND` elements can be replaced with a single `SBEND` element. This example would then be rewritten as:

```
DRIFT,  L = 10.0  ;
SBEND,  L = 5.0,  B = 10.0,  N = 0.5  ;
ALIGN,  RZ = 2.0  ;
DRIFT,  L = 10.0 ;
```

Example No. 3: A Misaligned Quadrupole Triplet

One typical use of both the R1 and R2 matrices is to permit the misalignment of a triplet. For example, an uncertainty in the positions within the following triplet:

```
QUAD,  L = 1.,  B = -8.,  APER = 10.   ;
QUAD,  L = 2.,  B = 7.,  APER = 10.   ;
QUAD,  L = 1.,  B = -8.,  APER = 10.   ;
```

may be induced by appropriate `ALIGN` element as noted:

```
UPDATE,  R  ;
QUAD,  L = 1.,  B = -8.,  APER = 10.   ;
UPDATE,  R2  ;
QUAD,  L = 2.,  B = 7.,  APER = 10.   ;
QUAD,  L = 1.,  B = -8.   APER = 10.   ;
ALIGN,  UNCERTAIN ;
ALIGN,  UNCERTAIN, R2 ;
ALIGN,  UNCERTAIN, R1 ;
```

The first `ALIGN` element in the list refers to the misalignment of the third magnet only. The second `ALIGN` element refers to the misalignment of the second and third magnets as a single unit via the R2 matrix update (the (`UPDATE, R2 ;`) element). The last `ALIGN` element refers to the misalignment of the whole triplet as a single unit via the R1 matrix update (the (`UPDATE R ;`) entry).

The comments about the `BEAM` card in Example 2 above are applicable here also.

Example No. 4: Misaligned Quadrupoles in a Triplet

Individual uncertainties in the positions of the quadrupoles in the triplet in Example 3 above may be induced by a single misalignment element as follows:

```
ALIGN,  -  -  -  -  -  -  UNCERTAIN, TABLE, QUADS  ;
QUAD,  L = 1.,  B = -8.,  APER = 10.   ;
QUAD,  L = 2.,  B = 7.,  APER = 10.   ;
QUAD,  L = 1.,  B = -8.,  APER = 10.   ;
```

The effect of each misalignment coordinate on each quadrupole will be stored separately in a table. This table is printed wherever a (`PRINT, ALIGN ;`) command is inserted.

A likely place to insert the (`PRINT, ALIGN ;`) command is at a focus. The misalignment table, as printed, then will show separately what the beam matrix would be for each quadrupole misplaced in each of six different ways. The entire table will then contain eighteen replications of the set of six beam centroid coordinates and beam half widths. Each may differ from the unaffected set of beam parameters by the effect of a single quadrupole misplaced along a single misalignment coordinate.

This example could equally well have been made up using known misalignments. The keyword `UNCERTAIN` would then be replaced by `KNOWN`. The displacements and rotations of the quadrupoles would cause shifts in the centroid replications in the misalignment table. If the initial beam matrix dimensions were nonzero, then rotations about the quadrupole axes and displacements along the axes would cause a change in the beam dimension columns in the misalignment matrix.

Another meaningful set of alignment specification keywords would be `UNCERTAIN, BEAM, QUADS`. The misalignment of the individual quadrupoles would be uncertain. The effects on the beam ellipse would be added in quadrature, giving a range of positions of the misaligned centroid. Since the keywords `UNCERTAIN` and `BEAM` are the default, the same specification can be specified via the keyword `QUADS` alone.

Example No. 5: A Randomly Aligned Quadrupole Triplet

The net effect of random errors in the placement of quadrupoles in a triplet may also be induced by a single misalignment element as follows:

```
ALIGN,  -   -   -   -   -   -   RANDOM, QUADS  ;
QUAD,  L = 1.,  B = -8.,  APER = 10.   ;
QUAD,  L = 2.,  B = 7.,  APER = 10.   ;
QUAD,  L = 1.,  B = -8.,  APER = 10.   ;
```

The quadrupoles in the triplet will be given displacements and rotations chosen randomly from the ranges specified on the `ALIGN` element. The qudrupoles will be misaligned independently of each other. The results of the misalignment will be stored in the beam matrix, including the centroid.

Different configurations can be created by changing the starting point of the random number generation. (See page 77.) The randomly aligned configurations can then be used to investigate different correction schemes for the effect of the misalignments.

Example No. 6: Quadrupoles in a Triplet Individually Misaligned by Name

The quadrupoles in a triplet may be individually misaligned as follows;

```
Q1:   QUAD,  L = 1.,  B = -8.,  APER = 10.   ;
Q2:   QUAD,  L = 2.,  B = 7.,  APER = 10.   ;
Q3:   QUAD,  L = 1.,  B = -8.,  APER = 10.   ;
ALIGN, Q1,  -  -  -  -  -  -  KNOWN  ;
ALIGN, Q2,  -  -  -  -  -  -  KNOWN  ;
ALIGN, Q3,  -  -  -  -  -  -  KNOWN  ;
```

Here the label of the magnetic element to be misaligned is used as a keyword in the `ALIGN` element. The `ALIGN` elements may now all be grouped near the end of the data after the beam line specification. This arrangement makes it much easier to do a variety of misalignment studies on an otherwise fixed beam line.

Example No. 7: A Misaligned Quadrupole Triplet

In example 3 we showed the use of both the R1 and R2 matrices is to permit the misalignment of a triplet. In that case the `UPDATE` elements and the `ALIGN` elements were mixed in with the beam line components. Using the same example, we can show how the `UPDATE` commands and the `ALIGN` elements can be placed after all the beam line components.

Markers are inserted in the beam line, and the `UPDATE` and `ALIGN` elements are referred to the markers via the `LOCA` keyword. For example, an uncertainty in the positions within the following triplet:

```
QUAD,  L = 1.,  B = -8.,  APER = 10.   ;
QUAD,  L = 2.,  B = 7.,  APER = 10.   ;
QUAD,  L = 1.,  B = -8.,  APER = 10.   ;
```

may be induced by appropriate `ALIGN` element as noted:

```
M1:   MARKER  ;
Q1:   QUAD,  L = 1.,  B = -8.,  APER = 10.   ;
M2:   MARKER  ;
Q2:   QUAD,  L = 2.,  B = 7.,  APER = 10.   ;
Q3:   QUAD,  L = 1.,  B = -8.   APER = 10.   ;
M3:   MARKER  ;
UPDATE,  R,  LOCA = M1  ;
UPDATE,  R2,  LOCA = M2  ;
ALIGN,  UNCERTAIN,  LOCA = M3  ;
ALIGN,  UNCERTAIN, R2,  LOCA = M3  ;
ALIGN,  UNCERTAIN, R1,  LOCA = M3  ;
```

```
'RECOMBINED MODE HIGH RESOLUTION BEAM'
0
UNIT, ANGLE, MR  ;
UNIT, L, FT ;
ALIGN, X=0.02, RX=0.1, Y=0.02, RY=0.1, Z=0.1, RZ=1.0, TABLE, QUAD ;
BEAM,  X = 0.05, XP = 0.562, Y = 0.05, YP = 0.867, P0 = 200.0 ;
DRIFT,  L = 80.74998 ;
SROT,  ANGLE = 180.  ;
B1: BEND,  L = 10.25, B = 9.18192  ;
SROT,  ANGLE =  -180.  ;
DRIFT,  L = 1.75  ;
SROT,  ANGLE = 180.  ;
B1: BEND,  L = 10.25, B = 9.18192  ;
SROT,  ANGLE = -180.  ;
DRIFT,  L = 27.68999  ;
Q1: QUAD,  L = 10.0,  B = -4.0006,  APER = 2.54  ;
DRIFT,  L = 7.7  ;
PRINT, BEAM, TRANS ;
DRIFT,  L = 3.6  ;
Q2: QUAD,  L = 10.0,  B = 3.4551,  APER = 2.54  ;
DRIFT,  L = 5.0 ;
PRINT, BEAM, TRANS ;
DRIFT,  L = 1.5 ;
SROT,  ANGLE = 180.  ;
ROTAT,  ANGLE = 8.53  ;
BM1: BEND,  L = 20.0,  B = 18.66998  ;
ROTAT,  ANGLE = 8.53  ;
SROT,  ANGLE = -180.  ;
DRIFT,  L = 1.5 ;
SROT,  ANGLE = 180.  ;
ROTAT,  ANGLE = 8.53  ;
BM1: BEND,  L = 20.0,  B = 18.66998  ;
ROTAT,  ANGLE = 8.53  ;
SROT,  ANGLE = -180.  ;
DRIFT,  L = 1.5 ;
SROT,  ANGLE = 180.  ;
ROTAT,  ANGLE = 8.53  ;
BM1: BEND,  L = 20.0,  B = 18.66998  ;
ROTAT,  ANGLE = 8.53  ;
SROT,  ANGLE = -180.  ;
PRINT, BEAM, TRANS ;
DRIFT,  L = 5.5 ;
Q3: QUAD,  L = 10.0,  B = 3.551,  APER = 2.54  ;
DRIFT,  L = 2.0 ;
Q4: QUAD,  L = 10.0,  B = -3.6962,  APER = 2.54  ;
DRIFT,  L = 289.8396  ;
PRINT, BEAM  ;
PRINT, ALIGN ;
PRINT, TRANS ;

    SENTINEL
```

Quadrupoles to be misaligned

print instruction for misalignment table

more beam line including more quadrupoles and misalignment table print instructions if desired

Input for a Misalignment Table. Shown is the input for a misalignment run of the early part of a beam line. The misalignment element specified that all subsequent quadrupoles are to be given an uncertain misalignment by the amount specified and the results (for up to ten quadrupoles) are to be entered in a table. The portion of the output produced by the indicated print instruction is shown in the next figure.

548.830 FT

Unperturbed beam ellipse ⟶

0.000	0.086 CM					
0.000	0.329 MR	0.043				
0.000	0.124 CM	0.000	0.000			
0.000	0.351 MR	0.000	0.000	0.031		
0.000	0.209 CM	-0.019	-1.000	0.000	0.000	
0.000	0.000 PC	0.000	0.000	0.000	0.000	0.000

*MISALIGNMENT EFFECT TABLE FOR MISALIGNMENTS OF

0.020 CM		0.100 MR		0.020 CM		0.100 MR		0.100 CM		1.000 MR	
*MISALIGNMENT OF Q1 * (130.690 FT TO 140.690 FT) MAGNET ENTRANCE FACE COORDS											
0.000	0.136 CM	0.000	0.086 CM	0.000	0.086 CM	0.000	0.119 CM	0.000	0.086 CM	0.000	0.093 CM
0.000	0.329 MR	0.000	0.329 MR	0.000	0.329 MR	0.000	0.329 MR	0.000	0.329 MR	0.000	0.329 MR
0.000	0.124 CM	0.000	0.164 CM	0.000	0.188 CM	0.000	0.124 CM	0.000	0.124 CM	0.000	0.128 CM
0.000	0.351 MR	0.000	0.351 MR	0.000	0.351 MR	0.000	0.351 MR	0.000	0.351 MR	0.000	0.351 MR
0.000	0.209 CM	0.000	0.209 CM	0.000	0.209 CM	0.000	0.209 CM	0.000	0.209 CM	0.000	0.209 CM
0.000	0.000 PC	0.000	0.000 PC	0.000	0.000 PC	0.000	0.000 PC	0.000	0.000 PC	0.000	0.000 PC
*MISALIGNMENT OF Q2 * (151.990 FT TO 161.990 FT) MAGNET ENTRANCE FACE COORDS											
0.000	0.164 CM	0.000	0.085 CM	0.000	0.085 CM	0.000	0.138 CM	0.000	0.085 CM	0.000	0.093 CM
0.000	0.329 MR	0.000	0.329 MR	0.000	0.329 MR	0.000	0.329 MR	0.000	0.329 MR	0.000	0.329 MR
0.000	0.124 CM	0.000	0.142 CM	0.000	0.155 CM	0.000	0.124 CM	0.000	0.124 CM	0.000	0.128 CM
0.000	0.351 MR	0.000	0.351 MR	0.000	0.351 MR	0.000	0.351 MR	0.000	0.351 MR	0.000	0.351 MR
0.000	0.209 CM	0.000	0.209 CM	0.000	0.209 CM	0.000	0.209 CM	0.000	0.209 CM	0.000	0.209 CM
0.000	0.000 PC	0.000	0.000 PC	0.000	0.000 PC	0.000	0.000 PC	0.000	0.000 PC	0.000	0.000 PC
*MISALIGNMENT OF Q3 * (236.990 FT TO 246.990 FT) MAGNET ENTRANCE FACE COORDS											
0.000	0.167 CM	0.000	0.085 CM	0.000	0.085 CM	0.000	0.138 CM	0.000	0.085 CM	0.000	0.093 CM
0.000	0.329 MR	0.000	0.329 MR	0.000	0.329 MR	0.000	0.329 MR	0.000	0.329 MR	0.000	0.329 MR
0.000	0.124 CM	0.000	0.144 CM	0.000	0.156 CM	0.000	0.124 CM	0.000	0.124 CM	0.000	0.128 CM
0.000	0.351 MR	0.000	0.351 MR	0.000	0.351 MR	0.000	0.351 MR	0.000	0.351 MR	0.000	0.351 MR
0.000	0.209 CM	0.000	0.209 CM	0.000	0.209 CM	0.000	0.209 CM	0.000	0.209 CM	0.000	0.209 CM
0.000	0.000 PC	0.000	0.000 PC	0.000	0.000 PC	0.000	0.000 PC	0.000	0.000 PC	0.000	0.000 PC
*MISALIGNMENT OF Q4 * (248.990 FT TO 258.990 FT) MAGNET ENTRANCE FACE COORDS											
0.000	0.150 CM	0.000	0.086 CM	0.000	0.086 CM	0.000	0.126 CM	0.000	0.086 CM	0.000	0.093 CM
0.000	0.329 MR	0.000	0.329 MR	0.000	0.329 MR	0.000	0.329 MR	0.000	0.329 MR	0.000	0.329 MR
0.000	0.124 CM	0.000	0.152 CM	0.000	0.169 CM	0.000	0.124 CM	0.000	0.124 CM	0.000	0.129 CM
0.000	0.351 MR	0.000	0.351 MR	0.000	0.351 MR	0.000	0.351 MR	0.000	0.351 MR	0.000	0.351 MR
0.000	0.209 CM	0.000	0.209 CM	0.000	0.209 CM	0.000	0.209 CM	0.000	0.209 CM	0.000	0.209 CM
0.000	0.000 PC	0.000	0.000 PC	0.000	0.000 PC	0.000	0.000 PC	0.000	0.000 PC	0.000	0.000 PC

TRANSFORM 1

-1.71164	-0.00014	0.00000	0.00000	0.00000	-6.35395
-0.27937	-0.58426	0.00000	0.00000	0.00000	-0.60101
0.00000	0.00000	-2.47238	-0.00003	0.00000	0.00000
0.00000	0.00000	-0.21507	-0.40447	0.00000	0.00000
0.07464	0.37123	0.00000	0.00000	1.00000	-0.02255
0.00000	0.00000	0.00000	0.00000	0.00000	1.00000

LENGTH 548.82953 FT

Example of a Misalignment Table. The misalignment table, the unperturbed complete beam matrix, and the first-order transfer matrix are all shown at the same point in the beam line. The misalignment element (not shown) has indicated an uncertain misalignment, so the beam centroid is unaffected. The magnitudes of the misalignments in each coordinate are shown above the columns to which they pertain. The results of independently misaligning each magnet are indicated by the label for that magnet.

Assembling Beam Lines

In TRANSPORT beam lines may be assembled from individual elements in two different ways.

1. The elements are all defined first, then assembled into beam lines and sublines by explicit instructions. This procedure is the one used in the MAD program.
2. The elements are listed in the order in which they occur in the beam line. An element is then defined as it is placed in the sequence of elements which make up the beam line.

Assembling Beam Lines with MAD Notation

A particle accelerator typically has a lattice structure which is highly repetitive. The MAD program was designed with particle accelerators in mind. Any of the elements is likely to be used many times.

The MAD method of assembling beam lines takes this repetitive structure into account. The elements are all defined first, then assembled into beam lines and sublines by explicit statements. Sublines can be treated as elements in assembling larger sublines or complete lines. The magnetic system over which the calculation is done is determined by the `USE` statement. It is this statement which signals the program that the MAD method of assembling beam lines is being used. If it is not present when the MAD procedure of assembling lines is being used, the user should not expect to get any meaningful results.

Simple Beam Lines

Using MAD notation, a simple beam line to be studied may be defined as a sequence of elements. As an example we define a simple beam line and give it the label `'SIMP'`. It is composed from previously defined drifts `D1` and `D2`, quadrupoles `Q1` and `Q2`, and bending magnet `B1`.

```
SIMP:  LINE = (D1,Q1,D2,Q2,D2,B1,D2,B1,D2,B1,D2,Q2,D2,Q1,D1)
```

The alternative notation, used in earlier versions of MAD, will be equivalent

```
LINE  SIMP = (D1,Q1,D2,Q2,D2,B1,D2,B1,D2,B1,D2,Q2,D2,Q1,D1)
```

Sub-Lines

Lines themselves may serve as elements of other lines. Such a sub-line must be defined in a separate statement. Thus one can easily include a subline several times in a given beam line. Defining two such sublines, the above example may be written as

```
DOUB:  LINE = (D1,Q1,D2,Q2)
BPD:  LINE = (B1,D2)
SIMP:  LINE = (DOUB,D2,BPD,BPD,BPD,Q2,D2,Q1,D1)
```

Repetition and Reflection

A repeated element can be indicated by preceding it with an integer followed by an asterisk Thus our line `SIMP` becomes

```
SIMP:  LINE = (DOUB,D2,3*BPD,Q2,D2,Q1,D1)
```

Several consecutive elements may also be enclosed in parentheses and repeated. We could forgo the definition of `BPD` and write our beamline as

```
SIMP:  LINE = (DOUB,D2,3*(B1,D2),Q2,D2,Q1,D1)
```

A subline which is to be reflected (reverse order of elements) is indicated by preceding it with a minus sign. Thus our beam line `SIMP` may finally be shortened to

```
SIMP:  LINE = (DOUB,D2,3*(BPD),-DOUB)
```

In the TRANSPORT implementation of MAD notation, a repetition indication may not be preceded with a negative sign. Nothing is lost by this restriction since a reflected repeated sub-line is the same as a repeated subline with its elements reversed and in reverse order.

Replaceable Arguments

A beam line definition may include formal arguments. Our sub-line `DOUB`, for example, may not have `Q2` included explicitly, but have a formal argument in its place. It is then defined as

```
DOUB:  LINE(X) = (D1,Q1,D2,X)
```

or

```
DOUB(X):  LINE = (D1,Q1,D2,X)
```

The formal arguments are included in the sequence of elements as if they were labels representing physical elements or sub-lines. They must also be included in parentheses before the equal sign.

When the sub-line `DOUB` is used, the formal argument must be replaced by the actual argument in parentheses following the label `DOUB`. Thus the line `SIMP` becomes

```
SIMP:  LINE = (DOUB(Q2),D2,3*(BPD),-DOUB(Q2))
```

Formal arguments may be single elements or sub-lines. When a subline has a single formal argument, it must be used with a single actual argument. When the subline has two formal arguments, it must be used with two actual arguments. In all cases, the number of formal arguments must be the same as the number of actual arguments.

The section `DOUB` may be defined with two formal arguments:

```
DOUB(X,Y):  LINE = (D1,X,D2,Y)
```

The line `SIMP` would then be:

```
SIMP:  LINE = (DOUB(Q1,Q2),D2,3*(BPD),-DOUB(Q1,Q2))
```

USE Statement

The `USE` statement specifies which of the defined beam lines and sublines is to be used in the calculation. In our example, the most obvious candidate is the complete assembled beam line `SIMP`. The `USE` command would then read

```
USE  SIMP ;
```

If, however, we wished to investigate the properties of the entrance doublet without creating a new data deck, then we could simply use the command

```
USE  DOUB ;
```

TRANSPORT would then run through only the beamline named `DOUB`.

It is the `USE` statement which differentiates between the MAD procedure of defining beam

lines explicitly and the original TRANSPORT procedure of just listing the elements in order. If no `USE` statement is included in the data, TRANSPORT considers the beam line to consist of the elements in the order listed.

Assembling Beam Lines by Listing the Elements

When the beam line is defined by simply listing the elements in order, the individual elements may be defined in any mannner described in this manual. Any complete set of parameters may be used to specify a given element. It is recommended for clarity that a user not mix types of parameters more than is necessary. For example, it would be awkward to have the quadrupole component of the field of some bending magnets specified by the `N` parameter, while that of others are given by `K1`.

The two commands `REPEAT` and `SECTION` are used only when a beam line is assembled by simply listing the elements. When MAD notation is used to assemble a beam line, there are other ways to represent the same functions as `REPEAT` and `SECTION`.

Repetition

Many systems include a set of elements that are repeated several times. To minimize the chore of input preparation, a 'repeat' facility is provided. A typical repeat element might look like:

```
REPEAT,  N = 5  ;
```

The only parameter is `N`. If `N` is nonzero, it states the number of repetitions desired from the point where it appears. If `N` is zero it marks the end of a repeating unit.

For example, a total bend of 12 degrees composed of four 3-degree bending magnets each separated by 0.5 metres could be represented by

```
REPEAT,  N = 4.   ;
BEND,  - - -  ;
DRIFT,  L = .5  ;
REPEAT,  N = 0.   ;
```

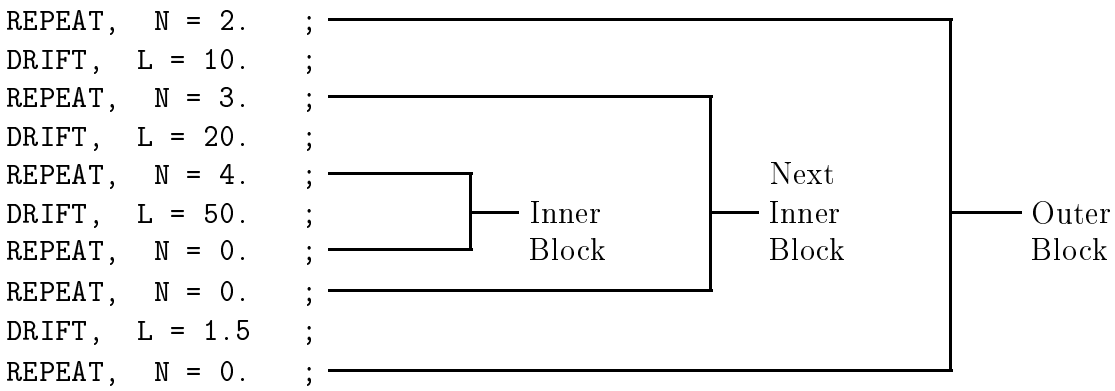
Those elements (in this case a bend and drift) between the (`REPEAT,  N = 4.  ;`) and (`REPEAT,  N = 0. ;`) would be used four times.

`REPEAT` elements do not appear explicitly in the output TRANSPORT prints when calculating. The only indication of their presence is the repeated listing of the elements they control.

The parameters of an element within a repeating unit may be varied in the usual fashion. However <u>all</u> repetitions of a given varied element will be coupled.

Repeat cards may be nested up to four deep. By "nesting" we mean a repeat within a repeat. An example is given below.

Example of Nesting



The total length of this sequence is:

$2 * (10. + 3 * (20 + 4 * \ 50) + 1.5) = 1343$.

Defined Section

A system may contain a section which is repeated at some later point. The repeated section may not occur immediately after the original section, so that the `REPEAT` element in not appropriate. It may also be useful to repeat a section, but with the elements listed in reverse order.

The keywords which can be used in defining a section of beam line are `BEGIN`, `END`, `FORWARD`, and `BACKWARD`. If a section of a beam line is to be identified to be used again at some later point, it is preceded by the command

```
FRED:  SECTION, BEGIN  ;
```

and followed by the command

```
FRED:  SECTION, END  ;
```

The label, in this case `FRED:`, identifies the section for later reference. If it is desired to repeat the same section at some later point in the beam line, one uses the command

```
FRED:  SECTION, FORWARD  ;
```

If the section is to be repeated but in reverse order, use the command

```
FRED: SECTION, BACKWARD  ;
```

A number of rules apply to the use of the SECTION element. Each defined section must have both its beginning and its end indicated, and the beginning must precede the end. The definition of a section must completely precede its use. A given name can be used only once to define a section, although it can be used many times to indicate a repeat of a section. The number of defined sections is limited to ten. The total z rotation (SROT element) within a defined section must equal zero. Finally, defined sections must nest properly with the REPEAT element. Defined sections, however, need not nest properly with each other.

Example of a Defined Section

```
DOUB:  SECTION,  BEGIN  ; ———————— Begin defined section
QUAD,  L = 10.,  B = 5.,  APER = 2.   ;
DRIFT,  L = 8.   ;
QUAD,  L = 10.,  B = -5.,  APER = 2.   ;
DOUB:  SECTION,  END,  ; ———————— End defined section
          ⋮
DOUB:  SECTION,  FORWARD  ; ———————— Repeat defined section
          ⋮
DOUB:  SECTION,  BACKWARD,  ; ———————— Repeat defined section in reverse order
```

The difference between the REPEAT command and the SECTION command is that the SECTION command does not require that the repetitions immediately follow the original. They may be separated by other portions of beam line. Thus we have that the sequence

```
REPEAT,  N = 5.   ;
(section of beam line)
REPEAT,  N = 0.   ;
```

is identical to the sequence

```
SAM:  SECTION,  BEGIN  ;
(section of beam line)
SAM:  SECTION,  END  ;
SAM:  SECTION  FORWARD  ;
SAM:  SECTION  FORWARD  ;
SAM:  SECTION  FORWARD  ;
SAM:  SECTION  FORWARD  ;
```

However, the example

```
SAM:  SECTION,  BEGIN  ;
(section of beam line)
SAM:  SECTION,  END  ;
(other elements)
SAM:  SECTION,  FORWARD  ;
```

cannot be done with the `REPEAT` element alone.

Operations

TRANSFORM 1 UPDATE *

To re-initialize the matrix `TRANSFORM 1` (R1 – the product of the R matrices), use the `UPDATE` element. An `UPDATE` element effects an update of the R1 matrix and initiates the accumulation of a new product matrix at the point of the update. This facility is occasionally useful for misaligning a set of magnets or fitting only a portion of a system. The element which updates R2 is usually more useful for these purposes. A description can be found on page 214.

The `UPDATE` element represents an operation and is not MAD compatible. It is expressed in keyword notation.

Notation

To update the transfer matrix R1, use the element.

```
UPDATE,  R1 ;
```

If the beam line is described simply by listing the elements in the order in which they occur physically, then the `UPDATE` instruction is placed at the location where the update is to be made.

If the beam line is assembled from elements and specified in a `LINE` statement, the `UPDATE` command may be placed after the beam line specification. It may refer to a marker which is included in the beam line specification. This is done with the `LOCA(TION)` keyword. Assume the marker

```
MAR1:   MARKER  ;
```

is included in the beam line description. Assume further that the user wishes to update the transfer matrix at the location of this marker. The command

```
UPDATE,  R1,  LOCA = MAR1 ;
```

will perform the desired update operation.

*By "updating" we mean initiating a new starting point for the accumulation (multiplication) of the R1 matrix. At the point of update the previous accumulation is discontinued. When the next element possessing a transfer matrix is encountered, the accumulated transfer matrix R1 is set equal to the individual transfer matrix R for that element. Accumulation is resumed thereafter.

Usage

An update may be used no matter what the order of the run. However, the beam matrix will be unaffected by the update only if the order of the transfer matrix being printed is no greater than first. The order order of the transfer matrix being printed is the second number on the `ORDER` element. In second and higher orders the aberrations cause the ellipse to become distorted. The distribution can still be described in terms of its first and second moments. An update causes the program to assume the distribution at the update is an ellipsoid with the given first and second moments. The subsequent behavior of this ellipsoid may differ from what would occur without this refitting procedure.

The matrix R1 is updated by no other element. It is not used in the calculation of the beam matrix. The beam matrix is calculated from the auxiliary transfer matrix R2 described on the next page.

A `TRANSFORM 1` matrix will be printed at any position in the data set where a

```
PRINT,  TRANS  ;
```

or

```
PRINT,  R1  ;
```

element is inserted.

See the following section for the introduction of an auxiliary transformation matrix R2 (`TRANSFORM 2`) to avoid the need for `TRANSFORM 1` updates.

The (`UPDATE,  R1 ;`) card also causes an update of the R2 matrix.

AUXILIARY TRANSFORMATION MATRIX (R2)

The R1 matrix represents the accumulated transfer matrix from either the beginning of the beam line or the last explicit R1 update (`UPDATE, R1 ;`). The transfer matrix is used to carry the beam matrix, i.e. from the initial beam matrix to calculate the beam matrix at some later point. However several elements in TRANSPORT which affect the beam matrix cannot be represented in any transfer matrix. To avoid update complications with R1 an auxiliary transfer matrix R2 exists. The beam matrix at a given point is then calculated from the R2 matrix and the beam matrix at the last R2 update.

Both the R1 and R2 matrices are normally available for printing. However there is no redundancy in computer use, since, internally to the program, only R2 is calculated at each element. The matrix R1 is calculated from R2 only as needed. An update of R2 does not update R1.

Notation

To update the auxilliary transfer matrix R2, one uses the element

```
UPDATE,  R2 ;
```

Usage

The R2 matrix may be printed by a

```
PRINT,  TRANS2  ;
```

or

```
PRINT,  R2  ;
```

entry. Constraints on R2 are imposed similarly to those on R1. For details see page 255 or page 263.

The complete list of elements which automatically update `TRANSFORM 2` is:

1. A beam (`BEAM`) element.
2. An R1 update (`UPDATE, R1`).
3. An R2 update (`UPDATE, R2`).
4. An unknown misalignment element (`ALIGN`).
5. A stray field type code `21.0` entry. (This element is not currently operative. There will be no description of it elsewhere in this manual.)

Please note that automatic updates of `TRANSFORM 2` occur when an `ALIGN` element is inserted specifying the uncertain misalignment of all subsequent bending magnets. These `TRANSFORM 2` updates take place immediately before and immediately after any bending magnet which has either the entrance or exit fringe fields specified via a `ROTATE` element.

An example of the data for a section of a beam line might be the following sequence of elements.

```
BEAM, .   .   .   ;
D1:   DRIFT,  .   ;
Q1:   QUAD,  .   .   .   ;
D2:   DRIFT,  .   ;
Q2:   QUAD,  .   .   .   ;
D3:   DRIFT,  .   ;
UPDATE,  R2  ;
D4:   DRIFT,  .   ;
Q3:   QUAD,  .   .   .   ;
D5:   DRIFT,  .   ;
Q4:   QUAD,  .   .   .   ;
D6:   DRIFT,  .   ;
```

From the beginning of the beam line through the end of the drift D3, the two transfer matrices R1 and R2 will be the same. The R2 matrix is reinitialized by the (`UPDATE,  R2  ;`) command and accumulation starts again with the drift D4. The R1 matrix continues to accumulate unaffected by the R2 update. After traversing the drift D6, R1 and R2 will be given by:

R1 = R(D6)*R(Q4)*R(D5)*R(Q3)*R(D4)*R(D3)*R(Q2)*R(D2)*R(Q1)*R(D1)

while

R2 = R(D6)*R(Q4)*R(D5)*R(Q3)*R(D4)

ALMARK – Alignment Marker

A known misalignment does not change the calculated phase space volume of the beam envelope. Therefore, no R1 or R2 update need occur at the end of a section which is given a known misalignment. It is also not necessary to use an `UPDATE` element to demarcate the beginning of a section with a known misalignment. The beginnings of sections to be misaligned may also be indicated by the alignment marker element. No `UPDATE` will result from the insertion of the alignment marker if the misalignment referring to the marker is known. The alignment marker will perform an (`UPDATE, R2 ;`) if the misalignment referring to it is uncertain.

Notation for the alignment marker is exactly the same as for the `UPDATE` element. The keyword `ALMARK` simply replaces the keyword `UPDATE`. The keyword (R1 or R2) on the `ALIGN` element will pertain to the most recent `ALMARK` element if it is more recent than the most recent `UPDATE` element of the same type.

Notation

The keyword `R1` on the misalignment element refers to an update of the R1 matrix. It can also refer to an `ALMARK` element if one occurs more recently than any R1 update. The `ALMARK` element should then read:

```
ALMARK,  R1  ;
```

If the beam line is assembled from elements and specified in a `LINE` statement, the `ALMARK` command may be placed after the beam line specification. It may refer to a marker which is included in the beam line specification. This is done with the `LOCA(TION)` keyword. Assume the marker

```
MAR1:   MARKER  ;
```

is included in the beam line description. Assume further that the user wishes to begin a misaligned section at the location of this marker. The command

```
ALMARK,  R1,  LOCA = MAR1  ;
```

will demarcate the location as a possible reference for subsequent `ALIGN` elements.

The keyword `R2` on a misalignment element refers to an update of the R2 matrix. Similarly, the command:

```
ALMARK,  R2 ;
```

can substitute for an update of the auxiliary R2 matrix.

Example

A bending magnet (including fringe fields) misaligned by a known amount was shown as an example on page 195. Since the misalignment in that case was known, the example might now be represented as follows:

```
DRIFT,  -  ;
ALMARK,  R2  ;
ROTATE,  ANGLE = 0.   ;
BEND,  -  -  -  ;
ROTATE,  ANGLE = 0.   ;
ALIGN,  RZ = 2.,  KNOWN,  R2  ;
DRIFT,  -  ;
```

This represents a known rotation of the bending magnet about the incoming beam direction (z axis) by 2.0 mr. The result of this misalignment will be a definite shift in the beam centroid, and a mixing of the horizontal and vertical coordinates. The use of the (`ALMARK, R2 ;`) alignment marker and the keyword `R2` is necessary because the magnetic array (bending magnet + fringing fields) consists of three elements instead of one. Because the `ALMARK` command was used and because the misalignment is known, there will be no R2 updates.

PRINT — Output Print Control Instructions

A number of control codes which transmit output print instructions to the program have been consolidated into the single element `PRINT`. Most `PRINT` elements produce a line or lines of output that advertise their existence.

Those that do not usually have an obvious effect upon the remainder of the output and thus make their presence known. Some other print controls which may be placed on the indicator card are described on page 54. They are of a more global nature and therefore not appropriate for inclusion among the `PRINT` instructions.

The `PRINT` instruction is an operation and not part of the physical description of the beam line. It is therefore specific to TRANSPORT and not expressible in MAD notation. The keywords listed below are used to indicate the different options.

In the descriptions the `PRINT` instructions are placed in parentheses to isolate them from the text. In actual use, the parentheses are omitted.

Summary of `PRINT` Instructions

The mnemonic used in specifying print control instructions is `PRINT`. The keywords and which can be used are:

Keyword	Prints or Activates
`FLOOR`	The floor coordinates of the reference trajectory.
`BP`	The floor coordinates of the effective bend points of all the bending magnets.
`TRANS` or `R1`	The transfer matrix R1 (and possibly T1 and U1)
`TRAN2` or `R2`	The auxilliary transfer matrix R2.
`R1INV`	The inverse of the transfer matrix R1 (and possibly T1 and U1).
`R2INV`	The inverse of the transfer matrix R2 (and possibly T2 and U2).
`BEAM`	The beam (σ) matrix
`WAIST`	The positions and sizes of the nearby waists
`CENTROID`	The beam centroid when the beam is off axis.
`TWISS`	The beam matrix in accelerator notation (β and α parameters) and the beam phase shift (ψ).
`ALIGN`	The misalignment effect table
`REFER`	Refer the transfer and beam matrices to the unrotated coordinate system.
`LEVEL`	Refer the transfer and beam matrices to transversely levelled coordinate system.
`ONELINE`	The printing of transfer and beam matrices on a single line.
`NOPARA`	The suppression of physical parameters when printing

ELEMENTS	The listing of the physical elements.
ALL	The printing of physical elements and results of calculation.
MARKER	Markers are included among the elements to be printed
ONLY	The printing only of varied elements and constraints
NARROW	Printing of the beam (σ) matrix and FIT elements within the first 80 columns.
PRECISE	Precise values of varied parameters
ON	Certain print instructions are to be executed after every physical element.
OFF	Turns off the automatic print after each element.

Notation

There are no values associated with the print instruction. The command consists of the word PRINT followed by a keyword giving the nature of the print instruction. To print the transfer matrix at a particular location, for example, one would use the instruction:

```
PRINT,  TRANS  ;
```

To print the beam matrix at a particular location, use the instruction

```
PRINT,  BEAM  ;
```

Several keywords may now be placed on a single line with the single mnemonic PRINT. An example might be

```
PRINT,  BEAM,  TRANS  ;
```

This single instruction causes both the beam (sigma) matrix and the transfer matrix to be printed at the specified location. *If* BEAM *and* TRANS *are included in the same element the beam matrix will be printed before the transfer matrix, regardless of the order of the keywords.*

If it is desired to have the transfer matrix printed after each physical element, then one uses the instruction

```
PRINT,  TRANS,  ON  ;
```

To terminate the automatic printing of the transfer matrix, use the command

```
PRINT,  TRANS,  OFF  ;
```

Placement

Some of the PRINT commands are applied globally and affect the entire beam line. Such a command should be placed before the BEAM element at the beginning of the beam line description. It then need not have a location parameter referring to any marker. Its effect will be invoked before the trace through the beam line is begun.

An example is the command

```
PRINT,  TRANS,  ON  ;
```

This command indicates that transfer matrix is to be printed after each physical element.

Other PRINT commands have their effect at a single place in the beam line. The placement of such a PRINT command depends on how the beam line is specified. A beam line can be specified either by simply listing the elements in order, or by first defining the elements then assembling them in to a beam line by LINE and USE statements.

If the beam line is specified simply by listing the elements, then the PRINT command is inserted at the location where it is to have its effect. For example, if it is desired to print the transfer matrix immediately after a particular 10 meter drift space, then the line

```
PRINT,  TRANS  ;
```

would be inserted immediately after the line

```
DRIFT,  L = 1.0  ;
```

If the beam line is assembled from elements and specified in a LINE statement, a PRINT instruction is normally placed after the LINE and USE instructions. It may refer to a marker which is included in the beam line specification. The reference is given by the LOCA(TION) keyword. Assume the marker

```
MAR1:   MARKER  ;
```

is included in the beam line description. Assume further that the user wishes to print the transfer matrix at the location of this marker. The command

```
PRINT,  TRANS,  LOCA = MAR1  ;
```

will print the transfer matrix at the location MAR1. The matrix will be printed to the order specified on the ORDER element, found near the beginning of the data.

The LOCA keyword can also be used if the beam line is specified simply by listing the elements. The PRINT command is now placed after the beam line definition, just as if it had

been defined by `LINE` and `USE` statements. When TRANSPORT encounters a marker, it looks for any `PRINT` or other elements which may refer to that marker. Such `PRINT` or other elements are then executed and the program returns to the element following the marker.

A `PRINT` command may be given a label. The purpose of the label is so that the print command can be activated or deactivated in going from one problem step to another. It has no relation to any marker.

Floor Coordinate Layout Control

(`PRINT, FLOOR ;`): The floor coordinates of the beam reference trajectory are printed after every beam line component. This instruction is placed before the `BEAM` element. One can produce a layout of a beam line in any Cartesian coordinate system one chooses. If no additional elements are inserted, the reference trajectory of the beam line will be assumed to start at the origin and proceed along the positive z-axis. One can also specify other starting coordinates and orientations by the placement of special parameter elements before the beam element. For a description of these particular special parameters (the `SPECIAL` element) see page 73.

The coordinates printed provide a global reference system for the entire beam line. They give the X, Y and Z position, and the yaw, pitch, and roll angles, respectively, of the reference trajectory at the interface between two elements. The Y-axis will point up and the X-axis to the left. The *yaw* angle is between the floor projection of the reference trajectory and the floor Z axis. The *pitch* angle is the vertical pitch, the angle the reference trajectory makes with a horizontal plane. The *roll* angle is a rotation about the reference trajectory. In the printed output the values given are those at the exit of the element listed above and at the entrance of the element listed immediately below. A figure showing the meaning of the three coordinates and the three angles is shown on the next page.

The calculation of the coordinates is done from the parameters of the physical elements as given in the data. Therefore, if effective lengths are given for magnetic elements, the coordinates printed will be those at the effective field boundary.

At lower energies, the fringe fields of bending magnets must also be included. The extended nature of fringing fields produces a transverse shift to the reference trajectory at a bending magnet entrance and exit faces in comparison with an idealized bending magnet where the field begins and ends abruptly. This transverse shift can become important when the quantity $g^2/2\rho$ is a large enough transverse dimension to have a detectable effect on the optical system. Here g represents the magnet gap, and ρ the trajectory radius of curvature. More details can be found in the section of fringing fields starting on page 129.

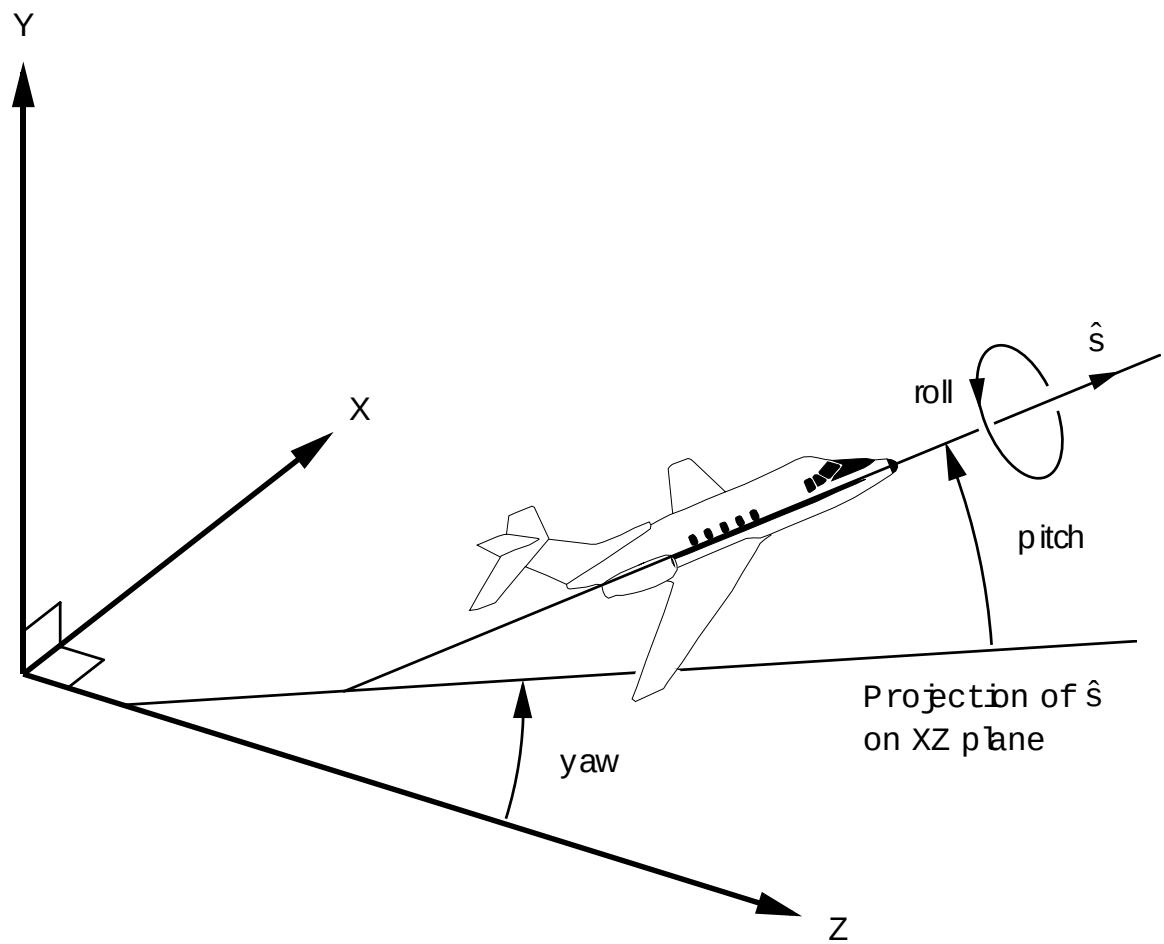
The floor coordinates will be printed after every physical element. The accumulated length L_c of the reference trajectory, the three Cartesian floor positions and the three orientation angles will be printed on a single line, along with the units in which these quantities

are expressed. The single line will have the following appearance:

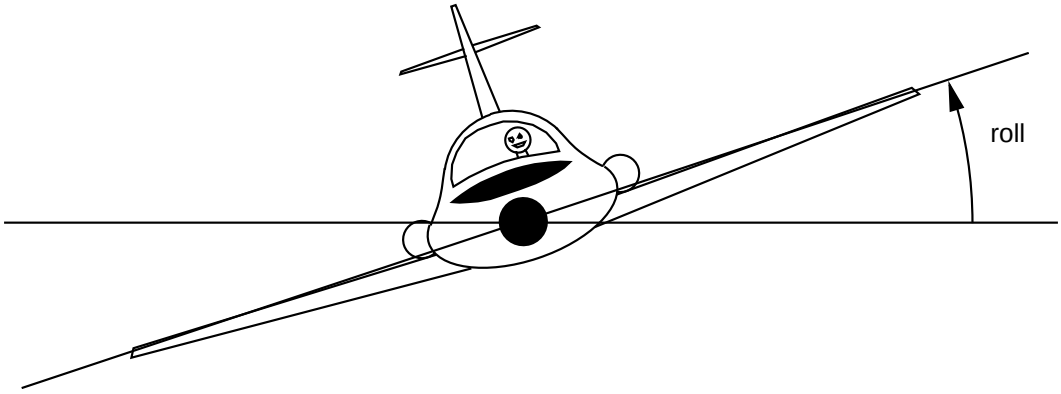
$$L_c \ \texttt{M} \qquad Xfloor \ \ Yfloor \ \ Zfloor \ \texttt{M} \qquad yaw \ \ pitch \ \ roll \ \texttt{R}$$

Here the units used are meters (`M`) and radians (`R`). These are the units used by the global unit specification `UMETER`. The units are meters (`M`) and milliradians (`DEG`) for the global unit set `UMM`. Other global unit sets or individual units can also be used and are described on pages 64 and 68.

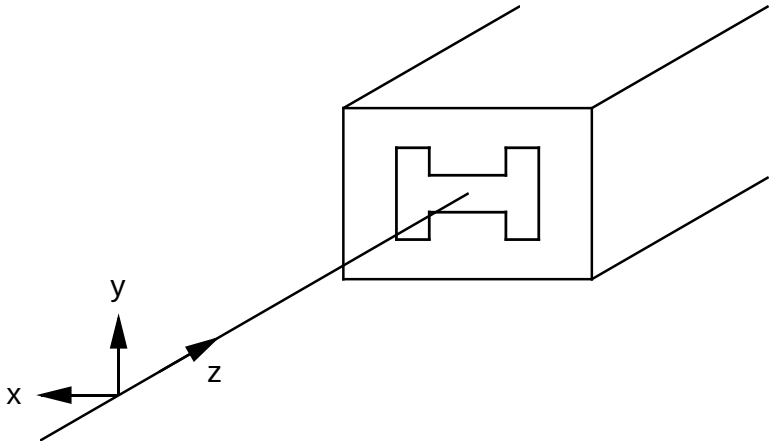
The following figure illustrates the meaning of the three angles used to specify the orientation of the reference trajectory.



The Global Reference System.



The Meaning of the Roll Angle



The Local Reference System

The beam and transfer matrices are referred to the local coordinate system. The z coordinate is the direction of travel of the airplane in the previous illustrations. The x coordinate points down the left wing of the airplane, and the y coordinate points through the roof of the cabin.

Effective Bend Points

(PRINT, BP ;): The effective bend points of all bending magnets are included in the floor coordinates. The effective bend point of a bending magnet is the intersection of the straight projections of the entering and exiting reference trajectories. It is if the magnet were shrunk to the limit of zero length, while keeping the bend angle constant.

The floor coordinates of the effective bend point are printed immediately upon exiting the magnet. They are followed by the floor coordinates of the reference trajectory at the exit face of the magnet. The (PRINT, BP ;) will be effective only if the (PRINT, FLOOR ;) command is also present.

Transformation Matrix Print Controls

(PRINT, TRANS ;) or (PRINT, R1 ;): The current transformation matrix R1 (TRANSFORM 1) is printed at the location of this element. The location of the element can be determined either by direct placement in the beam line, or by reference to a MARKER through the use of the LOCA keyword.

If the program is computing a second-order matrix, this second-order transformation matrix will be included in the print-out. If the program is computing a third-order matrix, the third-order matrix will also be included. This matrix is cumulative from the last R1 (TRANSFORM 1) update. The units of the elements of the printed matrix are consistent with the input units associated with the BEAM element.

(PRINT, TRANS, ON ;) or (PRINT, R1, ON ;): The transformation matrix R1 will be printed after every physical element which follows this code. Normally this element is included near the beginning of the data, before the BEAM element. If, for some reason, it is desired to have the transfer matrix printed after each element starting at some later point in the beam line, then the (PRINT, TRANS, ;) or (PRINT, R1, ;): element can be inserted at that later point.

The second order matrix will be printed automatically after every physical element only if the one-line form (PRINT, ONELINE ;) of the transformation is selected. The second order matrix will, however, always be printed at each location of a (PRINT, TRANS ;) element. The third-order matrix will be printed only at a location of the (PRINT, TRANS ;) or (PRINT, R2, ;) element.

The first-order matrix will be printed only once at a (PRINT, TRANS ;) or (PRINT, R1 ;) element. In other words, if (PRINT, TRANS, ON ;) or (PRINT, R1, ON ;) has been placed at the beginning of the beam line, and a (PRINT, TRANS ;) element is placed at some later point, the first-order matrix will not be printed again. If a second- or third order calculation has been specified, the higher-order matrices will be printed after the first-order matrix.

(PRINT, TRANS, OFF ;) or (PRINT, R1, OFF ;): The automatic printing of R1 will be suppressed and R1 will be printed only when subsequently requested.

(PRINT, TRAN2 ;) or (PRINT, R2 ;): The TRANSFORM 2 matrix, R2, will be printed by this code. It should be placed in the beam line at the point where the printing is to occur. The format and units of R2 are identical with those of R1, which is printed by the (PRINT TRANS ;) code. For a list of elements which update the R2 matrix, see the UPDATE element.

(PRINT, R1INV ;) The inverse of the current transfer matrix R1 will be printed by this instruction. If the program is computing a second-order matrix, the inverse of the second-order transfer matrix will be included in the print out. If the program is computing a third-order matrix, the inverse of the third-order matrix will be included.

(PRINT, R2INV ;) The inverse of the current transvermatrix R2 will be printed by this instruction. Comments about higher orders are the same as for the (PRINT, R1INV ;), described immediately above.

The units of the tabulated matrix elements in either the first-order R or σ matrix, second-order T matrix, or third-order U matrix of a TRANSPORT print-out will correspond to the units chosen for the BEAM element. For example, in the unit set UMM the $R_{12} = (x|x_0')$ matrix element will have the dimensions of mm/mr, which is the same as m/r. The $T_{236} = (x'|y_0\delta)$ matrix element will have the dimensions mr/(mm-pm) and so forth.

The six coordinates of the transfer matrix correspond to x, x', y, y', ℓ, and δ. The matrix will then have the appearance

$$\begin{array}{cccccc}
(x|x_0) & (x|x_0') & (x|y_0) & (x|y_0') & (x|\ell) & (x|\delta) \\
(x'|x_0) & (x'|x_0') & (x'|y_0) & (x'|y_0') & (x'|\ell) & (x'|\delta) \\
(y|x_0) & (y|x_0') & (y|y_0) & (y|y_0') & (y|\ell) & (y|\delta) \\
(y'|x_0) & (y'|x_0') & (y'|y_0) & (y'|y_0') & (y'|\ell) & (y'|\delta) \\
(\ell|x_0) & (\ell|x_0') & (\ell|y_0) & (\ell|y_0') & (\ell|\ell) & (\ell|\delta) \\
(\delta|x_0) & (\delta|x_0') & (\delta|y_0) & (\delta|y_0') & (\delta|\ell) & (\delta|\delta)
\end{array}$$

If the beam centroid is off axis, the appearance of what is printed is slightly different. The initial beam centroid is the initial center of the beam ellipse. It is given by a set of coordinates which can be taken as representing a trajectory and therefore traced through the beam line. The default phase-space position of the centroid is that it coincide with the reference trajectory. If a CENTROID element is included in the data, then the initial phase-space coordinates of the centroid are given on the CENTROID element. The centroid may, at some place in the beam line, deviate from the reference trajectory when it encounters a known misalignment, a bending magnet with excess horizontal bend or an error vertical bend field, a kicker magnet HKICK, VKICK, or KICKER, an electrostatic septum SEPTUM, or a shift in coordinates SHIFT.

The transfer matrix represents an expansion about the centroid. Thus the centroid

position represents a zeroeth order term in the transformation of an arbitrary trajectory. If the centroid coincides with the reference trajectory, it is "on axis" and its coordinates are zero. It must be emphasized that not all trajectory coordinates are geometrically defined. An initial beam centroid may be geometrically on the reference trajectory but have a different defining momentum, so that the coordinate δ for the centroid is not zero. In that case, the centroid is still considered to be "off axis."

If the centroid is off axis, an extra column will appear to the right, giving the position of the beam centroid. The centroid given is the optical image of the centroid at the beginning of the beam line. In first order it will coincide with the center of gravity of the beam ellipse. In higher orders the two will differ due to nonlinear distortions of the beam ellipse.

$$\begin{array}{cccccc c}
(x|x_0) & (x|x'_0) & (x|y_0) & (x|y'_0) & (x|\ell) & (x|\delta) & x_c \\
(x'|x_0) & (x'|x'_0) & (x'|y_0) & (x'|y'_0) & (x'|\ell) & (x'|\delta) & x'_c \\
(y|x_0) & (y|x'_0) & (y|y_0) & (y|y'_0) & (y|\ell) & (y|\delta) & y_c \\
(y'|x_0) & (y'|x'_0) & (y'|y_0) & (y'|y'_0) & (y'|\ell) & (y'|\delta) & y'_c \\
(\ell|x_0) & (\ell|x'_0) & (\ell|y_0) & (\ell|y'_0) & (\ell|\ell) & (\ell|\delta) & \ell_c \\
(\delta|x_0) & (\delta|x'_0) & (\delta|y_0) & (\delta|y'_0) & (\delta|\ell) & (\delta|\delta) & \delta_c
\end{array}$$

For a static magnetic system with midplane symmetry, a good deal of simplification occurs. Many of the matrix elements become zero. For the case of an on-axis centroid, the transfer matrix then becomes:

$$\begin{array}{cccccc}
(x|x_0) & (x|x'_0) & 0.0 & 0.0 & 0.0 & (x|\delta) \\
(x'|x_0) & (x'|x'_0) & 0.0 & 0.0 & 0.0 & (x'|\delta) \\
0.0 & 0.0 & (y|y_0) & (y|y'_0) & 0.0 & 0.0 \\
0.0 & 0.0 & (y'|y_0) & (y'|y'_0) & 0.0 & 0.0 \\
(\ell|x_0) & (\ell|x'_0) & 0.0 & 0.0 & 0.0 & (\ell|\delta) \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & (\delta|\delta)
\end{array}$$

If the beam line is midplane symmetric and there is no vertical bending, then the transfer matrix can be printed on a single line. To do this, one should include the command (`PRINT, ONELINE ;`) described below. Only those matrix elements which can be nonzero under the stated conditions are printed. The quantities printed are:

$$L_c \ \texttt{M} \qquad R_{11} \ R_{12} \ R_{21} \ R_{22} \ R_{33} \ R_{34} \ R_{43} \ R_{44} \ R_{16} \ R_{26}$$

The units for only the accumulated length L_c are shown and are here taken to be meters `M`.

If second order is specified, then the second-order matrix elements which are nonzero in a midplane symmetric situation will be printed in a compact form. At this point, there is no abbreviated form for the third-order transfer matrix.

Beam Matrix Print Controls

(`PRINT, BEAM ;`): The beam (σ) matrix is printed at the location of this element. The location of this element can either be determined directly by its placement in the beam line, or indirectly by reference to a `MARKER` element using the `LOCA` keyword.

(`PRINT, BEAM, ON ;`): The beam (σ) matrix will be printed after every physical element which follows this element. This element is normally placed near the beginning of the data, before the `BEAM` element.

(`PRINT, BEAM, OFF ;`): The effect of a previous (`PRINT, BEAM, ON ;`) code or its equivalent is cancelled and the beam (σ) matrix is printed only when a (`PRINT, BEAM ;`) code is encountered or when another (`PRINT, BEAM, ON ;`) code is inserted. The suppression of the beam matrix is the normal default.

The beam matrix, as printed will have the following appearance:

x_g	Δx	CM					
x'_g	$\Delta x'$	MR	r_{21}				
y_g	Δy	CM	r_{31}	r_{32}			
y'_g	$\Delta y'$	MR	r_{41}	r_{42}	r_{43}		
ℓ_g	$\Delta \ell$	CM	r_{51}	r_{52}	r_{53}	r_{54}	
δ_g	$\Delta \delta$	PC	r_{61}	r_{62}	r_{63}	r_{64}	r_{65}

The first column, with the subscript "g", represents the coordinates of the center of gravity of the beam phase space. In first order the center of gravity is the same as the centroid, which is the geometric center of the phase ellipsoid. In second and higher order, the ellipsoid may become distorted. In that case, the center of gravity is no longer the beam centroid. If the centroid does not coincide with the reference trajectory, then the centroid coordinates are printed along with the transfer matrix. (see page 224 above for a description of the printing of the transfer matrix and a definition of the beam centroid).

The second column gives the half widths of the beam ellipsoid in each of the six beam coordinates. Once again, in higher orders the beam phase space is not an ellipsoid, but some distorted shape like a banana. The quantities given are then the square roots of the second moments of the beam phase space, taken about the beam center of gravity. In terms of the diagonal elements σ_{ii} of the beam matrix the beam half widths are given by:

$$\Delta X_i = \sqrt{\sigma_{ii}}$$

The units used are given in the third column. Those illustrated here are millimeters `MM`, milliradians `MR`, and permil `PM`. These units are used in the global unit set `UMM`. The use of unit sets or individual units is described on pages 64 and 68.

The remaining quantities r_{ij} are the correlations between coordinates. They are defined

in terms of the beam matrix elements σ_{ij} by:

$$r_{ij} = \frac{\sigma_{ij}}{[\sigma_{ii}\ \sigma_{jj}]^{1/2}} \quad .$$

Their initial values may be specified by the `CORRELATION` element described on page 109.

The beam matrix may also be printed in a single line. The command which causes the printing to occupy a single line is (`PRINT, ONELINE ;`). This command, described below, does not print anything, but simply affects the style of how quantities are printed. A single line printing of the beam matrix would look like:

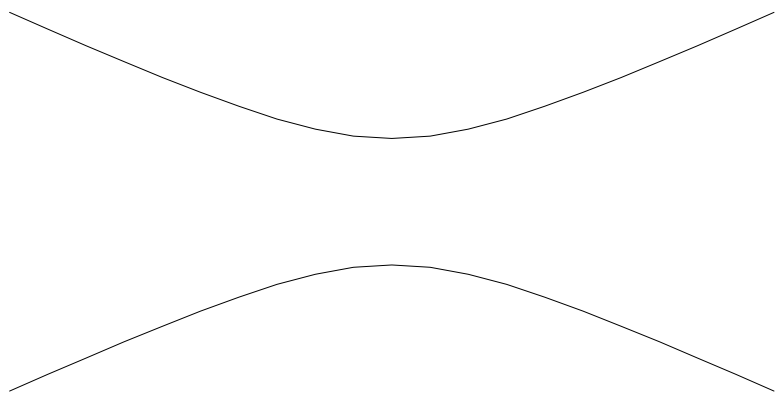
L_c `M` Δx `MM` $\Delta x'$ `MR` Δy `MM` $\Delta y'$ `MR` $\Delta \ell$ `MM` $\Delta \delta$ `PM` r_{21} r_{43}

The accumulated length L_c along the reference trajectory is shown first. The units shown are from the global unit set `UMM`. Other global unit sets are described on page 64. Individual unit changes are given on page 68.

A diagram of the beam phase ellipse is shown later in this section.

Positions of Waists

(`PRINT, WAIST ;`): The longitudinal positions and transverse dimensions of (real or virtual) waists in both transverse planes are printed at the point of insertion of this element. The waist location and characteristics are computed from the dimensions of the current location assuming infinitely long drift spaces both upstream and downstream. An illustration of a waist is shown below.



A Waist in a Beam.

Printing of the Beam Centroid

The beam centroid is taken as the center of the beam ellipsoid. If misalignments, centroid shifts, or mistuned bending magnets are in the beam, the beam centroid may not coincide with the reference trajectory. Normally the centroid position is printed as part of the beam matrix output. If the beam matrix is printed on a single line, the centroid position is printed only if the beam is off axis. Two lines of output are then printed after each element: the first being the centroid position and the second being the σ matrix. The output then has alternating lines of different meaning, and scanning down single columns is visually unpleasant.

For this reason, there is a command which will suppress the printing of the centroid.

```
PRINT,  CENTROID,  OFF  ;
```

This element should be placed near the beginning of the data, before the `BEAM` element. Restoring the printing of the centroid is done with the command

```
PRINT,  CENTROID,  ON  ;
```

If this latter element is placed at the beginning of the data, before the `BEAM` element, then the centroid will still be printed after each physical element, even though all its entries may be equal to zero.

In a second or higher-order calculation, the beam centroid will differ from the beam center of gravity, as explained above. In this case, it is actually the printing of the center of gravity that is controlled by this element. The element therefore, in higher orders, is slightly misleadingly named. The exact meaning of the centroid is described later on page 224.

Beam Matrix Print in Accelerator Notation

(`PRINT, TWISS ;`): The beam matrix in accelerator parameters (β, α, and η) and the beam phase shift (ψ) are printed at the location of this element. The location of this element can either be determined directly by its placement in the beam line, or indirectly by reference to a `MARKER` element using the `LOCA` keyword.

(`PRINT, TWISS, ON ;`): The beam matrix in accelerator parameters (β, α, and η) and the beam phase shift (ψ) are printed after every physical element which follows this code. This element is normally placed near the beginning of the data, before the `BEAM` element.

(`PRINT, TWISS, OFF ;`): The effect of a previous (`PRINT, TWISS, ON ;`) is cancelled and the beam matrix in accelerator parameters (β, α, and η) and the beam phase shift (ψ) are printed only when a (`PRINT, TWISS ;`) code is encountered or when another (`PRINT, TWISS, ON ;`) code is inserted. The default in TRANSPORT is not to print the accelerator parameters.

The use of the accelerator parameters for the beam matrix makes sense only in first order. In this case, the term "first order" applies to the printing of the transfer matrix. If the beam centroid is off axis, then the transfer matrix can be calculated to first, second, or third order and then linearized about the centroid position. The linear transformation then preserves the elliptical shape of the beam phase space. The elliptical shape allows the representation in terms of accelerator parameters. The inclusion of higher orders in the expansion about the centroid position causes the ellipse to become distorted and precludes the meaningful use of the accelerator parameters.

When the beam matrix is printed in accelerator notation after each physical element, TRANSPORT will first print a heading identifying the quantities printed. The printing of accelerator parameters occurs on a single line, so that the progress of each quantity as it passes down the beam line can be read in a single column. The single line begins with the accumulated length L_c, and then appears as:

$$L_c \quad \mathtt{M} \qquad \psi_x \;\; \psi_y \;\; \beta_x \;\; \beta_y \;\; \alpha_x \;\; \alpha_y \;\; \eta_x \;\; \eta_y \;\; \eta'_x \;\; \eta'_y$$

The quantities ψ_x and ψ_y are the phase advances in the two transverse planes and are defined by

$$\psi = \int \frac{ds}{\beta}$$

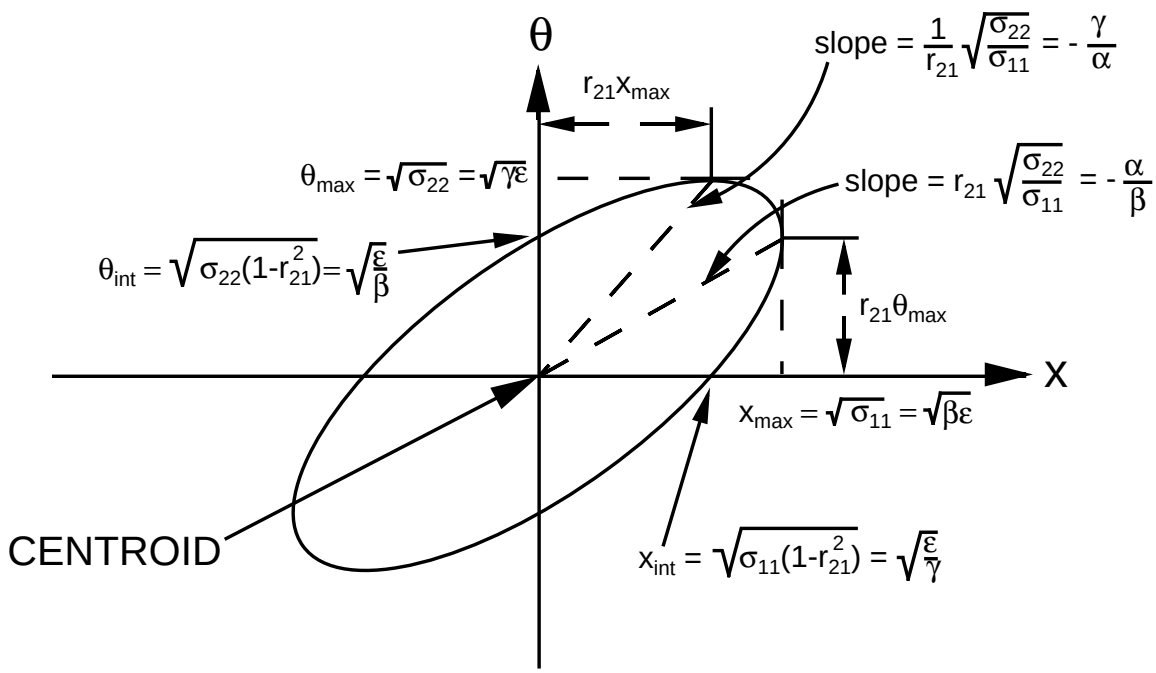
The quanitites β and α are shown in the figure below. The η vector and its derivatives represent the momentum behavior of the beam line and are described in the section on ETA. (See page 112.)

The accelerator parameters calculated and shown describe the phase space of the beam. They are not a description of the lattice, although they are affected by the lattice. The α and β are calculated from the values given on the `BEAM` element. They can be calculated

even if the beam description is given in σ matrix notation, as long as there is no mixing of transverse planes.

The units for only the accumulated length L_c are given, and here they are shown to be meters (`M`). The units used should always be such that the ratio of transverse distance to transverse angle is meters, and the ratio of transverse distance to momentum difference is also meters. Otherwise the components of β and η will come out in strange units like dekameters or hectinches. The global unit sets `UMAD`, `UMETER`, `UMM`, and `UMICRON` all have this desirable characteristic. The global unit set `UTRANS` does not and is not suitable for use with accelerator parameters for the beam ellipse.

The only violation of midplane symmetry that is permitted with the use of accelerator parameters is a vertical bend. The use of accelerator parameters assumes a separation of the optics of the two transverse planes. Skew quadrupoles, or other skew elements violate this assumption. Should TRANSPORT encounter such an element when accelerator notation is being used, it will refuse to proceed.



A Two-Dimensional Beam Phase Ellipse

Misalignment Table Print Control

(`PRINT, ALIGN ;`): The misalignment summary table is printed wherever this element is inserted. Its contents are the effects of all previously specified misalignments whose results were to be stored in a table. A full description of the table and its contents is to be found on page 204. The location of insertion may be explicit by placing the command at the point in the beam line where the table is to be printed. It can also be given by reference to a `MARKER` element through the `LOCA` keyword.

Refer Transfer Matrix to Original Coordinate System

.

(`PRINT, REFER ;`): The interpretation of the beam rotation element `SROT` is affected by this element. The transverse coordinates can be rotated about the longitudinal axis by an angle via the `SROT` element. The default interpretation is that the beam phase space is rotated. The transfer and beam matrices and the floor coordinates are printed in the rotated system. This element should be near the beginning of the data, after the `BEAM` element.

When the (`PRINT, REFER ;`) element is used, the rotation is taken as specifying only the orientation of any ensuing magnets. The transfer matrix, whenever printed, is given in the unrotated coordinate system. The transfer matrix then gives only the effect of the beam line on a particle trajectory, and not the effect of specifying the trajectory components in a different coordinate system. This element should be placed near the beginning of the data, before the `BEAM` element.

Refer Transfer Matrix to Level Coordinate System

(`PRINT, LEVEL ;`): The transfer matrix, beam matrix, and floor coordinates are caused by this element to be referred to a coordinate system whose x axis is level. A combination of vertical and horizontal bends can produce a beam line where the z axis is level, but the x axis is not. This is often the coordinate system in which the original x and y coordinate systems remain uncoupled as we progress down the beam line. but the floor coordinates have a slight roll. This command removes the roll, levelling the x axis, but introduces a coupling between the two transverse planes. This command should be inserted at the location where it is to have its effect.

General Output Format Controls

(PRINT, ONELINE ;): The beam (σ) and transformation (R1 or R2) matrices, when printed, will occupy a single line. Only those elements are printed which will be nonzero if horizontal midplane symmetry is maintained. The second- and third-order transfer matrices will obviously take several lines of printing for all the nonzero elements. However, this command also causes these higher-order matrices to be printed in a more compact format. This command should be placed near the beginning of the data, before the BEAM element .

In a first-order run, the (PRINT, ONELINE ;) element, in conjunction with the (PRINT, NOPARA ;) element and either the (PRINT, BEAM, ON ;) element or the (PRINT, TRANS, ON ;) element, will produce output in which the printed matrix elements will occupy single uninterrupted columns. For visual appearances it is recommended that, if both beam (σ) and transformation matrices are desired, they be printed in separate steps of a given problem.

(PRINT, NOPARA ;): The subsequent printing of the physical parameters of all physical elements will be suppressed. Only the mnemonic and the label will remain. This element is useful in conjunction with the (PRINT ONELINE ;) element which restricts the beam (σ) matrix and the transformation (R1) matrix each to a single row. The elements of these matrices then appear in uninterrupted columns in the output. This element should also be placed near the beginning of the data, before the BEAM element.

(PRINT, ELEMENTS, OFF ;) and (PRINT, ELEMENTS, ON ;): The first of these elements completely suppresses the printing for the elements in a portion of the beam line. The second element restores that printing. The transfer matrix and beam matrix are still printed wherever an explicit print instruction is found. If one has a very long beam line, but is investigating only a small portion of it, one need print only that portion in which one is interested. One then places the command ((PRINT, ELEMENTS, OFF ;)) near the beginning of the data before the BEAM element. The (PRINT, ELEMENTS, ON ;) is placed before the first physical element to be printed. Finally, another (PRINT, ELEMENTS, OFF ;) element is placed after the last physical element to be printed.

(PRINT, ALL, OFF;) and (PRINT, ALL, ON ;): The first of these elements completely suppresses all the printing in a portion of the beam line. The second element restores that printing. With this element, one can produce an even more compact output than with the previous element. The placement of this element is the same as that for the preceding element.

(PRINT, MARKER ;): Any markers inserted in the beam line will now be included in the printed output. Markers are not physical elements, but simple designations of position to which PRINT or FIT coommands can refer. Some users have found it useful to have them printed out when they occur. The default is not to have the markers printed, but simply execute any operations referring to the particular marker. This command should be placed before the BEAM element.

(`PRINT, ONLY ;`): Only varied elements and constraints will be printed. This element, in conjunction with the various options on the indicator card, can produce a very abbreviated output. The entire output of a multistep problem can now easily be viewed on a terminal or printed on a laser printer. This element should be placed near the beginning of the data, after any unit changes, but before the `BEAM` element.

(`PRINT, NARROW ;`): Some terminals display only the first 80 characters of a line. It can sometimes be both awkward and annoying to try to view anything printed to the right of the first 80 characters. The above command causes the beam matrix to be moved to the left in the output, so that it is entirely within the first 80 characters. The beam matrix printing is affected only in the σ matrix notation. The printing of the `FIT` element is now printed on two lines so that the current value of the constrained quantity also fits within the first 80 characters. This element should be placed near the beginning of the data, before the `BEAM` element.

Precise Values of Varied Parameters

(`PRINT, PRECISE ;`): The values of any varied parameters will be printed in F18.10 format when this element is used. This option is useful primarily for investigating the mathematical characteristics of a solution. It should be inserted near the beginning of the deck before the `BEAM` element.

STORE – Storage of Matrix Elements

Any matrix element or quantity that can be constrained can also be stored for use in a subsequent constraint. The stored quantity is given its own proper name on the `STORE` command. The quantity is referred to by this proper name at any later point. For example, the initial value of the synchrotron function `BETAX` may be stored under the name `BX0`.

```
STORE,  BX0 = BETAX  ;
```

If a constraint is to be imposed at the end of the beam line, setting the final beta value equal to the initial beta value, the user could impose the constraint.

```
FIT,  NAME = BETAX - BX0,  VALUE = 0.0,  TOLER = 0.001  ;
```

The difference `BETAX - BX0` would then be fit to zero. The quantity `BETAX` would be the horizontal value of beta at the point of the `FIT` element. The quantity `BX0` would be the initial horizontal value of beta.

If the beam line is assembled from elements and specified in a `LINE` statement, the `STORE` instruction may be placed after the beam line specification. It may refer to a marker which is included in the beam line specification. This is done with the `LOCA(TION)` keyword. Assume the marker

```
MAR1:   MARKER  ;
```

is included in the beam line description. Assume further that the user wishes to fit an element of the transfer matrix at the location of this marker. The command

```
STORE,  BXN = BETAX,  LOCA = MAR1  ;
```

will store the horizontal beta value under the name `BXN` at the location `MAR1`.

PLOT — Data for Plotting

Data for plotting may be output in a separate file on logical unit 8. The job running TRANSPORT should contain the appropriate computer instructions to identify logical unit 8 with the file the user wishes to contain the output. These data may then be used to create a plot with a separate plotting program. Examples in this section were made with the plotting package TOPDRAWER. The procedure for using TOPDRAWER is described later in this section.

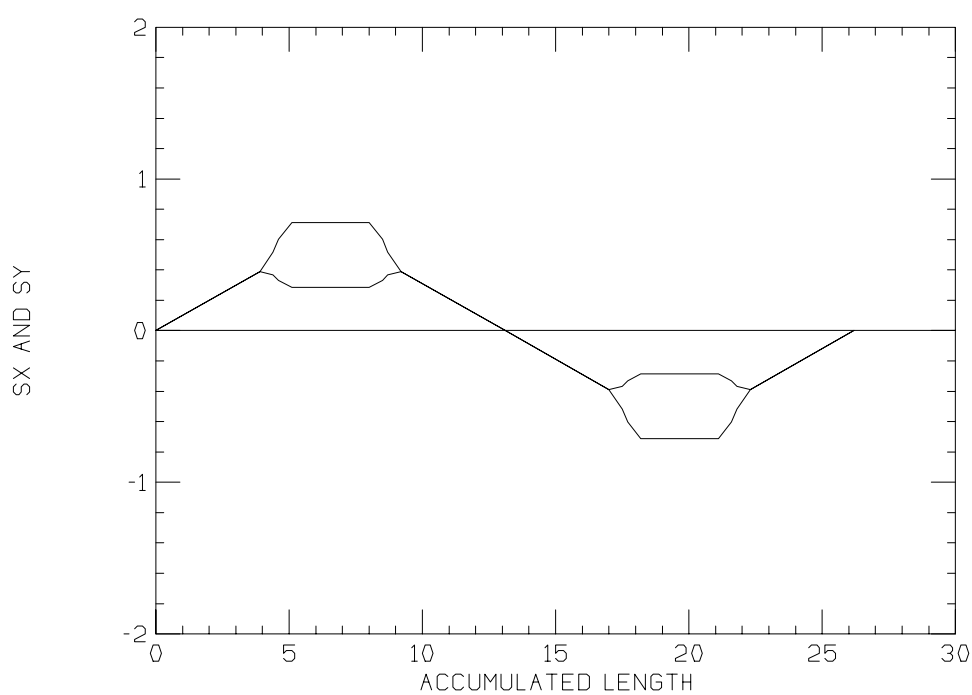
Plots vs Accumulated Length

There are currently four types of plot that can be made with TRANSPORT. The first is a plot of something vs accumulated length along the reference trajectory. For example, a file to be used to plot the matrix element R_{12} against accumulated length would be created by placing the element:

```
PLOT,  L,  R12  ;
```

before the beam card.

A plot of the transfer matrix elements R_{12} and R_{34} for a beam line is shown below. The transfer matrix elements, R_{12} and R_{34}, are also respectively known as the horizontal and vertical sinelike trajectories, s_x and s_y.



TRANSPORT will produce output to the separate file for each run where a `PLOT` instruction is included. A heading will first be printed with the names of the quantities to be plotted. If one of the items to be plotted is the accumulated length L, then the values of all the indicated items will be printed on unit 8 after each physical element. Up to eight items may be included on the list. The printing will be done in format 8F10.5. The items that may be included on the list are as follows:

Symbol	Item for Plotting
`L`	The beam line accumulated length.
`Rij`	An element of the first-order transfer matrix $R1$.
`RAij`	An element of the auxilliary first-order transfer matrix $R2$.
`XC`	The beam centroid position in x.
`XPC`	The beam centroid position in x'.
`YC`	The beam centroid position in y.
`YPC`	The beam centroid position in y'.
`DLC`	The beam centroid position in ℓ.
`DELC`	The beam centroid position in δ.
`XBEAM`	The beam half width in x.
`XPBEAM`	The beam half width in x'.
`YBEAM`	The beam half width in y.
`YPBEAM`	The beam half width in y'.
`DLBEAM`	The beam half width in ℓ.
`DELBEAM`	The beam half width in δ.
`Sij`	An element of the beam matrix σ. If a diagonal element is specified, then the square root of that element will be printed. The square root of a diagonal element is the beam half width in a particular coordinate. Any beam half width can also be represented by one of the mnemonics immediately preceding Sij.
`Cij`	An element of the beam correlation matrix r.
`BETAX`	The accelerator function β_x.
`ALPHAX`	The accelerator function α_x.
`BETAY`	The accelerator function β_y.
`ALPHAY`	The accelerator function α_y.
`ETAX`	The accelerator function η_x.
`DETAX`	The accelerator function η'_x.
`ETAY`	The accelerator function η_y.
`DETAY`	The accelerator function η'_y.
`Tijk`	An element of the second-order transfer matrix $T1$.
`TAijk`	An element if the auxilliary second-order transfer matrix $T2$.
`Uijk`ℓ	An element of the third-order transfer matrix $U1$.
`UAijk`ℓ	An element of the auxilliary third-order transfer matrix $U2$.

The plotting package TOPDRAWER can be used to make a plot from the data in the separate file. Details are given later in this section.

It is possible also to plot algebraic combinations of plottable items. Here, as with fitting, an algebraic expression must be on the right side of an equation. The plottable items are then given keywords `NAME1` ... `NAME8`. A command to plot the product $R_{11} * R_{12}$ vs accumulated length might then read:

```
PLOT,  NAME1 = L,  NAME2 = R11*R12  ;
```

Floor Coordinate Layouts

The second type of plot that one can make is a beam layout. The beam layout is a plot of the floor coordinates of the reference trajectory. The coordinates that may be plotted are either x vs z or y vs z. Unless otherwise specified the reference trajectory initially is along the z axis. Alternate specifications can be made with the `SPECIAL` element described on page 73. The perpendicular horizontal direction is x and the vertical direction is y.

The `PLOT` command for a floor layout should be placed before the `BEAM` element. For a view of the beam line as seen from above, use

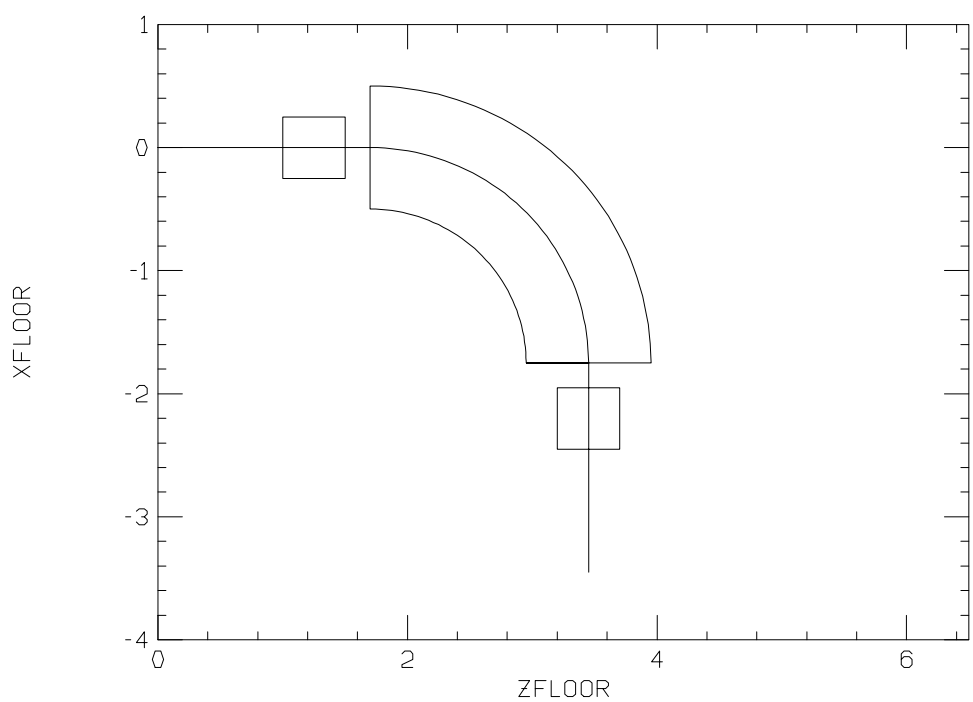
```
PLOT,  ZFLOOR,  XFLOOR  ;
```

For a view as seen from the side, use

```
PLOT,  ZFLOOR,  YFLOOR  ;
```

Magnets will be drawn also if their dimensions are specified via the `MAGNET` element. The use of the `MAGNET` element is described on page 95.

A simple beam line as seen from the top is shown below. The plotting program TOPDRAWER has been used to do the actual plotting.



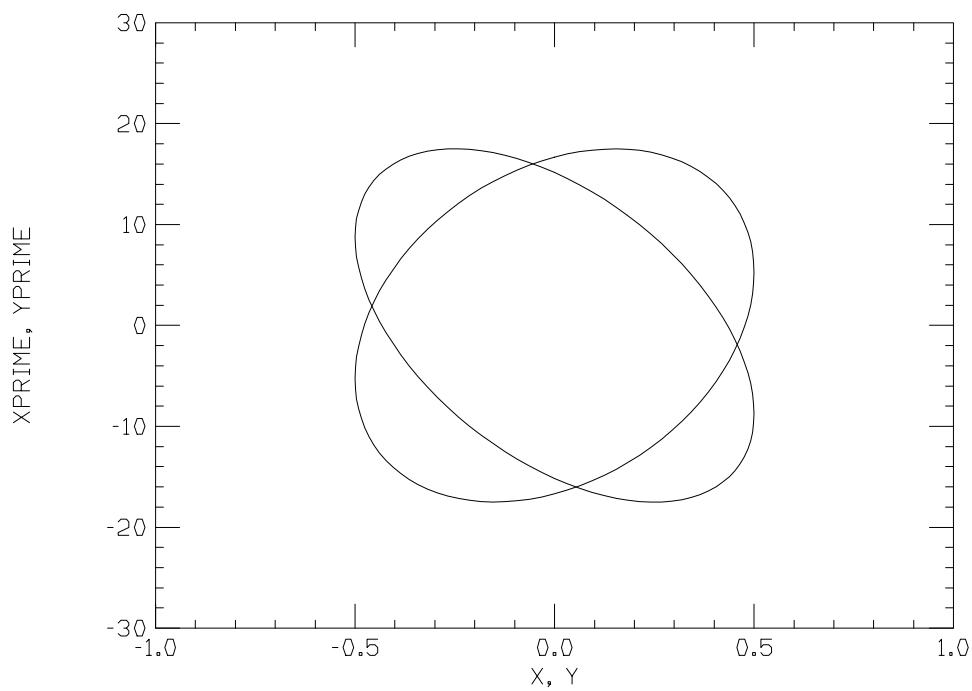
Plots of the Beam Ellipse

The third type of plot is of the beam ellipse. The beam ellipse in x vs x' at a given point in the beam line may be plotted by including the command:

```
PLOT,  XBEAM,  XPBEAM  ;
```

at that particular point. The output will consist of the x and x' coordinates of 100 points about the circumference of the ellipse. The output will be presented in two columns with headers. Each of the two columns will be 101 items long. The last numbers are a repetition of the first in order to close the ellipse. The output format will be 2F10.5.

A combined plot of the horizontal and vertical phase ellipses might appear as follows:



Anything vs Anything Else

The STEP command allows any one of the physical parameters used to described the beam line to be stepped over a specified interval. Each step may or may not include a fitting process. Using the stepping process, any physical parameter or matrix element may be plotted against any other physical parameter or matrix element.

For example, a parameter DS may be given a value by

```
DS: = 0.5  ;
```

The parameter may be used to give the length of a drift space.

```
DR1:   DRIFT,  L = 12.0 - DS  ;
```

A fitting procedure may adjust a quadrupole pole tip field B1 to obtain a focus.

To examine a range of values for the parameter DS, one inserts the command

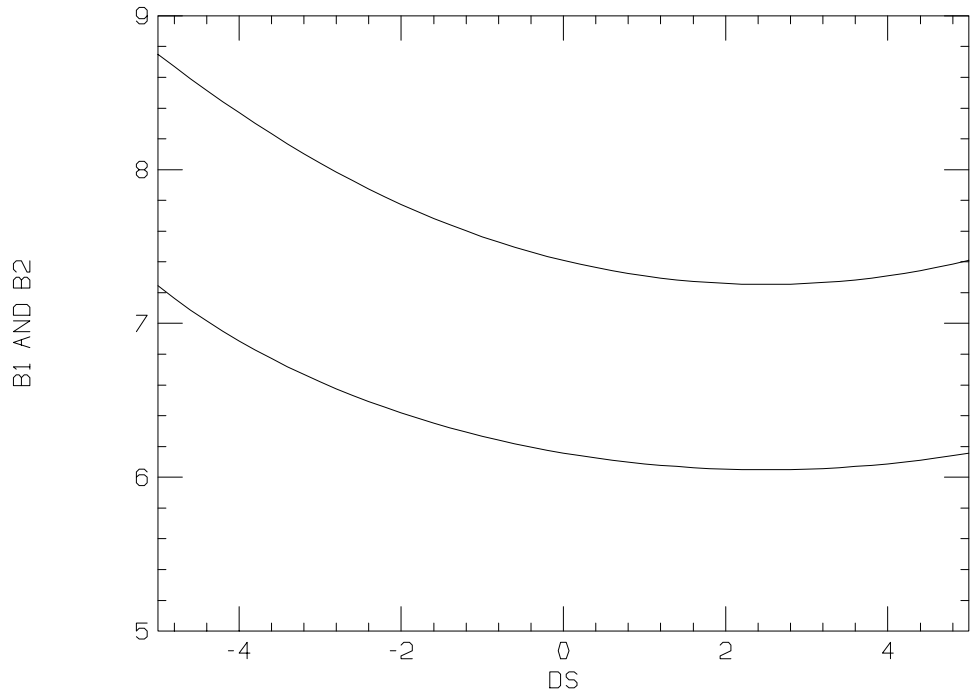
```
STEP,  DS,  LOWER = -5.0,  UPPER = 5.0,  STEP = 0.2  ;
```

The STEP command overrides the value given earlier to the parameter DS. The STEP command causes the parameter DS to step from the lower value (-5.0) to the upper value (5.0) in step sizes of 0.2. Any specified fitting procedure is then carried out for each of the values assumed by the parameter DS. Then by using the command

```
PLOT,  DS,  B1  ;
```

a plot of the pole tip field B1 against the parameter DS can be made.

Here we show a plot of `B1` and `B2` vs the parameter `DS`. The quantity `B2` is the other independent pole-tip field in a symmetric quadrupole triplet. Plots of both quantities together may be made with sequential runs described below.



If one of the items in the plot is a matrix element, any of the elements on the list on page 238 can be used. The `PLOT` element is inserted in the data at the point where the matrix element is to be evaluated. If a plot, is to be made of one matrix element vs another, then the `STORE` command can be used to retain the value of the first until the location for the second is encountered. More information about the stepping process can be found on page 279.

Sequential Runs

Different plot data may be output in sequential steps of a run so as to be included in the same data set. The first step (with indicator card 0) may contain the specification:

```
PLOT:  PLOT,  L,  R12  ;
```

In a subsequent step (with indicator 1) the items on the `PLOT` card may be altered to plot another quantity. An example might be:

```
PLOT:  PLOT,  L,  R34  ;
```

Another header would be printed followed by another two columns of numbers. The lengths of the columns would be the same as in the first step. The first columns, being L in both cases, would be identical. The second column would be different, since R_{34} is being printed instead of R_{12}.

Use with TOPDRAWER

The output feature described has been used with the plotting package TOPDRAWER. A skeleton data set for TOPDRAWER might look like:

```
SET LIMITS X 0.0 1100.0
SET LIMITS Y -10.0 10.0
SET SCALE X LINEAR
SET SCALE Y LINEAR
TITLE TOP SIZE 2 'R12 AND R34 VS L'
SET ORDER X Y
DATA
JOIN 1
DATA
JOIN 1
STOP
```

The word `DATA` is not a TOPDRAWER command, but rather indicates where the data generated by TRANSPORT are to be inserted. The deck shown would be appropriate for plotting R_{12} and R_{34} vs accumulated length L, as described above. The replacement of the word `DATA` by the TRANSPORT generated data need not be done by hand, but can be performed automatically. An auxiliary program exists which combines the TRANSPORT generated data with the TOPDRAWER skeleton to produce a complete topdrawer data set. This data set may then be run through TOPDRAWER to generate plots. Once the TRANSPORT data deck and the TOPDRAWER skeleton data set are set up, the whole procedure

may be automated on a computer file. A single command can then run TRANSPORT and TOPDRAWER to produce plots.

Variation of Parameters for Fitting

Some (not all) of the physical parameters of the elements comprising a beam line may be varied in order to fit selected matrix elements or algebraic combinations of matrix elements. In a first-order calculation one might fit elements of the $R1$ or $R2$ transformation matrices or the beam (σ) matrix. In second order one might constrain an element of the second-order matrix $T1$ or minimize the net contribution of aberrations to a given beam coordinate. Similarly, in third order one might constrain an element of the third-order matrix $U1$. Special constraints are also available.

One may mix orders in fitting. Any lower order vary code or constraint may be imposed in a higher-order run as long as it is meaningful. Exceptions will be described below.

In MAD notation the physical parameters to be varied are selected by the `VARY` command. The fitting constraints on matrix elements are selected via `FIT` elements placed in the system where the constraint is to be imposed.

The use of the permissive 'may' rather than the imperative 'will' in discussing variables is meaningful. The program will choose the parameters it <u>will</u> vary from among those that it <u>may</u> vary. In general it chooses to vary those parameters that have the greatest influence upon the conditions to be fit.

The `VARY` Command

The explicit `VARY` command is suited to keyword notation of MAD. Strictly speaking the `VARY` command is in MAD notation only when specifying the variation of parameters which are themselves expressed in MAD notation. Since TRANSPORT has many alternative parameterizations of elements which are characterized as MAD-like, corresponding occurrances of the `VARY` command must also be characterized as MAD-like. The `VARY` command is not an element, but rather an instruction as to how to modify an element already specified. The `VARY` command occurs in the deck when any parameter is to be declared to be variable. <u>The `VARY` command need never occur in the data preceded by a minus sign</u>.

If a quadrupole is specified as:

```
Q1:   QUAD,  L = 10.,  B = 5.,  APER = 25.4  ;
```

then the magnetic field of that quadrupole can be made to be variable by the statement

```
VARY,  NAME = Q1[B]  ;
```

The keyword `NAME` is followed by a designation of the parameter to be varied. An alternate formulation of the same command is:

```
VARY,  Q1[B]  ;
```

In the case of an element parameter the parameter is designated by the element label followed by the parameter keyword in brackets.

A parameter can be varied only if it is explicitly included in the data. In other words, the above command to vary the `B` parameter of quad `Q1` has no meaning if quad `Q1` is described in terms of its length and gradient, e.g.

```
Q1:   QUAD,  L = 10.,  GRAD = 2., ;
```

Parameters declared on `PARAMETER` elements may also be varied. In this case the item following the keyword `NAME` is the parameter name. If the value of the magnetic field were declared in a separate parameter statement, the quadrupole specification might look like.

```
PARAM,  B1 = 5.0  ;
Q1:   QUAD,  L = 10.0,  B = B1,  APER = 2.54  ;
```

The parameter `B1` could be varied by means of the statement

```
VARY,  NAME = B1  ;
```

or simply

```
VARY,  B1  ;
```

The use of the `VARY` statement allows physical separation of the data describing the mathematical operations to be performed from the specification of the beam line. The `VARY` statements can all be placed together, near the other operations, after the beam line specification is complete. This grouping of `VARY` statements is possible both in the case when the beam line is defined explicitly with `LINE` and `USE` statements and in the original TRANSPORT situation when the beam line is simply the sequence of elements listed.

The `FIX` Command

A variable parameter may be fixed again by the command

```
FIX,  NAME = name  ;
```

Whatever parameters are named are then held fixed and are not available for achieving the desired constraints. An alternative formulation of the same command is

```
FIX, name ;
```

If the previously varied field of quadrupole `Q1` were to be fixed, the user could insert the command:

```
FIX, Q1[B] ;
```

The `FIX` command is not an element, but rather an instruction as to how to modify an element already specified. The `FIX` command occurs in the deck when any formerly varied parameter is now to be held fixed. The `FIX` command need never occur in the data preceded by a minus sign.

If all previously varied parameters of any type are to be fixed, the single command

```
FIX, ALL ;
```

may be included in the data. The user now need not go back through the data determining which parameters might have been varied, in order to instruct the program no longer to vary them. A single command will now fix all previously varied parameters.

Parameters Variable Only in First Order

In a first-order run the following parameters may be varied.

`BEAM`	`X`	One-half the horizontal beam extent.
	`XP`	One-half the horizontal beam divergence.
	`Y`	One-half the vertical beam extent.
	`YP`	One-half the vertical beam divergence.
	`L`	One-half the longitudinal beam extent.
	`DEL`	One-half the fractional momentum spread.
`CORRELATION`	`CIJ`	Any element of the beam correlation matrix may be varied. The indices I and J must be replaced by their numerical values, so that the varied quantity will be, for example, `C43`.
`ALIGN`	`X`	The magnet displacement in x.
	`RX`	A rotation about the x axis.
	`Y`	The magnet displacement in y.
	`RY`	A rotation about the y axis.
	`Z`	The magnet displacement in z.
	`RZ`	A rotation about the z axis.

Parameters Variable in Any Order

SPECIAL	XBEGIN	The initial floor x coordinate
	YBEGIN	The initial floor y coordinate
	ZBEGIN	The initial floor z coordinate
	YAW	The initial floor horizontal angle θ
	PITCH	The initial floor vertical angle ϕ
	RMPS	The fractional excess bend field on a bending magnet.
	RNMS	The overall scaling factor for the midplane-symmetry-violating field component of a bending magnet
	VR	The fractional vertical bend field of a bending magnet.
	NP	The midplane-symmetry-violating normalized gradient of a bending magnet.
PARAM		Any defined parameter may be varied.
CENTROID	X	The shift in the x coordinate
	XP	The shift in x'.
	Y	The shift in y.
	YP	The shift in y'.
	L	The shift in ℓ.
	DEL	The shift in δ.
ETA	ETAX	The x component of η.
	DETAX	The x' component of η.
	ETAY	The y component of η.
	DETAY	The y' component of η.
	ETAL	The longitudinal displacement of η.
	ETAP	The fractional momentum deviation for η.
DRIFT	L	The drift length.
ROTAT	ANGLE	Angle of pole-face rotation.
BEND	L	The magnet length.
	ANGLE	The angle of bend.
	K1	The quadrupole coefficient.
	RMPS	The fractional excess magnetic field.
	RNMS	Scaling factor for midplane-symmetry-violating field components.
	VR	The midplane-symmetry-violating dipole field.
	K1P	The midplane-symmetry-violating quadrupole coefficient.
	B	The magnetic field.
	RADIUS	The radius of curvature of the reference trajectory.
	N	The normalized field derivative.
	NP	The midplane-symmetry-violating normalized field derivative.

	TILT	The roll of the coordinate system about the bend entrance and exit axes.
RBEND, SBEND	L	The magnet length.
	ANGLE	The angle of bend.
	K1	The quadrupole coefficient.
	E1	The entrance pole-face rotation angle.
	E2	The exit pole-face rotation angle.
	RMPS	The fractional excess magnetic field.
	RNMS	Scaling factor for midplane-symmetry-violating field components.
	VR	The midplane-symmetry-violating dipole field.
	K1P	The midplane-symmetry-violating quadrupole coefficient.
	B	The magnetic field.
	RADIUS	The radius of curvature of the reference trajectory.
	N	The normalized field derivative.
	NP	The midplane-symmetry-violating normalized field derivative.
	TILT	The roll of the coordinate system about the bend entrance and exit axes.
HKICK, VKICK	L	The magnet length.
	B	The magnetic field.
	ANGLE	The deflection angle.
	TILT	The roll of the coordinate system about the bend entrance and exit axes.
QUAD	L	The magnet length.
	B	The magnetic field.
	APER(TURE)	The magnetic aperture.
	GRADIENT	The gradient of the magnetic field.
	TILT	The roll of the coordinate system about the quadrupole axis.
PLASMA	L	The magnet length.
	K1	The normalized quadrupole component.
	B	The magnetic field.
	GRAD	The gradient.
SOLENOID	L	The magnet length.
	KS	The rotation angle.
	B	The magnetic field.
SROT	ANGLE	The angle of rotation.

In a second- or higher-order run variation of a drift length or any first-order element should be done with caution as it may affect the first-order properties of the beam line. But inversely coupled drift spaces straddling a sextupole will, for example, show only second-

and higher-order effects.

Parameters Variable Only in Second or Higher Order

In a second-order run the following parameters may be varied:

SPECIAL	EPS	The normalized quadratic term (sextupole component) in the midplane expansion for the field of a bending magnet.
	H1	The pole face curvature of a bending magnet entrance
	H2	The pole face curvature of a bending magnet exit.
	EPSP	The midplane-symmetry-violating sextupole component of a bending magnet.
ROTAT	H	Pole face curvature.
BEND	K2	The sextupole coefficient.
	K2P	The midplane-symmetry-violating sextupole component.
	EPS	The quadratic field dependence.
	EPSP	The midplane-symmetry-violating quadratic field dependence.
RBEND, SBEND	K2	The sextupole coefficient.
	H1	The entrance pole-face curvature.
	H2	The exit pole-face curvature.
	K2P	The midplane-symmetry-violating sextupole component.
	EPS	The quadratic field dependence.
	EPSP	The midplane-symmetry-violating quadratic field dependence.
SEXTUPOLE	B	The magnetic field.
	K2	The normalized sextupole component.
	TILT	The roll of the coordinate system about the sextupole axis.

Parameters Variable Only in Third Order

SPECIAL	EPS3	Cubic term for the field of a bending magnet.
BEND	K3	The octupole coefficient.
	EPS3	The cubic field dependence.
RBEND, SBEND	K3	The octupole coefficient.
	EPS3	The cubic field dependence.

OCTUPOLE	K3	The normalized octupole component.
	B	The magnetic field.
	TILT	The roll of the coordinate system about the octupole axis.

The special parameter cards (SPECIAL) once introduced apply to all subsequent magnets in a beam line until another SPECIAL element specifying the same parameter is encountered. Thus if such a parameter is varied, the variation will apply simultaneously to all subsequent magnets to which it pertains. The variation will persist until the parameter is changed by the introduction of another SPECIAL element specifying the same parameter.

The total number of independent variables in a first-order run is limited to 20 by reasons of the mathematical method of fitting and to 10 for a second-order run. Parameters and element parameters may be defined in terms of other parameters and element parameters. Only independent parameters and element parameters can be declared as variable. Any dependent parameter which depends on a varied parameter, receives its variation secondarily from the varied parameter. The limit of 20 counts only against independently varied parameters. Variables within repeat elements (REPEAT) also count only one.

Available Constraints – The FIT Command

Types of Constraints

A variety of possible constraints is available. Fitting may be done in either first , second, or third order. The order of the run must be at least as great as the order of the constraint. A list of constraints available is given below. They are explained more fully on later pages.

Layout Constraints

1. The total system length.
2. The reference trajectory floor coordinates.
3. The elevation (including the effect of the earth's curvature).

To fit the reference trajectory floor coordinates, the floor coordinate calculation must be activated via a (`PRINT, FLOOR ;`) command. (See page 221.)

First-Order Constraints

1. An element of the first-order transfer matrix $R1$.
2. An element of the auxiliary first-order matrix $R2$.
3. An AGS machine constraint.
4. The first moments of the beam.
5. A σ (BEAM) matrix element.
6. The correlations r in the beam coordinates.
7. The accelerator beam parameters β and α.
8. The accelerator η function.
9. The accelerator beam phase advance ψ.

Second-Order Constraints

1. An element of the second-order transfer matrix $T1$.
2. An element of the second-order auxiliary transfer matrix $T2$.
3. The net contributions of aberrations to a given coordinate of the beam matrix σ.

The second-order matrices are actually computed using the auxiliary matrix T2. Therefore, when activating second-order fitting, one must not include any element which causes an update of the R2 matrix. For a complete list of such elements see the `UPDATE` element description on page 212.

Third-Order Constraints

1. An element of the third-order transfer matrix $U1$.
2. An element of the third-order auxiliary transfer matrix $U2$.
3. The net contribution of aberrations to a given coordinate of the beam matrix σ.

The present value of the constrained quantity, as well as the desired value, is printed in the output. In the case of transfer matrix elements this value may be checked by printing the transfer matrix itself. Certain other constrained quantities may be checked similarly. Exceptions are noted in the explanations following.

Notation

The keywords which can be used in specifying a fitting constraint are

Keyword	Description
`VALUE`	Desired value of matrix element.
`TOLERANC`	Desired accuracy of fit (standard deviation).
`NAME`	Name of matrix element to be constrained.
`LOWER`	If the constraint is only a lower limit.
`UPPER`	If the constraint is only an upper limit.

A constraint of the R_{12} transfer matrix element may be written as

```
FIT,  NAME = R12,  VALUE = 0.0,  TOLER = 0.001  ;
```

This notation is somewhat inconvenient because it requires the user repeatedly to type in the names of all three keywords. For fitting simple matrix elements another form may be more desirable. The same constraint can be represented as:

```
FIT,  R12 = 0.0,  TOLER = 0.001  ;
```

The matrix element to be fit is now used as a keyword. A complete list of matrix elements which may be identified by keyword follows. The lower-case letters i, j, k, and ℓ stand for integers which specify the particular matrix element. The TRANSPORT output for the fit element is printed using the same symbols.

Symbol	Matrix Element
`L`	The beam line accumulated length.
`XFLOOR`	The floor coordinate x value.
`YFLOOR`	The floor coordinate y value.
`ZFLOOR`	The floor coordinate z value.
`YAW`	The floor coordinate plan view reference trajectory angle.
`PITCH`	The angle the reference trajectory makes with the horizontal.
`ROLL`	The net rotation angle about the reference trajectory.
`ELEVATION`	The elevation, including corrections for the earth's curvature.
`Rij`	An element of the first-order transfer matrix $R1$.
`RAij`	An element of the auxilliary first-order transfer matrix $R2$.
`XC`	The beam centroid position in x.
`XPC`	The beam centroid position in x'.
`YC`	The beam centroid position in y.

YPC	The beam centroid position in y'.
DLC	The beam centroid position in ℓ.
DELC	The beam centroid position in δ.
XBEAM	The beam half width in x.
XPBEAM	The beam half width in x'.
YBEAM	The beam half width in y.
YPBEAM	The beam half width in y'.
DLBEAM	The beam half width in ℓ.
DELBEAM	The beam half width in δ.
Sij	An element of the beam matrix σ.
Cij	An element of the beam correlation matrix r.
BETAX	The accelerator function β_x.
ALPHAX	The accelerator function α_x.
BETAY	The accelerator function β_y.
ALPHAY	The accelerator function α_y.
ETAX	The accelerator function η_x.
DETAX	The accelerator function η'_x.
ETAY	The accelerator function η_y.
DETAY	The accelerator function η'_y.
PSIX	The accelerator phase shift in the horizontal plane
PSIY	The accelerator phase shift in the vertical plane
Tijk	An element of the second-order transfer matrix $T1$.
TAijk	An element if the auxilliary second-order transfer matrix $T2$.
Uijkℓ	An element of the third-order transfer matrix $U1$.
UAijkℓ	An element of the auxilliary third-order transfer matrix $U2$.

Constraints on Algebraic Expressions

In addition any algebraic combination of matrix elements may also be constrained. The algebraic combination is written, as in a FORTRAN statement, after the keyword NAME. For example, a fit of the tilt of the focal plane angle, might be

```
FIT,  NAME = R16/(R11*T126),  VALUE = 0.3,  TOLER = 0.001  ;
```

A number of FORTRAN supplied intrinsic functions may also be used in an algebraic expression. The intrinsic functions now available are

SQRT	Square root
ALOG	Natural logarithm (written Ln in engineering notation).
EXP	Exponential function
SIN	Sine function
COS	Cosine function
SINH	Hyperbolic sine function

COSH	Hyperbolic cosine function
ASIN	Inverse sine function
ACOS	Inverse cosine function

Additional functions can easily be added if there is a need for any of them.

Upper and Lower Limits

If the keyword LOWER is present, the constraint is taken to be a lower limit. If the keyword UPPER is present, the constraint is taken to be an upper limit. Thus if the constraint:

```
FIT,  R12 = 0.0,  TOLER = 0.001,  UPPER  ;
```

were to appear in the data, the matrix element R12 would be constrained to be no greater than 0.0. If the value of R12 were anything less than 0.0, the constraint would be considered to be satisfied.

Placement

If the beam line is assembled from elements and specified in a LINE statement, the FIT instruction may be placed after the beam line specification. It may refer to a marker which is included in the beam line specification. This is done with the LOCA(TION) keyword. Assume the marker

```
MAR1:   MARKER  ;
```

is included in the beam line description. Assume further that the user wishes to fit an element of the transfer matrix at the location of this marker. The command

```
FIT,  R12 = 0.0,  TOLER = 0.001,  LOCA = MAR1  ;
```

will constrain the matrix R_{12} to equal 0.0 at the location given by the marker MAR1.

If all constraints from a previous step in a multi-step problem are to be cancelled, the single command:

```
FIT,  NONE  ;
```

can be used. This command can then be followed by any new constraints to be activated. These new constraints must have been specified in the earlier steps, but might simply have been inactive due to a minus sign (−) in front of the mnemonic FIT.

Individual Examples

In principle, we have now give a complete description of how to impose constraints in TRANSPORT. However, it has proved very useful to give specific examples of how to impose certain focal conditions. For this reason we describe separately the focal conditions listed above, and show how different optical conditions can be imposed.

System Length Constraint

A running total of the lengths of the various elements encountered is kept by the program and may be fit. The code digits are i = 0., j = 0.

Thus the element

```
FIT,  L = 150.,  TOLER = 5.   ;
```

would make the length of the system prior to this element equal to 150 ± 5 meters. Presumably there would be a variable drift length somewhere in the system. By redefining the cumulative length via the (`SPEC,  LENGTH = (value)  ;`) element, partial system lengths may be accumulated and fit.

Floor Coordinate Fitting Constraint

Six keywords may be used to specify a floor coordinate constraint:

Symbol	Matrix Element
`XFLOOR`	The floor coordinate x value.
`YFLOOR`	The floor coordinate y value.
`ZFLOOR`	The floor coordinate z value.
`YAW`	The floor coordinate plan view reference trajectory angle.
`PITCH`	The angle the reference trajectory makes with the horizontal.
`ROLL`	The net rotation angle about the reference trajectory.

The floor projection of the reference trajectory makes the angle *yaw* with the floor z axis. The angle between the reference trajectory and the horizontal plane is *pitch*. The rotation about the reference trajectory is *roll*. This is also the order in which coordinates are printed in the floor layout activated by the (`PRINT, FLOOR ;`) element. Initial coordinates are given on the `SPEC` elements with the keywords `XBEGIN`, `YBEGIN`, `ZBEGIN`, `YAW`, and `PITCH`, and the `SROT` element.

The floor coordinates are actually zeroeth rather than first order properties of a beam line. However, in TRANSPORT, they may be constrained in a fitting run of any order, and therefore are included here. However, simultaneous fitting of floor coordinates and focal conditions should be done with some caution. TRANSPORT has no way of determining which parameters are intended to be varied to satisfy which constraints. It might vary a parameter, such as the length of a bending magnet, to satisfy a focal condition, thereby producing a very strange result.

To fit the reference trajectory floor coordinates, the floor coordinate calculation <u>must</u> be activated via a (`PRINT, FLOOR ;`) command. (See page 221.)

Elevation Fitting Constraint

The elevation of the reference trajectory above the curved surface of the earth may be constrained. The elevation is initially the same as the vertical floor coordinate. It assumes that a beam with no pitch is initially tangent to the surface of the earth. The elevation then differs from the vertical floor coordinate by an increasing term quadratic in the distance away from the initial point. The radius of the earth is taken to be 6.3714×10^6 meters or 3959 miles. The keyword indicating elevation is `ELEV(ATION)`.

Thus the element

```
FIT,  ELEV = 150.,  TOLER = 0.01  ;
```

would make the elevation of the system at a given point equal to 150 meters $\pm$ 1 centimeter. Presumably there would be a some variable quantity in the system which would affect the altitude. Examples might be the initial vertical floor angle, or the strength of a vertical bend.

R1 Matrix Fitting Constraints

The keyword indicating an element of the R1 matrix, consists of the letter "R" followed, with no spaces, by the two indices of the matrix element.

Thus a constraint to zero of the R_{12} matrix element would be indicated by:

```
FIT,  R12 = 0.0,  TOLER = 0.001  ;
```

Note that any fitting constraint on an R1 matrix element is from the preceding update of the R1 matrix. An R1 matrix is updated only by a (`UPDATE,  R1  ;`) entry.

Some typical R1 matrix constraints are as follows:

Desired Optical Condition	Typical Fitting Constraint
<u>Point to Point Imaging:</u>	
Horizontal plane $R_{12} = 0$	`F1: FIT, R12 = 0.0, TOLER = .0001 ;`
Vertical plane $R_{34} = 0$	`F2: FIT, R34 = 0.0 TOLER = .0001 ;`
<u>Parallel to point focus:</u>	
Horizontal plane $R_{11} = 0$	`F3: FIT, R11 = 0., TOLER = .0001 ;`
Vertical plane $R_{33} = 0$	`F4: FIT, R33 = 0., TOLER = .0001 ;`
<u>Point to parallel transformation:</u>	
Horizontal plane $R_{22} = 0$	`F5: FIT, R22 = 0., TOLER = .0001 ;`
Vertical plane $R_{44} = 0$	`F6: FIT, R44 = 0., TOLER = .0001 ;`
<u>Achromatic beam:</u>	
Horizontal plane	`F7: FIT, R16 = 0., TOLER = .0001 ;`
$R_{16} = R_{26} = 0$	`F8: FIT, R26 = 0., TOLER = .0001 ;`
<u>Zero dispersion beam:</u>	
Horizontal plane $R_{16} = 0$	`F9: FIT, R16 = 0., TOLER = .0001 ;`
<u>Simultaneous point to point and waist to waist imaging:</u>	
Horizontal plane	`F10: FIT, R12 = 0., TOLER = .0001 ;`
$R_{12} = R_{21} = 0$	`F11: FIT, R21 = 0., TOLER = .0001 ;`
Vertical plane	`F12: FIT, R34 = 0., TOLER = .0001 ;`
$R_{34} = R_{43} = 0$	`F13: FIT R43 = 0., TOLER = .0001 ;`
<u>Simultaneous parallel to point and waist to waist transformation:</u>	
Horizontal plane	`F14: FIT, R11 = 0., TOLER = .0001 ;`
$R_{11} = R_{22} = 0$	`F15: FIT, R22 = 0., TOLER = .0001 ;`
Vertical plane	`F16: FIT, R33 = 0., TOLER = .0001 ;`
$R_{33} = R_{44} = 0$	`F17: FIT, R44 = 0., TOLER = .0001 ;`

R2 Matrix Fitting Constraints

The keyword indicating an element of the R1 matrix, consists of the letters "RA" followed, with no spaces, by the two indices of the matrix element.

Thus a constraint to zero of the $R2_{12}$ matrix element would be indicated by:

```
FIT,  RA12 = 0.0,  TOLER = 0.001  ;
```

Some typical R2 matrix constraints are as follows:

Desired Optical Condition		Typical Fitting Constraint
<u>Point to point imaging:</u>		
Horizontal plane	$R_{12} = 0$	`F1:  FIT,  RA12 = 0.,  TOLER = .001  ;`
Vertical plane	$R_{34} = 0$	`F2:  FIT,  RA34 = 0.,  TOLER = .001  ;`
<u>Parallel to point focus:</u>		
Horizontal plane	$R_{11} = 0$	`F1:  FIT,  RA11 = 0.  TOLER = .001  ;`
Vertical plane	$R_{33} = 0$	`F2:  FIT,  RA33 = 0.,  TOLER = .001  ;`
<u>Achromatic beam:</u>		
Horizontal plane		`F3:  FIT,  RA16 = 0.,  TOLER = .001  ;`
$R_{16} = R_{26} = 0$		`F4:  FIT,  RA26 = 0.,  TOLER = .001  ;`

Note that any fitting constraint on an R1 matrix element is from the preceding update of the R1 matrix. See the description of the `UPDATE` element on page 212 for a complete list of elements which update the R2 matrix.

AGS Machine Constraint

Provision has been made in the program for fitting the betatron phase shift angle μ, associated with the usual AGS treatment of magnet systems. †

Here the phase shift angle μ is in units of the number of betatron oscillations. The unit for phase shift angle in the unit sets `UMETER`, `UMM`, and `UMICR` are degrees. Only in the unit set `UMAD` is the unit for phase shift angle in the number of betatron oscillations. However, in any unit set, the unit for phase shift angle can be converted to number of betatron oscillations by use of the command.

```
UNIT,  PHASE,  TUNE  ;
```

In the example below, we assume that `TUNE` is the unit used.

In the horizontal plane: use the keyword `MUX`, and specify:

$$\begin{aligned} \Delta = \frac{1}{2\pi}\ \cos^{-1}\left[0.5\ (R_{11} + R_{22})\right] &= \frac{\mu}{2\pi}\ \text{(horiz)} \\ &= \text{freq./(No. of periods)}\ . \end{aligned}$$

In the vertical plane the keyword is `MUY`, and

$$\Delta = \frac{1}{2\pi}\ \cos^{-1}\left[0.5\ (R_{33} + R_{44})\right] = \frac{\mu}{2\pi}\ \text{(vert)} \quad .$$

The factor of 2π is included in the equation for Δ since, mathematically, inverse trigonometric functions are usually considered to be expressed in radians.

For example, if there are 16 identical sectors to a proposed AGS machine and the betatron frequencies per revolution are to be 3.04 and 2.14 for the horizontal and vertical planes respectively, then the last element of the sector should be followed by the constraints:

```
FIT,  MUX = .190,  TOLER = .001  ;
FIT,  MUY = .134,  TOLER = .001  ;
```

$$\text{i.e.} \quad \frac{3.04}{16} = 0.190 \quad \text{and} \quad \frac{2.14}{16} = 0.134 \quad .$$

†See Courant and Snyder [1]. Also note that this constraint is valid only when the unit cell structure <u>and</u> the corresponding beta functions are both periodic.

For example: A typical data listing might be:

```
Q1:  QUAD - - - ;
DRIFT - - - ;
Q2:  QUAD - - - ;
DRIFT - - - ;
FIT, MUX = 0.190, TOLER = .001 ;
FIT, MUY = 0.134, TOLER = .001 ;
VARY, Q1[B] ;
VARY, Q2[B] ;
```

First Moment Constraint

In first order, known misalignments, centroid shifts, and mistuned magnets cause the center (centroid) of the phase ellipsoid to be shifted from the reference trajectory, i.e., they cause the beam to have a nonzero first moment. The first moments appear in a vertical array to the left of the vertical array giving the $\sqrt{\sigma_{ii}}$. The units of the corresponding quantities are the same.

It is perhaps helpful to emphasize that the origin always lies on the reference trajectory. First moments refer to this origin. However, the ellipsoid is defined with respect to its center, so the covariance matrix, as printed, defines the second moment about the mean.

First moments may be fit. The keywords used to indicate the coordinates of the beam centroid are:

Symbol	Matrix Element
`XC`	The beam centroid position in x.
`XPC`	The beam centroid position in x'.
`YC`	The beam centroid position in y.
`YPC`	The beam centroid position in y'.
`DLC`	The beam centroid position in ℓ.
`DELC`	The beam centroid position in δ.

Thus

```
FIT,  XC = 1.0,  TOLER = .1  ;
```

constraints the horizontal (1.) displacement of the ellipsoid to be 1.0 ± 0.1 mm. Here it is assumed that the unit set selected is `UMM`.

This constraint is useful in deriving the alignment tolerances of a system or in warning the system designer to offset the element in order to accommodate a centroid shift.

σ (BEAM) Matrix Fitting Constraints

The keyword indicating an element of the beam (sigma) matrix, consists of the letter "S" followed, with no spaces, by the two indices of the matrix element.

Thus a constraint to zero of the S_{21} matrix element would be indicated by:

```
FIT,  S21 = 0.0,  TOLER = 0.001  ;
```

Some typical σ matrix constraints are as follows:

Desired Optical Condition		Typical Fitting Constraint
Horizontal waist σ_{21}	$= 0$	`F1:  FIT,  S21 = 0.,  TOLER = .001  ;`
Vertical Waist σ_{43}	$= 0$	`F2:  FIT,  S43 = 0.,  TOLER = .001  ;`
Fit beam size to x_{max}	$= 1$ cm	`F3:  FIT,  S11 = 1.,  TOLER = .001  ;`
Fit beam size to y_{max}	$= 2$ cm	`F4:  FIT,  S33 = 2.,  TOLER = .001  ;`
Limit max beam size to x	$= 2$ cm	`F5:  FIT,  S11 = 2.,  TOLER = .01,  UPPER  ;`
Limit min beam size to y	$= 1$ cm	`F6:  FIT,  S33 = 1.,  TOLER = .01,  LOWER  ;`

In general, it will be found that achieving a satisfactory 'beam' fit with TRANSPORT is more difficult than achieving an R matrix fit. When difficulties are encountered, it is suggested that the user 'help' the program by employing sequential (step by step) fitting procedures when setting up the data for his problem. More often than not a "failure to fit" is caused by the user requesting the program to find a physically unrealizable solution. An often encountered example is a violation of Liouville's theorem.

Beam Correlation Matrix (r) Fitting Constraints

The keyword indicating an element of the beam correlation matrix consists of the letter "C" followed, with no spaces, by the two indices of the matrix element. The letter "C" is used here as the letter "R" is already used for the first-order transfer matrix. In terms of elements of the σ matrix, the correlation r_{ij} is given by:

$$r_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii}}\sqrt{\sigma_{jj}}}$$

A constraint to zero of the r_{21} matrix element would be indicated by:

```
FIT,  C21 = 0.0,  TOLER = 0.001  ;
```

TRANSPORT does not print the beam (σ) matrix directly. Instead it prints the beam half widths and represents the off-diagonal elements by the correlation matrix. If one wishes to fit an element of this matrix to a nonzero value it is convenient to be able to constrain the matrix element directly.

Some typical r matrix constraints are as follows:

Desired Optical Condition		Typical Fitting Constraint
Horizontal waist	$r_{21} = 0$	`F1:  FIT,  C21 = 0.,  TOLER = .001  ;`
yy' correlation	$r_{34} = 0.2$	`F2:  FIT,  C34 = 0.2,  TOLER = .001  ;`

Accelerator Function β and α Fitting Constraints

If the two transverse planes are independent, the beam phase space in each plane can be described in terms of the accelerator functions β, α, and ϵ. The meaning of these parameters is given by the diagram on page 107. The horizontal and vertical β and α parameters may be fit. The emittances ϵ in the two planes may not be.

The keywords used to indicate the various accelerator functions are as follows:

Symbol	Keyword	Function
β_x	`BETAX`	The beta function in the horizontal plane.
β_y	`BETAY`	The beta function in the vertical plane.
α_x	`ALPHAX`	The alpha function in the horizontal plane.
α_y	`ALPHAY`	The alpha function in the vertical plane

A constraint on the horizontal beta function might be written as:

```
FIT,  BETAX = 1.0,  TOLER = 0.001  ;
```

Here the β_x function is constrained to the value of 1.0 meters, with a tolerance of one millimeter.

In most accelerator applications the unit of the beta function is taken to be meters. This would seem to imply that the correct unit set for any accelerator problem would be `UMETER`. However, a more precise statement would be that the units of the beta function are meters per radian. This latter unit is the same as millimeters per milliradian, or microns per microradian. Therefore the unit sets `UMM` or `UMICR` can be used equally as well.

A constraint on the vertical alpha function could be written as:

```
FIT,  ALPHAY = 0.0,  TOLER = 0.001  ;
```

Since the alpha functions are unitless, the values are the same for any unit set.

Accelerator Eta Function ($\eta_{x,y}$) Fitting Constraints

The accelerator eta ($\eta_{x,y}$) function is very close in concept to the dispersion trajectory. The difference is that in a system which closes on itself, such as a circular accelerator, eta is taken to be a periodic function. In a single pass system, it is often taken to have an initial nonzero value. This value is the eta function of some preceding accelerator or the result of a preceding beam transport system. The eta function is a six-component vector, where it is usual for the sixth component to have a value of unity.

The components of the eta function may be fit. The keywords used to indicate the coordinates of the eta function are:

Symbol	Matrix Element
`ETAX`	The accelerator function η_x.
`DETAX`	The accelerator function η'_x.
`ETAY`	The accelerator function η_y.
`DETAY`	The accelerator function η'_y.

A constraint on the horizontal eta function η_x would be written as:

```
FIT,  ETAX = 0.5,  TOLER = 0.01  ;
```

It is customary in accelerator calculations to use meters and radians for the transverse units. If this is done in the present case, then the desired value for the above `FIT` is 0.5 meters. The tolerance is 1 cm. The units taken for eta are whatever transverse units are being used. For a description on the input value of eta, see the description of eta on page 112.

Phase Advance ($\psi_{x,y}$) Fitting Constraints

The accelerator phase advance is defined as

$$\psi = \int \frac{ds}{\beta}$$

It differs from the betatron phase shift defined earlier in that it is a function on the input beam as well as of the intervening lattice. It can also be derived from the initial and present beta functions and the transfer matrix as

$$\psi_x = \sin^{-1} \frac{R_{12}}{\sqrt{\beta_o \beta_1}}$$

The phase advance in either transverse plane may be fit. The keywords used to indicate horizontal and vertical phase advances are:

Symbol	Matrix Element
`PSIX`	The accelerator phase shift in the horizontal plane
`PSIY`	The accelerator phase shift in the vertical plane

Thus a constraint on the horizontal phase advance ψ_x would be written as:

```
FIT,  PSIX = 90.0,  TOLER = 0.01  ;
```

Here the phase advance is set to 90 degrees.

If the user specifies that the beam matrix is to be printed out in accelerator notation, then the phase advance in both transverse planes will be printed in the output along with the beta, alpha, and eta functions.

The units of phase advance are degrees for all the unit sets except for `UMAD`. For `UMAD` the units of phase advance are radians over 2π. The initial values of the phase shift in both planes are always taken to be zero.

T1 Matrix Fitting Constraints

The keyword indicating an element of the T1 matrix, consists of the letter "T" followed, with no spaces, by the three indices of the matrix element.

Note that upper and lower limit constraints are not available for second order fitting.

Some typical T1 matrix constraints are as follows:

Desired Optical Condition	Typical Fitting Constraint
Geometric aberration $T_{122} = 0$	`F1: FIT, T122 = 0.0, TOLER = .001 ;`
Chromatic aberration $T_{346} = .5$	`F2: FIT, T346 = .5 TOLER = .001 ;`

There must be <u>no</u> updates of the R2 matrix when constraining an element of the T1 matrix. There is no limit on the number of constraints which may be imposed.

If only sextupoles or sextupole equivalents are varied the problem will be linear and the absolute size of the tolerances will be unimportant. Only their relative magnitude will be significant. Sometimes only a subset of the elements of the matrix T_{ijk} which give significant contributions to beam dimensions need be eliminated. In such cases one may wish to minimize the effect of this subset, by weighing each matrix element according to its importance. One does this by including a constraint for each such matrix element, and setting its tolerance equal to the inverse of the phase space factor which the matrix element multiplies. For a matrix element T_{ijk} acting on an uncorrelated initial phase space, the tolerance factor would be $1/(x_{0j}x_{0k})$, where x_{0j} and x_{0k} are the initial beam half widths specified by the `BEAM` element.

T2 Matrix Fitting Constraints

The keyword indicating an element of the T2 matrix, consists of the letters "TA" followed, with no spaces, by the three indices of the matrix element.

Note that upper and lower limit constraints are not available for second-order fitting.

Some typical T2 matrix constraints are as follows:

Desired Optical Condition	Typical Fitting Constraint
Geometric aberration $T_{122} = 0$	`F1: FIT TA122 = .0 TOLER = .001 ;`
Chromatic aberration $T_{346} = .5$	`F2: FIT TA346 = .5 TOLER = .001 ;`

By using a T2 constraint the user may fit an element of the second-order transfer matrix which pertains to any section of the beam. One causes an R2 update at the beginning of the section with a (`UPDATE R2 ;`) element. One then places the T2 constraint at the end of the section. Any number of such constraints may be imposed. This is the only second-order constraint that may be used in conjunction with an R2 update.

If a printing of the T1 matrix is requested via a (`PRINT, TRANS ;`) element it will be the second-order transfer matrix from the last R1 update. The comments about phase space weighing, made in connection with the T1 constraint, are equally valid for the T2 constraint, provided the phase space factors are obtained from the beam matrix at the position of the R2 update.

Second-Order Sigma (BEAM) Matrix Fitting Constraint

The keyword indicating an element of the beam (sigma) matrix, consists of the letter "S" followed, with no spaces, by the two indices of the matrix element.

If, for example, one wished to minimize the net contributions of second-order aberrations to the horizontal divergence, one would insert the following element:

```
FIT, S22 = .0,  TOLER = .01  ;
```

The quantity that is minimized is the net increase due to second-order terms in the second moment of the beam about the origin. This quantity is treated as the chi-squared of the problem, so the only meaningful desired value for the fit is zero. The square root of this quantity is printed in the output. It is computed using the R2 matrix. Therefore, once again, one must not include any element which updates the R2 matrix.

The second-order image of the initial beam centroid at some later point in the beam is not necessarily the beam centroid at the later point. The parameters printed by TRANSPORT are the new centroid position and the beam matrix about the new centroid. One must therefore look at both of these to observe the effects of the fitting procedure. It may even happen that an improvement in one parameter will be accompanied by a slight deterioration in the other.

The beam profile at any point is a function of the initial beam parameters. One may therefore impose weights on the effect of the various aberrations by the choice of parameters on the `BEAM` element. One might, for example, adjust the strength of the correction of the chromatic aberrations by the choice of the $\Delta p/p$ parameter. In particular, when using a BEAM constraint, one should not attempt to minimize or eliminate chromatic aberrations if $\Delta p/p$ is set equal to zero on the `BEAM` element.

Correlations (the `CORR` element) may also be included in the initial beam specification.

U1 or U2 Matrix Fitting Constraint

The keyword indicating an element of the U1 matrix, consists of the letter "U" followed, with no spaces, by the four indices of the matrix element.

The third-order transfer matrix constraint has a form very similar to the second-order constraint. The third-order matrix element $U_{ijk\ell}$ has four indices, i, j, k, and ℓ. The first of the two index codes on the constraint element is the negative of the first index i. The second index code is made by mushing together the last three indices. To fit the third-order matrix element $U_{ijk\ell}$ to zero, one would use the `FIT` element

```
FIT,  Uijkℓ = 0.0,  TOLER = 0.001  ;
```

To fit the auxiliary transfer matrix U2 to zero, use the command

```
FIT  UAijkℓ = 0.0,  TOLER = 0.001  ;
```

Comments about updating with reference to the T matrix also apply to the U matrix.

Third-Order Sigma (BEAM) Matrix Fitting Constraint

Just as with second order, the keyword indicating an element of the beam (sigma) matrix, consists of the letter "S" followed, with no spaces, by the two indices of the matrix element.

The command one would use to minimize the net contributions of third-order aberrations to the horizontal divergence is the same as is used for second order:

```
FIT, S22 = .0,  TOLER = .01  ;
```

The difference is that the minimization of third-order aberrations requires making a third-order run, and the variation of octupoles or their equivalent.

The quantity that is minimized is then the net increase due to third-order terms in the second moment of the beam about the origin. This quantity is treated as the chi-squared of the problem, so the <u>only</u> meaningful desired value for the fit is zero. The square root of this quantity is printed in the output. It is computed using the R2, T2, and U2 matrices. Therefore, once again, one must <u>not</u> include any element which updates the R2 matrix.

The comments about the printed beam centroid and the weights imposed according to the initial phase-space dimensions apply in third order just as in second.

Internal Constraints

A set of upper and lower bounds on the value of each type of parameter is in the memory of the program. If a correction is computed for a parameter which would take its value outside this range, it is reset to the limit of the range. The current limit are:

Element type		Limits
`BEAM`	0	$<$ input beam
`ROTAT`	-60	$<$ pole-face rotation $<$ 60 (deg)
`DRIFT`	0	$<$ drift
`BEND`	0	$<$ magnet length
`QUAD`	0	$<$ quad length
`SOLENOID`	0	$<$ magnet length
`SROT`	-360	$<$ beam rotation $<$ 360 (deg)
`RBEND, SBEND`	0	$<$ magnet length
`PLASMALENS`	0	$<$ magnet length
`HKICK, VKICK`	0	$<$ magnet length

These limits apply only when a parameter is being varied. Fixed values that exceed this range may be used as desired.

These constraints were included to avoid physically meaningless solutions. The limits for these and many other physical parameters can be reset via the `LIMIT` element. A complete list of parameters for which limits can be set is found under the description for that element.

Corrections and Covariance Matrix

When the program is fitting, it makes a series of runs through the beam line. From each run it calculates the chi-squared and the corrections to be made to the varied parameters. For each iteration a single line is printed containing these quantities.

The program calculates the corrections to be made using a matrix inversion procedure. However, because some problems are difficult, it proceeds with caution. The corrections actually made are sometimes reduced by a scaling factor from those calculated. This scaling factor is the first item appearing on the line of printed output. The second factor is the chi-squared before the calculated corrections are made. Following are the corrections to be made to the varied parameters. They are in the order in which they appear in the beam line. If several parameters are coupled, they are considered as one and their position is determined by the first to appear.

When convergence has occured, the final value of the chi-squared and the covariance matrix are printed. The covariance matrix is symmetric, so only a triangular matrix is shown. The diagonal elements give the change in each varied parameter needed to produce a unit increase in the chi-squared. The off-diagonal elements give the correlations between the varied parameters.

The appearance of the chi-squared and covariance matrix is:

*COVARIANCE (FIT χ^2)

$$
\begin{array}{lllll}
\sqrt{C_{11}} & & & & \\
r_{12} & \sqrt{C_{22}} & & & \\
\cdot & \cdot & & & \\
\cdot & & \cdot & & \\
\cdot & & & \cdot & \\
r_{1n} & \cdot & \cdot & \cdot & r_{n,n-1}\sqrt{C_{nn}}
\end{array}
$$

For more details on the mathematics of the fitting, the user should consult the Appendix. For an example of the output of the program he (or she) should look on page 40.

Stepping of Parameters and Broad-Band Fitting

Stepping

Stepping is when one of the parameters used in the specification of the beam line is given a set of uniformly spaced values over a given interval. Either a parameter defined on a `PARAMETER` element or a parameter from a physical element (such as `BEAM`, `BEND`, or `QUAD`) may be stepped.

For example, a parameter `DS` can be defined which gives the length of a drift space. The parameter definition is given by the statement:

```
DS: = 10.0
```

The use as the length of a drift space, could be represented as:

```
D1:   L = DS  ;
```

A `STEP` command, which steps the value of `DS` over an interval might look like:

```
STEP, DS, LOWER = 10.0, UPPER = 12.0, STEP = 0.1 ;
```

TRANSPORT will make a total of 21 passes through the beam line. In the first pass the parameter `DS` will be assigned a value of 10.0. In each successive pass the value of `DS` will be increased by 0.1. In the final pass, the value will be 12.0. If a parameter is subject to a `STEP` command, then the value given directly on the parameter specification is not used. However, a parameter must be defined in order to be stepped. The stepping command must, of course, be located after the parameter to which it applies.

The length of the drift could also have been given on the `DRIFT` specification. The `DRIFT` element would then have been:

```
D1:   DRIFT,  L = 10.0,  ;
```

The `STEP` command would now be:

```
STEP,  D1[L],  LOWER = 10.0,  UPPER = 12.0,  STEP = 0.1 ;
```

The Purpose of Stepping

There are two basic situations where stepping is useful. They are:

1. The determination of functional dependence. This usage of the `STEP` command is useful in conjunction with the `PLOT` command. One of the coordinates on the `PLOT` command can be the value of the parameter that is being stepped. The other can be any transfer or beam matrix element. The functional dependence of the matrix element on the value of the parameter can then be plotted. The beam line can also contain varied parameters and constraints. TRANSPORT will then perform a fitting process for each separate value of the stepped parameter. The value of the matrix element will be that for a beam line where the constraints are satisfied.

2. Broad band fitting. Here all the separate runs in the stepping process will be combined into a single fitting process with a single chi-squared. Such a procedure is useful, for instance, in eliminating the chromatic aberration in a beam line, including all significant higher-order contributions. Rather than constrain higher-order matrix elements, one might use the higher-order expansion described in the section on the Mathematical Formulation of TRANSPORT. One then has a first-order transfer matrix which is a function of relative momentum $\Delta p/p$. A single constraint may be imposed on this momentum- dependent matrix element as one steps across an interval in $\Delta p/p$.

Functional Dependence

If there is fitting in a data set which has stepping, then the default procedure is that the fitting is done separately for each value of the stepped parameter. There is no printout of the beam line during the calculation, as there would have to be a number of such printouts equal to the number of different values of the stepped parameter. Instead there is a table giving the different values of the stepped parameter and the corresponding chi-squared values which are the result of the fitting procedure. If there is stepping, but no fitting, then there is no such table. In the printed output, there is no evidence that there have been any runs through the beam line.

The useful output of TRANSPORT is the file that is used for plotting. From it, and a plotting program, such as TOPDRAWER, an plot can be made from the TRANSPORT file. For details, see page 237.

The following is an example of stepping with fitting. A quadrupole triplet is used to produce a point-to-point focus in both transverse planes. The fitting procedure adjusts the pole-tip magnetic fields of the three quadrupoles to satisfy the constraint. The pole-tip fields of the first and third quadrupoles are tied together by defining them in terms of parameters. The field of the center quadrupole is allowed to vary independently.

The lengths of the initial and final drift spaces are defined in terms of a parameter `DS`, such that the total length of the system is independent of the value of `DS`. Stepping `DS` over an interval then moves the position of the triplet within the available space.

The purpose of the run is to determine the dependence of the pole-tip magnetic fields of the quadrupoles on the longitudinal position of the triplet. Here there are two steps to the

problem. A plot of B1 vs DS is made in the first step, and a plot of B2 vs DS is made in the second.

```
' (1) USE TRIPLET TO FOCUS BEAM FROM SOURCE ONTO TARGET'
0
UNIT, X, IN ;
UNIT, L, FT ;
ORDER, N1=2., N2=1.  ;
B1: = 7.4  ;
B2: = 6.2  ;
DS: = 0.5 ;
BEAM, X=0.5, XP=17.5, Y=0.5, YP=17.5, DEL=3.0, P0=1.05 ;
DR1: DRIFT, L = 12.0 - DS ;
Q1:  QUAD, L = 1.5, B = - B1, APER = 4.0  ;
DR2: DRIFT, L = 0.5 ;
Q2:  QUAD, L = 3.0, B = B2, APER = 4.0  ;
DR3: DRIFT, L = 0.5 ;
Q3:  QUAD, L = 1.5, B = - B1, APER = 4.0  ;
DR4: DRIFT, L = 7.0 + DS ;
FIT1: FIT, R12 = 0.0, TOLER = .001 ;
FIT2: FIT, R34 = 0.0, TOLER = .001 ;
PRINT, BEAM, TRANS ;
PLOT: PLOT, DS, B1 ;
VARY  B1  ;
VARY  B2  ;
STEP  DS, LOWER =  -5.0, UPPER = 5.0, STEP = 0.2  ;
SENTINEL
'SECOND STEP'
 1
PLOT: PLOT, DS, B2 ;
SENTINEL
SENTINEL
```

In this case the value of a parameter (the magnetic fields) describing the beam line has been plotted against the value of another parameter. The fitting procedure causes the two parameters to be related. One could also plot any transfer or beam matrix element against a parameter describing the beam line.

Broad-Band Fitting

If broad-band fitting is indicated, then runs with all values of the stepped parameter are combined into a single fitting problem. Broad-band fitting is specified by use of the

```
BROAD
```

command, in conjunction with the STEP command.

Single runs with printing of elements and relevant matrices are made before and after the fitting procedure. However, there is a difference in the handling of the fitting constraints. Without stepping, the current values of any constrained quantities are printed along with the desired values. With broad-band fitting, the current values are really an ensemble of values, one value for each value of the stepped parameter. Two numbers corresponding to the current value are printed out for the `FIT` element. The first of these numbers can be taken as the mean value of the constrained quantity, averaged over the different values of the stepped parameter. The second of these numbers can be taken as the standard deviation of the ensemble of actual values of the constrained quantity.

A single chi-squared is printed for each iteration of the fitting procedure. This chi-squared is the sum of the chi-squareds for all the different values of the stepped parameter.

The following is an example of broad-band fitting.

```
"FOUR CELL TEST OF CHROMATIC ABERRATIONS IN PERIODIC ARRAYS"
 0
UNIT, X, MM  ;
PRINT  PRECISE  ;
NORD: ORDER, N1=2., N2=1. ;
BROAD  ;
BEAM, X=4.666, XP=.1286, Y=4.666, YP=.1286, DEL=5., P0=10.  ;
CENT: CENT, DEL = 1. ;
REPEAT, N = 4.  ;
QY: QUAD, L = .375, B = -9.080013, APER = 100. ;
SY1: SEXT, L = .1, B = 0.0, APER = 100. ;
DRIFT, L = .4125  ;
RBEND, L = 5.4, B = 2.01525  ;
DRIFT, L = .4125  ;
SX1: SEXT, L = .1, B = 0., APER = 100. ;
QX: QUAD, L = .75, B = 9.088963, APER = 100. ;
SX2: SEXT, L = .1, B = SX1[B], APER = 100. ;
DRIFT, L = .4125  ;
RBEND, L = 5.4, B = 2.01525  ;
DRIFT, L = .4125  ;
SY2: SEXT, L = .1, B = SY1[B], APER = 100. ;
QY: QUAD, L = .375, B = -9.080013, APER = 100. ;
REPEAT, N = 0.  ;
PRINT  BEAM  ;
PRINT  TRANS  ;
FITA: FIT  R12 = 0.0, TOLER = 0.001 ;
FITB: FIT  R34 = 0.0, TOLER = 0.001 ;
VARY, SY1[B] ;
VARY, SX1[B] ;
STEP  CENT[DEL], LOWER =  -1.0, UPPER = 1.0, STEP = 0.5  ;
SENTINEL
SENTINEL
```

Here the transfer matrices are evaluated to second order, but accumulated only to first order. The presence of a centroid displacement in $\Delta p/p$ in the data means that the first-order transfer matrix is actually a function of $\Delta p/p$. Producing a first-order transfer matrix which is a function of $\Delta p/p$, and then stepping over $\Delta p/p$ while imposing constraints, effectively eliminates the problem of truncation of the transformation to a given order. For mathematical details, the reader should consult the section on off-axis expansions under Mathematical Formulation of TRANSPORT.

While the value of $\Delta p/p$ is stepped over an interval, a first-order point-to-point constraint is imposed in both transverse planes. Because we are stepping over an interval in $\Delta p/p$, as well as constraining the matrix elements, we are also minimizing the chromatic aberration. In this case, the fields of the quadrupoles have already been set to produce a first-order focus. The reduction of the chromatic aberration comes from the variation of the sextupoles. The appearance is that we have used first-order constraints to solve a higher-order fitting problem.

SENTINEL

Each step of every problem in a TRANSPORT data set must be terminated with the word `SENTINEL`. The word `SENTINEL` need not be on a separate card. For a description of the form of a TRANSPORT data set see page 17.

An entire run, consisting of one or several problems, is indicated by an additional card containing the word `SENTINEL`. Thus, at the end of the entire data set the word `SENTINEL` will appear twice.

Original TRANSPORT Notation

Positional Format

Element Description

In the original TRANSPORT notation, the elements are written in positional notation. In positional notation, the element specification begins with a numerical type code. The physical parameters follow the numerical type code. They are given by a sequence of numbers, separated by spaces and terminated by a semicolon. The end of a line also serves as a semicolon.

The physical parameters are the quantities which describe the physical element represented. Their meaning depends on the numerical type code and their position in the sequence of numbers. There is no possibility of describing an element in an alternate set of parameters using positional notation. The label, if present, identifies the element. It may contain up to fifteen characters and is enclosed by single quotes.

In positional notation, a sector bending magnet would be specified as:

```
4.   10.   20.   0.5  'BM1'  ;
```

The first integer (4.) is a type code, indicating that this element is a bending magnet. The additional numbers here represent respectively the magnet length (10.) in meters, the magnetic field (20.) in kilogauss, and the dimensionless field index n (0.5). The sequence of characters `BM1` is the label. A summary, indicating the proper order for the physical parameters is given in Table 6.

For any element the numerical type code is considered to be the first parameter. The second parameter is also known as the first physical parameter. It is the second entry in Table 6. In some cases the parameters of an element do not really refer to physical quantities, but will nevertheless be referred to as such in this manual.

In positional notation the label may be placed anywhere among the numbers. Labels are useful in problems with many elements and/or when sequential fitting is used. They must be used to identify any element to be changed in succeeding steps of a given problem.

If the physical parameters of an element are specified in positional notation, then a vary field forms an extension of the type code. The vary field indicates which physical parameters of the element are to be adjusted if there is to be any fitting. It is placed immediately (no intervening blanks) to the right of the decimal point which follows the type code.

```
4.01  10.   20.   0.5  'BM1'  ;
```

Here the decimal `.01` following the type code `4` indicates that the second number in the description of the physical element is to be varied during fitting. Here the second number is the strength B of the magnetic field and initally has the value 20. See page 300 for an explanation of the use of vary codes.

Each element must be terminated by a semicolon (`;`). For a single element per line, the end of a line serves as a semicolon. If there is more than one element per line of data, then each element must be terminated by a semicolon. Thus the `4.` above may be followed by a drift as follows

```
4.   10.   20.   0.5  'BM1'  ;  3.   10.
```

Here the `4.` element must be followed by a semicolon. The end of the line serves as a semicolon for the drift element.

Spaces before and after the semicolon are allowed but not required. If the program encounters a semicolon before the expected number of parameters has been read in and if the indicator card was a zero (0), the remaining parameters are set to zero. Thus, the element

```
4.   10.   20.   'BM1'  ;
```

is the same as

```
4.   10.   20.   0.0  'BM1'  ;
```

For a bend, three physical parameters are to be specified in original TRANSPORT notation. If only two physical parameters are specified, a single additional parameter is assumed with the value 0.0.

If the indicator card was a one (1), then the numbers indicated on the card are substituted for the numbers remaining from the previous solution; all other numbers are unchanged. Thus if the first problem step contained the element

```
4.   10.   20.   0.5  'BM1'  ;
```

and a subsequent problem step had the element

```
4.   10.   20.   'BM1'  ;
```

the normalized field gradient n would continue to have the value 0.5. If the second problem step had the element

```
4.   10.   20.   0.0  'BM1'  ;
```

the value of the normalized field gradient would be changed to 0.0.

The "free-field" input format of the data cards makes it considerably easier to prepare input than the standard fixed-field formats of FORTRAN. Numbers may be placed anywhere on the card and must simply be in the proper order. They must be separated by one or more blanks. Several elements may be included on the same card. A single element may be continued from one card to the next by placing an ampersand `&` after all other characters on the card to be continued. A single number must be all on one card; it may not continue from one card to the next. The program storage is limited to a total of 12,000 locations (including representation of mnemonics and those parameters not entered but implied equal to zero) and 4000 elements.

A decimal number (e.g., 2.47) may be represented in any of the following ways:

```
2.47
.0247E+02
247E-2
```

Type Codes

In original TRANSPORT, the element types are designated by numerical type codes. Not all TRANSPORT elements can be expressed as a sequence of numbers. For those that can, the numerical type code and the meaning are given in the table following.

Type Code	Mnemonic	Element
1.	`BEAM`	Input phase space dimensions and reference trajectory momentum.
2.	`ROTATION`	Fringing fields and pole-face rotations for bending magnets
3.	`DRIFT`	Drift space – a field free region
4.	`BEND`	Sector bending magnet (without fringing fields)
5.	`QUADRUPOLE`	Quadrupole magnet
6.	`UPDATE`	Transfer matrix update
7.	`CENTROID`	Shift in the beam centroid
8.	`ALIGN`	Magnet alignment tolerances
9.	`REPEAT`	Repetition of a segment of a beam line
10.	`FIT`	Fitting constraint
11.	`ACCELERATOR`	Travelling wave linear accelerator
12.	`CORRELATION`	Beam rotated ellipse
13.	`PRINT`	Output print control instructions
14.	`MATRIX`	Arbitrary matrix input
15.	`UNITS`	Input-output units
16.	`SPECIAL`	Special input parameters

17.	ORDER	Zeroeth, first, second, or third order
18.	SEXTUPOLE	Sextupole magnet
19.	SOLENOID	Solenoid magnet
20.	SROT	Coordinate Rotation
24.	SECTION	Defined section of beam line
25.	OCTUPOLE	Octupole magnet
27.	ETA	Accelerator eta function
28.	RBEND	Rectangular bending magnet (with fringing fields)
29.	SBEND	Sector bending magnet (with fringing fields)
31.	MARKER	Position marker
34.	PLASMALENS	Plasma (lithium) lens
35.	HKICKER	Horizontal steering magnet
36.	VKICKER	Vertical steering magnet
37.	ALMARK	Alignment marker
41.	SEPTUM	Electrostatic septum
43.	SHIFT	A shift in the reference coordinate system

The Use of Labels

Labels are available for identification of individual elements. The use of labels is necessary if the parameters of an element are to be changed between steps of a given problem. The label identifies the element in the earlier step to which the changes specified in the latter step are to apply. Labels may also be inserted for the user's convenience. In this case the placement of a label on a given element is optional.

In positional notation the label may be placed anywhere among the parameters of a given element. It must be enclosed in quotes. Blanks within a label are ignored. The maximum length of a label is fifteen non-blank characters. It may contain letters, numerals, underscore, percent (%), and dollar sign ($).

As an example, the following all denote the same drift space:

```
'DRF' 3.  1.5      ;
3'DRF'15-1;
3.  .15E1 'D R F'  ;
```

On a unit specification 15 element, the label is the unit name. A unit specification indicating the longitudinal length (unit type 8) is to be measured in feet would then be:

```
15.   8.  'FT' ;
```

Labels may be used to change an element from one problem step to the next.

An element may be altered in any of several ways in passing from one step of a problem to the next. An element may be deactivated by placing a minus sign in front of its type code (`5.`, `4.`, etc.) or activated by removing the minus sign. New elements can be inserted and old elements removed from the beam line. The values of the parameters can also be changed and vary codes may be inserted or removed.

Activation and Deactivation of Elements

If an element is to be activated or decativated in passing from one step to the next of a given problem, the element must be included in both steps, having the same label each time. All elements to be activated in a later step must be included in the first step (indicator card 0) of that problem. In later steps (indicator card 1) of a problem only those elements to be changed are specified. The elements to be changed are identified by their labels.

If the type code of an element is immediately preceded by a minus sign in a given step of a problem, that element will be ignored when the calculation is performed. However, storage space in the computer will be allocated for the element for possible activation in later steps of the problem. For example, if a fitting constraint is to be ignored in the first step of a problem, but activated in a latter step, it should be indicated in both steps. In the first step such an element might appear as

```
-10  1  2  0.0  .001  'FIT1'  ;
```

in the first problem step, and as

```
10  'FIT1'  ;
```

in the later step.

Here the sigma matrix element σ_{12} is fit to zero, causing a waist constraint to be imposed on the beam.

The storage space used for a `10` element is fixed. Therefore, in the first step, we can simply indicate that storage is to be reserved for a `10` element. In an ensuing problem step, the remainder of the `10` element can be specified. The above procedure is therefore equivalent to placing the element:

```
-10  'FIT1'  ;
```

in the first problem step. In the later step one would then insert

```
10  1  2  0.0  .001  'FIT1' ;
```

Changing of Physical Parameters

The physical parameters of an element may be changed between steps of a problem. In the first step, all the parameters to be used should be specified. In the later step, only those parameters to be changed need be specified. The storage space allocated for a given element is determined by the element type and the set of parameters given. For the `BEAM` and `MATRIX` elements, included among the data may be continuation codes which will also affect the amount of storage space taken by the element.

In the first step a bending magnet may be given a length of 5 meters.

```
4.   5.0  10.0  0.0  'B1'  ;
```

In a succeeding step, its length could be increased to 10 meters by inserting the element

```
4.   10.0  'B1'  ;
```

All parameters, up to and including the one to be changed, must be specified. the remaining, if not specified, will be left unchanged from the previous step.

Variation of Physical Parameters

Vary codes are designed for use in positional notation. Vary codes may be inserted or removed in passing from one problem step to the next. For instance, one might wish to vary the field of a quadrupole in one step of a problem and then use the fitted value as data in the following step. The first step might then contain the element:

```
5..01  5.0  10.0  5.0  'Q1'  ;
```

indicating that the second physical parameter in the quadrupole specification is to be varied. The second parameter represents the magnet pole-tip field strength and has the present value of 10.0. The following step would contain the element

```
5.   'Q1'  ;
```

Since, in the second step, the first item on the card contains no vary code the vary code is deleted. All other parameters, not being respecified, are left unchanged.

Several elements may have the same label. If, as in the above example, one wished to vary the field of several quadrupoles in one step, then pass the final values to the next step, one could give all such elements the same label. There might be four quadrupoles, all labeled `'Q1'`, being varied simultaneously. If the data for the next step contain the single element

```
5.   'Q1'  ;
```

the vary code on all elements labeled `'Q1'` will be deleted.

Examples

The sample problem below contains two problem steps, each beginning with title and indicator cards and terminating with a SENTINEL. The first step causes TRANSPORT to do a first-order calculation with fitting. The second initiates a second-order calculation with the data that is a result of that fitting. Elements which are considered to be the same in the two steps are identified by having the same label.

Here we present two versions of the same example. The example describes the same beam line and set of operations as did the previous examples for MAD notation.

In the first example, elements themselves are given in original TRANSPORT notation. The beam line consists of a drift (3), bending magnet with pole face rotations (2, 4, 2), and another drift (3). It is prefaced by a specification of the beam phase space (1), and a print command(13). An order specification (-17) is present, but turned off for the first step. The beam line description is followed by another print commant (13), and the fit command (10).

For the two drift spaces, the numeral "3" appears after the decimal point which follows the type code. These two numerals signify that the two drifts are to be varied and that the changes made are always to be equal.

The type code `10` element which specifies the fitting condition is labeled `FIT1`. It is active for the first-order calculation, but is turned off for the second-order calculation. The vary codes for elements `DR1` are set to zero for the second-order problem. The second-order element, `SEC1`, is ineffective during the fitting, but causes the program to compute the second-order matrices in the second calculation.

In the second example, the numerical type codes are replaced by literal mnemonics, making the data representing the beam line easier to read.

An Example of a TRANSPORT Input Deck

Original TRANSPORT Notation

```
'FORTRAN H CHECK ON BETA FIT'                    Title card
0                                                Indicator card
1   .5   1   .5   1   .5   1   1   ;
-17   'SEC1'   ;   13   3   ;
3.3   2.745   'DRI'   ;
2   0   ;
4   9.879   10   .5   ;                          Elements
2   0   ;
3.3   2.745   'DRI'   ;
13   4   ;
10   -1   2   0   .0001   'FIT1'   ;
SENTINEL                                                          First problem step
'SECOND ORDER'                                   Title card
1                                                Indicator card
17   'SEC1'   ;
3   'DRI'   ;                                    Elements to be
-10   'FIT1'   ;                                 changed
SENTINEL                                                          Second problem step
SENTINEL              Second sentinel signifies end of run.
```

As many problems and problem steps as one wishes may be stacked in one job.

Note that in some previous versions of TRANSPORT a decimal point was required with every numerical entry except the indicator card (which must not have a decimal point in any version of TRANSPORT). The decimal point is <u>not</u> required in this version of TRANSPORT.

Table 6: Summary of Elements in Positional Notation

Preliminary Specifications in Original TRANSPORT Notation

Physical Element	Type & Vary	2nd Entry	3rd Entry	4th Entry
Units (Transport Dimensions)	15	Code	Unit Symbol	Scale Factor (if required)
Quadratic Term for Bend Field	16.0v	1.0	$\epsilon(1) = \beta \left(\frac{1}{\rho_0} \right)^2$ ρ_0 in units of transverse length	
Fringe Field Correction Coefficient	16	2.0	κ_0	
Mass of Particles in beam	16	3.0	M/m m = mass of electron	
Half Aperture of Bending Magnet in x Plane	16	4.0	$w/2$	
Half Aperture of Bending Magnet in y plane	16	5.0	$g/2$	
Length of System	16	6.0	L	
Fringe Field Correction Coefficient	16	7.0	κ_1	
Reference Trajectory Momentum	16	11.0	p_0	
Curvature of Entrance Face of Bending Magnet	16.0v	12.0	$(1/R_1)$	
Curvature of Exit Face of Bending Magnet	16.0v	13.0	$(1/R_2)$	
Random Number	16	14.0	Initial value of random number	
Focal Plane Rotation	16	15.0	Angle of focal plane rotation	

Physical Element	Type & Vary	2nd Entry	3rd Entry	4th Entry
Initial Beam Line x-Coordinate	`16.0v`	16.0	x_0	
Initial Beam Line y-Coordinate	`16.0v`	17.0	y_0	
Initial Beam Line z-coordinate	`16.0v`	18.0	z_0	
Initial Beam Line Horizontal Angle	`16.0v`	19.0	θ_0	
Initial Beam Line Vertical Angle	`16.0v`	20.0	ϕ_0	
Reference Momentum for Magnetic Fields	`16`	21.0	p_{ref}	
Fractional Excess bend	`16`	22.0	r_s	
Scaling Factor for Skew Field Components	`16`	23.0	r_a	
Skew Bend Field	`16`	24.0	v_r	
Skew Quadrupole Component for a Bending Magnet	`16`	25.0	n'	
Skew Sextupole Component for a Bending Magnet	`16`	26.0	ϵ'	
Octupole Component for a Bending Magnet	`16`	27.0	ϵ_3	
Order of Calculation	`17`	n_1	n_2	

Beam Description in Original TRANSPORT Notation

Physical Element	Type & Vary	2nd Entry	3rd Entry	4th Entry	5th Entry	6th Entry	7th Entry	8th Entry	9th Entry
Beam	`1.vvvvvv0`	x	x'	y	y'	ℓ	δ	p_0	
rms Addition to Beam Envelope	`1.vvvvvv00`	Δx	$\Delta x'$	Δy	$\Delta y'$	$\Delta \ell$	$\Delta \delta$	p_0	0
Beam (Rotated Ellipse)	`12`	The fifteen correlations among the six elements (This entry must be preceded by a `1` element.)							
Beam Centroid Shift	`7.vvvvvv`	Δx	$\Delta x'$	Δy	$\Delta y'$	$\Delta \ell$	$\Delta \delta$	p_0	
Accelerator Eta Function	`27`	Δx	$\Delta x'$	Δy	$\Delta y'$	$\Delta \ell$	$\Delta \delta$	p_0	

Physical Elements in Original TRANSPORT Notation

Physical Element	Type & Vary	2nd Entry	3rd Entry	4th Entry	5th Entry
Drift	`3.v`	Length			
Bending Magnet	`4.vvv`	Length	Field	Field gradient (n value)	
Pole Face Rotation	`2.v`	Angle of Rotation			
Rectangular Bending Magnet	`28.vvv`	Length	Field	Field gradient (n value)	
Sector Bending Magnet	`29.vvv`	Length	Field	Field gradient (n value)	
Horizontal Steering Magnet	`35.vvv`	Length	Field		
Vertical Steering Magnet	`36.vvv`	Length	Field		
Quadrupole	`5.vv0`	Length	Field	Half aperture	
Sextupole	`18.vv0`	Length	Field	Half aperture	
Octupole	`25.vv0`	Length	Field	Half aperture	
Plasma Lens	`34`	Length	Field	Half aperture	
Solenoid	`19.vv`	Length	Field		
Accelerator	`11`	Length	E (energy gain)	ϕ (phase lag)	Wavelength
Electrostatic Septum	`41`	Length			
Marker	`31`				

Other Transformations in Original TRANSPORT Notation

Physical Element	Type & Vary	2nd Entry	3rd Entry	4th Entry	5th Entry	6th Entry	7th Entry	8th Entry
Arbitrary Matrix	`14.vvvvvv0`	R_{j1}	R_{j2}	R_{j3}	R_{j4}	R_{j5}	R_{j6}	J
		Continuation codes also allow the input of second- and third-order terms.						
Coordinate Rotation	`20.v`	Angle of rotation						
Shift	`43.vvvvvv`	Δx	$\Delta x'$	Δy	$\Delta y'$	$\Delta \ell$	$\Delta \delta$	p_0
Alignment Tolerance	`8.vvvvvv0`	Δx	$\Delta \theta$	Δy	$\Delta \phi$	Δz	$\Delta \alpha$	Code Number

Assembling Beam Lines in Original TRANSPORT Notation

Physical Element	Type & Vary	2nd Entry
Repeat control	`9.`	Number of repetitions
Defined Section	`24.`	Code

Operations in Original TRANSPORT Notation

Physical Element	Type & Vary	2nd Entry	3rd Entry	4th Entry	5th Entry
Transform 1 Update	`6.`	0.	1.		
Transform 2 Update	`6.`	0.	2.		
Alignment Marker	`37.`	0.	1. or 2.		
Input/Output Options	`13.`	CODE			
Fitting Constraints	`10.`	I	J	Desired Value (i,j) Matrix Element	Accuracy of Fit

Elements with Code Digits

Several of the elements have parameters which are not a measure of any physical quantity, but rather a code indicating the action to be taken by the element. The code digits for such elements are explained in the following pages. The equivalent mnemonic is given for the numerical codes, along with a short explanation. Much more comprehesive explanations are given in the keyword descriptions of the various elements.

Magnet Alignment Tolerances – Type Code 8

The last numerical parameter on the type code 8. element in original TRANSPORT format is a code indicating the nature of the misalignment. It determines whether the misalignment is uncertain or known and displayed in the beam matrix or a table. It also specifies the element or section of the beam line to be misaligned.

It is much better formulated in keywords. The use of keywords also has allowed the expansion of the number of varieties of misalignment that can be represented. Code numbers that can be understood by TRANSPORT here are given, along with equivalent mnemonics and descriptions. The symbol "X", in a code digit, is not to be taken literally. It simply means that the position occupied by the "X" is irrelevant to the definition being given. It may take on any of the possibilities given when its position is being defined.

Code Digit	Corresponding Mnemonic	Description
0XX	`UNCERTAIN`	The magnet position is uncertain.
1XX	`KNOWN`	The magnet or section of beam line is misaligned by a known amount.
2XX.	`RANDOM`	Random numbers are used to calculate the displacement.
X0X	`BEAM`	The beam matrix contains the results of the misalignment.
X1X	`TABLE`	A table is used to store the results of misalignments.
XX0	`SINGLE`	The single magnet (element) immediately preceding the align card is to be misaligned.
XX1	`R1`	The last R1 matrix update or alignment marker marks the beginning of the section to be misaligned.
XX2	`R2`	The last R2 matrix update or alignment marker marks the beginning of the misaligned section.
XX3	`MAGNETS`	All subsequent bending magnets and quadrupoles are independently misaligned by the amount specified.
XX4	`BENDS`	All subsequent bending magnets, including fringing fields, are independently misaligned by the amount specified.
XX5	`QUADS`	All subsequent quadrupoles are independently misaligned by the amount specified.

Repeat – Type Code 9

The second parameter on the `REPEAT` element represents the number of repetitions through a section of beam line, provided the `REPEAT` element is demarcating the beginning of that section. This parameter is zero if the `REPEAT` element is demarcating the end of the section.

An example of the use of the `REPEAT` element might be:

```
REPEAT  4.   ;
BEND  - - -  ;
DRIFT  .5  ;
REPEAT  0.   ;
```

Those elements (in this case a bend and drift) between the (`REPEAT  4.  ;`) and (`REPEAT  0.  ;`) would be used four times.

More information about the repeat element can be found on page 208.

Vary Codes

The original positional parameter notation of TRANSPORT allows the variation of physical parameters to be specified by means of a vary code. This code occurs to the immediate right of the decimal point which follows the type code of the element. It has one digit for each parameter, the digits having the same order in the vary code as the physical parameters have on the card. A '`0`' indicates the parameter may not be varied; a '`1`' that it may be. For instance, `3.0` is the combined type and vary codes (`0`) for a drift length which is to remain fixed; `3.1` indicates a drift length that may be varied (by the virtue of the `.1`). `4.010` indicates a bending magnet with a variable magnetic field. In typing `3.0`, the zero is unnecessary. In typing `4.010` , the first zero must be included but the second zero need not be.

A quadrupole is specified in terms of its length, pole-tip field, and aperture. If the magnetic field is to be varied, the element is expressed as

```
5.01  10.   5.   2.54  'Q1' ;
```

The numerals following the type code `5` and the decimal point indicate that the second parameter, in this case the magnetic field, may be varied. A fitting procedure may cause its value to be changed from 5. to some other value, such as 5.47723.

First-Order Vary Codes

In a first-order run the following parameters marked v may be varied, those marked 0 may not be varied.

`BEAM` ...	`1.`vvvvvv0 – All components of the input beam may be varied, except the momentum.
R.M.S. ADDITION ...	`1.`vvvvvv00 – All components of an r.m.s. addition may be varied except the momentum change Δp.
`CORRELATION` ...	`12.`vvvvvvvvvvvvvvv – Any of the beam correlation ma-. trix elements may be varied.
`ALIGN` ...	`8.`vvvvvv0 – Any of the alignment parameters may be varied.

Vary Codes Usable in Any Order

INITIAL COORDINATES ...	`16.0`v – Any of the three initial position floor coordinates or two angle coordinates may be varied.
`CENTROID` ...	`7.`vvvvvv – Any of the axis shift parameters may be varied.
`ETA` ...	`27.`vvvvvv – Any of the components of the accelerator η function may be varied.
`DRIFT` ...	`3.`v – The drift length may be varied.
`ROTAT` ...	`2.`v – The pole face angle of a bending magnet may be varied.
`BEND` ...	`4.`vvv – The length, the field, and/or the n-value may be varied.
`RBEND` ...	`28.`vvv – The length, the field, and/or the n-value may be varied.
`SBEND` ...	`29.`vvv – The length, the field, and/or the n-value may be varied.
`HKICK` ...	`35.`v – The magnetic field may be varied.
`VKICK` ...	`36.`v – The magnetic field may be varied.
`QUAD` ...	`5.`vv0 – The length may be varied; the field may be, the aperture may <u>not</u> be.
`PLASMA LENS` ...	`34.`vv0 – The length may be varied; the field may be, the aperture may <u>not</u> be.
`SOLENOID` ...	`19.`vv – The length and/or field may be varied.
`MATRIX` ...	`14.`vvvvvv0 – Any of the first order matrix elements may be varied.
BEAM ROTATION ...	`20.`v – The angle of rotation may be varied.

Second-Order Vary Codes

In a second-order run the following parameters may be varied:

$\epsilon(1)$ ...	`16.0`v `1.` – The normalized quadratic term (sextupole component) in the midplane expansion for the field of a bending magnet may be varied.
1/R1 ...	`16.0`v `12.` – The pole face curvature of a bending magnet entrance may be varied.
1/R2 ...	`16.0`v `13.` – The pole face curvature of a bending magnet exit may be varied.
`SEXTUPOLE`	`18.0`v – The field strength may be varied.

Third-Order Vary Codes

`OCTUPOLE`	`25.0v` - The field strength may be varied.

Coupled Vary Codes

It is possible to apply the same correction to each of several variables. This may be done by replacing the digit `1` in the vary code with one of the digits `2` through `9`, or a letter `A` through `Z`. All such variables whose vary digits are the same, regardless of position will receive the same correction. For example, the three element-vary codes (`QUAD.0A, QUAD.01, QUAD.0A`) might represent a symmetric triplet. The same correction will be made to the first and third quadrupoles, guaranteeing that the triplet will remain symmetric.

If a vary digit is immediately preceded by a minus sign, the computed correction will be subtracted from, rather than added to, this variable. Thus parameters with the same vary digit, one of them being preceded by a minum sign, will be inversely coupled. For example the type-vary code sequence (`DRIFT.B, QUAD.01, DRIFT.-B`) will allow the quadrupole to move without altering the total system length.

Vary digits may also be immediately preceded by a plus sign without changing their meaning. Thus `QUAD.0A` is the same as `QUAD.0+A`. For historical reasons, the vary digits (`9` and `4`), (`8` and `3`), and (`7` and `2`) are also inversely coupled. Inverse coupling may not be used with the `BEAM` or `ALIGN` elements.

Constraints – Type Code 10

A fitting constraint which fits the R_{12} matrix element to zero is given in positional notation as

```
10.   -1.   2.   0.0  0.001  'F1'  ;
```

The first number (`10`) is a numerical type code indicating a constraint. The fourth number (`0.0`) is the desired value for the matrix element R_{12}. The fifth number (`0.001`) is the tolerance for the fit.

The second (`-1.`) and third (`2.`) numbers indicate what is to be constrained. The numbers `1.` and `2.` are the indices of the matrix element. The minus sign in front of the `1.` is used to indicate that the constraint is to be applied to the transfer matrix R. For a constraint on the beam (σ) matrix, the first integer is positive. Below we give a list of the meaningful combinations of code digits, their equivalent keywords, and a brief identification.

As a warning, the keyword cannot simply be substituted for the pair of code digits in positional notation. The usage of keywords is described in the earlier sections based on the keyword format. For more detail, the user should consult page 253.

First Code Digit	Second Code Digit	Equivalent Keyword	Description
0	0	`L`	The beam line accumulated length.
8	1	`XFLOOR`	The floor coordinate x value.
8	2	`YFLOOR`	The floor coordinate y value.
8	3	`ZFLOOR`	The floor coordinate z value.
8	4	`YAW`	The floor coordinate plan view reference trajectory angle.
8	5	`PITCH`	The angle the reference trajectory makes with the horizontal.
8	6	`ROLL`	The net rotation angle about the reference trajectory.
-i	j	`Rij`	An element of the first-order transfer matrix $R1$.
-(20+i)	j	`RAij`	An element of the auxilliary first-order transfer matrix $R2$.
-11	2	`MUX`	A horizontal AGS machine constraint.
-13	4	`MUY`	A vertical AGS machine constraint.
0	1	`XC`	The beam centroid coordinate x.
0	2	`XPC`	The beam centroid coordinate x'.
0	3	`YC`	The beam centroid coordinate y.
0	4	`YPC`	The beam centroid coordinate y'.

0	5	`DLC`	The beam centroid coordinate ℓ.
0	6	`DELC`	The beam centroid coordinate δ.
1	1	`XBEAM`	The beam half width in x.
2	2	`XPBEAM`	The beam half width in x'.
3	3	`YBEAM`	The beam half width in y.
4	4	`YPBEAM`	The beam half width in y'.
5	5	`DLBEAM`	The beam half width in ℓ.
6	6	`DELBEAM`	The beam half width in δ.
i	j	`Sij`	An element of the beam matrix σ.
10+i	j	`Cij`	An element of the beam correlation matrix r.
27	1	`ETAX`	The accelerator function η_x.
27	2	`DETAX`	The accelerator function η'_x.
27	3	`ETAY`	The accelerator function η_y.
27	4	`DETAY`	The accelerator function η'_y.
-15	1	`PSIX`	The accelerator phase shift in the horizontal plane.
-15	3	`PSIY`	The accelerator phase shift in the vertical plane.
-i	10j+k	`Tijk`	An element of the second-order transfer matrix $T1$.
-(20+i)	10j+k	`TAijk`	An element of the auxilliary second-order transfer matrix $T2$.
-i	100j+10k+ℓ	`Uijk`ℓ	An element of the third-order transfer matrix $U1$.
-(20+i)	100j+10k+ℓ	`UAijk`ℓ	An element of the auxilliary third-order transfer matrix $U2$.

Print – Type Code 13

In positional notation, the element

```
13.   4.   ;
```

causes the $R1$ transfer matrix to be printed at the point where the element is placed. Its equivalent in keyword notation is

```
PRINT,  R1  ;
```

To translate a `13.` type code element to a `PRINT` element, the user should substitute the word `PRINT` followed by a comma (`,`) for the number `13.`, and substitute the corresponding mnemonic for the code digit.

The various code digits and their equivalent keywords and short descriptions of their effect are given below. A few `13` elements do not have keyword equivalents, but they are given for historical reasons. There are quite a few keywords which have no positional equivalent. They can be found in the section on the keyword format for the `PRINT` command. If someone has an old data set in numerical notation, a `PRINT` command in keyword notation can be slipped into the data and will probably work. However, mixing notations is usually to be discouraged.

Code Digit	Equivalent Keyword and/or Description	
1	`BEAM`	Print the beam (σ) matrix.
2	`BEAM, OFF`	Turn off printing of the beam matrix.
3	`BEAM, ON`	Turn on printing of the beam matrix.
4	`R1`	Print the transfer matrix.
-4	`R1INV`	Print the inverse transfer matrix.
5	`R1, OFF`	Turn off printing of the transfer matrix.
6	`R1, ON`	Turn on printing of the transfer matrix.
7	`ACCELERA`	Interpret the beam as being in accelerator notation.
8	`ALIGN`	Print the misalignment effect table.
9	Equivalent to the `REALIGN` element. There are no other keywords.	
12	`FLOOR`	Print the floor coordinates.
13	`REFER`	Refer to unrotated coordinates.
14	`WAIST`	Print positions and sizes of waists.
16	`PRECISE`	Precise values of varied parameters.
17	`NOPARA`	Suppress printing of physical parameters.
18	`ONLY`	Print only varied elements and constraints.
19	`ONELINE`	Print matrices on a single line.
24	`R2`	Print auxilliary transfer matrix.

Punched Output Controls

(`13. 29.` through `36. ;`): This element is now used little if at all. Its original purpose was to allow another program to use the transfer matrices for ray tracing and histogram generation. The transfer matrices were punched on cards by this element. The transfer matrices can still be written to an output file by this element. The word "punch" is used in this description to emphasize the obselesence of this element.

Since 1972 the program TURTLE has existed which did this same operation starting with the TRANSPORT input data. However, every time any feature of TRANSPORT is eliminated, we discover some user somewhere who has been using that feature. It is therefore included for compatability with previous versions.

If the control is `29`, all of the terms in the first-order matrix and the x and y terms of the second-order matrix are punched.

If the control is `30`, all of the terms of the first-order matrix and all second-order matrix elements are punched out.

If the control, n, is greater than 30, all of the first-order terms are punched and the second-order matrix elements which correspond to $(n - 30.)$, i.e., if n = 32, the second-order theta matrix elements are punched out. If n = 31, the second-order x matrix elements are punched, and so forth.

Matrix – Type Code 14

For the arbitrary matrix with positional input, all the elements up to and including the ones to be specified must be included in the input. This means that not only does each first-order row contain six coefficients, but each second-order row contains 21 coefficients, and each third-order row contains 56 coefficients. To avoid having to allocate space for higher-order matrix elements when only lower-order ones are present, a continuation code is used. There is one continuation code to indicate the presence of second-order terms, and another for third-order terms.

The first-order representation of a single row of a matrix might look like:

```
14.   -.1  .9  0.   0.   0.   0.   2.   ;
```

There are eight parameters. The first is the type code (14), which indicates a matrix element. The second through seventh give the values of the matrix elements. The eighth is the row number.

Second Order

If it is desired to include the second-order matrix coefficients for the ith row, then the following 22 additional numbers may be read in.

9	continuation code `0`.
10 to 30	The 21 coefficients:

$$\begin{array}{cccccc} T_{i11} & T_{i12} & T_{i13} & T_{i14} & T_{i15} & T_{i16} \\ T_{i22} & T_{i23} & T_{i24} & T_{i25} & T_{i26} & T_{i33} \\ T_{i34} & T_{i35} & T_{i36} & T_{i44} & T_{i45} & T_{i46} \\ T_{i55} & T_{i56} & T_{i66} & & & \end{array}$$

in that order, where i is the row number.

Third Order

It is also possible to include the third-order matrix coefficients for the ith row by reading in the following 57 additional numbers.

31	continuation code `0`.
32 to 87	The 56 coefficients:

$$\begin{array}{cccccc} U_{i111} & U_{i112} & U_{i113} & U_{i114} & U_{i115} & U_{i116} \\ U_{i122} & U_{i123} & U_{i124} & U_{i125} & U_{i126} & U_{i133} \\ U_{i134} & U_{i135} & U_{i136} & U_{i144} & U_{i145} & U_{i146} \\ U_{i155} & U_{i156} & U_{i166} & U_{i222} & U_{i223} & U_{i224} \\ U_{i225} & U_{i226} & U_{i233} & U_{i234} & U_{i235} & U_{i236} \end{array}$$

U_{i244} U_{i245} U_{i246} U_{i255} U_{i256} U_{i266}
U_{i333} U_{i334} U_{i335} U_{i336} U_{i344} U_{i345}
U_{i346} U_{i355} U_{i356} U_{i366} U_{i444} U_{i445}
U_{i446} U_{i455} U_{i456} U_{i466} U_{i555} U_{i556}
U_{i566} U_{i666}

in that order, where i is the row number.

Units Changes – Type Code 15

To change length to feet, width to inches, and momentum to MeV/c, add to the front of the deck the elements

```
15.   8.   'FT'  0.3048;
15.   1.   'IN'  2.54;
15.   11.   'MEV'  0.001;
```

The name of the new unit is given by the label of the unit element (`FT`, `IN`, and `MEV`). The scale factors (0.3048, 2.54, and 0.001) give the size of the new unit (feet, inches, and MeV) in terms of the reference unit (meters, etc). The three code digits (8, 1, and 11) indicate which unit is to be altered. The code digit 8, for example, indicates the longitudinal distance along the reference trajectory.

Other code digits, their keyword equivalent, and a short description of the meaning of the unit type are given below. Users may not simply replace a code digit by a keyword and expect the unit element to be intelligible to TRANSPORT. An element must be either completely in keyword notation, or completely in positional notation.

The original TRANSPORT units and their meanings are:

Code Digit	Mnemonic	Quantity	Original TRANSPORT Unit
1.0	`X`	horizontal and vertical transverse dimensions, magnet apertures, and misalignment displacements	cm
2.0	`XP`	horizontal and vertical angles and misalignment rotation angles	mr
3.0		vertical beam extent (only) * and bending magnet gap height	cm
4.0		vertical beam divergence (only) *	mr
5.0	`DL`	pulsed beam length and wave length in accelerator	cm
6.0	`DP`	momentum spread	percent
7.0	`ANGLE`	bend, pole face rotation, and coordinate layout angles	degrees
8.0	`L`	length (longitudinal) of elements, layout coordinates, and bending magnet pole face curvatures	meters
9.0	`B`	magnetic fields	kG
10.0	`MASS`	mass	electron mass

11.0	`P`	momentum and	GeV/c
		energy gain in accelerator section	GeV
12.0	`PHASE`	Betatron phase shift	degrees
13.0	`ROLL`	Rotations about the reference trajectory	degrees
14.0	`V`	Electric fields and voltages	MV

∗ These codes should not be used if the coordinate rotation (`SROT`) element is used anywhere in the system.

Special Parameters – Type Code 16

Keywords and Code Digits

The specification of the quadratic dependence of a bending magnet field might be done through a special parameter element. The element would apply to all subsequent bending magnets, until reset, and, in positional notation, would look like

```
16.   1.   .001  ;
```

The code digit (`1`) would specify which special parameter is being given a new value. The following decimal number (`.001`) would give that value.

The code digits which can be used in specifying the special input parameters are given below. The equivalent keywords and short descriptions are given also. For more complete descriptions, the user should consult the section on the keyword formulation of Special Input Parameters.

The code digits are used only in original TRANSPORT notation. A warning is that the user cannot obtain a valid element simply by replacing the code digit by the keyword. The element must be written completely either in keyword format or in positional format.

Code Digit	Symbol	Keyword	Description
1.	$\epsilon(1)$	EPS	A second-order measure of magnetic field inhomogeneity in bending magnets.
2.	κ_o	FINT0	An integral used to calculate the reference orbit displacement.
3.	(M/m)	PMASS	Mass of the particles comprising the beam.
4.	$W/2$	HWIDTH	Horizontal half-aperture of bending magnet.
5.	$g/2$	HGAP	Vertical half-aperture of bending magnet.
6.	L	LENGTH	Cumulative length of system.
7.	κ_1	FINT	An integral related to the extent of the fringing field of a bending magnet.
8.	κ_2	FINT2	A second integral related to the extent of the fringing field.
11.	p_0	P0	The momentum of the central trajectory.
12.	$1/R_1$	H1	Where R_1 is the radius of curvature of the entrance face of bending magnets.
13.	$1/R_2$	H2	Where R_2 is the radius of curvature of the exit face of bending magnets.
14.		RANNO	The input value of the random number used for error calculation.
15.	α	FOTILT	The tilt of the focal plane.

16.	x_0	XBEGIN	Initial floor x value of the reference trajectory.
17.	y_0	YBEGIN	Initial floor y value of the reference trajectory.
18.	z_0	ZBEGIN	Initial floor z value of the reference trajecotyr.
19.	θ_0	YAW	Initial horizontal angle of the reference trajectory.
20.	ϕ_0	PITCH	Initial vertical angle of the reference trajectory.
21.	p_{ref}	PREF	The reference momentum (normally GeV/c) to which the magnetic fields are normalized.
22.	r_s	RMPS	The fractional excess bend field on a bending magnet.
23.	r_a	RNMS	Overall scaling factor for the midplane-symmetry-violating field components of a bending magnet.
24.	v_r	VR	The vertical (midplane-symmetry-violating) bend field of a bending magnet.
25.	n'	NP	The midplane-symmetry-violating normalized gradient of a bending magnet.
26.	ϵ'	EPSP	The midplane-symmetry-violating sextupole component of a bending magnet.
27.	ϵ_3	EPS3	Cubic term for the field of a bending magnet. The quantity ϵ_3 is equal to γh^3. The units are the inverse cubed of the horizontal beam width unit.

Defined Section – Type Code 24.

In positional notation, the second parameter of the Defined Section element is a numerical code. It indicates whether the section is being defined or used. The beginning of a section to be identified can be preceded by the element

```
24.   1.   'FRED'  ;
```

and followed by the element

```
24.   1.   'FRED'  ;
```

To repeat the same section later in the beam line, one inserts the element

```
24.   3.   'FRED'  ;
```

To insert the section, but in the reverse direction, use the element

```
24.   4.   'FRED'  ;
```

For more information, see page 209.

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Ray-Tracing Programs to Supplement TRANSPORT:

D. C. Carey, "TURTLE (Trace Unlimited Rays Through Lumped Elements," Fermilab Report No. NAL-64 (1971). This is a computer program using TRANSPORT notation and designed to be run using the same data cards as for a previous TRANSPORT run.

D. C. Carey, K. L. Brown and Ch. Iselin, "DECAY TURTLE (Trace Unlimited Rays Through Lumped Elements)," CERN Report 74-2 (1974). This is an extension of TURTLE to include particle decay calculations.

H. Enge and S. Kowalski have developed a Ray-Tracing program using essentially the same terminology as TRANSPORT. Any experienced user of TRANSPORT should find it easy to adapt to the M.I.T. program.

M. Berz, "COSY INFINITY," Michigan State University National Superconducting Cyclotron Laboratory Report MSUCL-771 (1991). This program uses differential algebraic methods allowing a systematic calculation of arbitrary order effects of arbitrary particle optical elements.

R. Servranckx, K.L. Brown, L. Schachinger, and D. Douglas, "Users Guide to the Program DIMAD," SLAC Report No. 285 (1990).